MA 3205 – Set Theory – Homework for Week 8

Frank Stephan, fstephan@comp.nus.edu.sg, 6516-2759, Room S14#04-13.

Homework. The homework follows the lecture notes. Below the list of the homeworks for the tutorials from 10.10.2006 onwards. It is not mandatory to hand in homework; but it is recommended to solve the questions by yourself before going to the tutorials. Homework can be corrected on request.

Exercise 10.6. Let $(A,<)$ be a linearly ordered set and $B = A^{\mathbb{N}}$. Define

$$f <_{lex} g \Leftrightarrow \exists k \in \mathbb{N}\,(f\restriction k = g\restriction k \wedge f(k) < g(k)).$$

Here $f\restriction k$ is the restriction of f to k: $f\restriction k = \{(x, f(x) \mid x \in k\}$.

Furthermore, let $C = A^*$. The lexicographic ordering on A^* is defined such that either the smaller word is shorter than the longer one or that the first word has a member of A strictly before the second one at the first position where they differ. That is, if m is the domain of f and n the domain of g, then

$$f <_{lex} g \Leftrightarrow \exists k \in S(m) \cap n\,((f\restriction k = g\restriction k) \wedge (k = m \vee (k < m \wedge f(k) < g(k)))).$$

Show that $(B, <_{lex})$ and $(C, <_{lex})$ are linearly ordered sets. Assuming that $A = \{0, 1, 2, \ldots, 9\}$ with the usual ordering, put the following elements of C into lexicographic order: 120, 88, 512, 500, 5, 121, 900, 0, 76543210, 15, 7, 007, 00.

Exercise 10.10. Determine which of the following subsets of the real numbers $\mathbb{R}$ have a lower and upper bound. If so, determine the infimum and supremum and check whether these are even the least and greatest element of these sets.

1. $A = \{a \in \mathbb{R} \mid \exists b \in \mathbb{R}\,(a^2 + b^2 = 1)\}$;
2. $B = \{b \in \mathbb{R} \mid b^3 - 4 \cdot b < 0\}$;
3. $C = \{c \in \mathbb{R} \mid \sin(c) > 0\}$;
4. $D = \{d \in \mathbb{R} \mid d^2 < \pi^3\}$;
5. $E = \{e \in \mathbb{R} \mid \sin(\frac{\pi}{2} \cdot e) = \frac{e}{101}\}$.

Exercise 10.22. Show that in a complete ordered set $(A,<)$ every nonempty subset which is bounded from below has an infimum in A.