MA 3205 – Set Theory – Homework for Week 5

Frank Stephan, fstephan@comp.nus.edu.sg, 6516-2759, Room S14#04-13.

Homework. The homework follows the lecture notes. Below the list of the homeworks for the tutorials from 12.09.2006 onwards. It is not mandatory to hand in homework; but it is recommended to solve the questions by yourself before going to the tutorials. Homework can be corrected on request.

Exercise 5.6. Determine the functions f_n given by the following recursive equations:

1. $f_1(0) = 0$, $f_1(S(n)) = f_1(n) + 2^n$,
2. $f_2(0) = 1$, $f_2(1) = 0$, $f_2(S(S(n))) = f_2(n) \cdot \frac{4 \cdot S(n)}{S(S(n))}$,
3. $f_3(n) = 1$ for $n = 0, 1, \ldots, 9$, $f_3(10n + m) = f_3(n) + 1$ for $n = 1, 2, \ldots$ and $m = 0, 1, \ldots, 9$,
4. $f_4(0) = 0$, $f_4(1) = 0$, $f_4(2) = 0$, $f_4(3) = 1$, $f_4(S(n)) = f_4(n) + \frac{1}{2}(n^2 - n)$ for $n > 2$,
5. $f_5(n) = 1$, $f_5(S(n)) = 256 \cdot f_5(n)$.

Give informal explanations what these functions compute, for example, consider f_6 given by $f_6(0) = 0$, $f_6(1) = 0$ and $f_6(S(n)) = f_6(n) + 2n$ for $n \geq 1$. Then $f_6(n) = n(n-1)$. One explanation would be to assume that there is a soccer league with n teams. Then there are $f_6(n)$ games per season, each pair $\{A, B\}$ of two different teams plays once at A's place and once at B's place.

Exercise 5.9. Let $H : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function and $h_m : \mathbb{N} \to \mathbb{N}$ be given by $h_m(n) = H(m, n)$ for all n. Show that there is a function f dominating every h_m.

Exercise 6.7. Prove by giving a one-to-one function that the set {Auckland, Christchurch, Dunedin, Wellington} of New Zealand's largest towns has a cardinality which is less than the set {Adelaide, Brisbane, Canberra, Melbourne, Perth, Sydney} of Australian towns. Furthermore, prove that it is not less or equal than the cardinality of the set {Singapore}.

Exercise 6.11. Show that if $|X| = |X \times \mathbb{N}|$ then $|\{0,1\}^X| = |\mathbb{N}^X|$.