MA 3205 – Set Theory – Homework for Week 10

Frank Stephan, fstephan@comp.nus.edu.sg, 6516-2759, Room S14#04-13.

Homework. The homework follows the lecture notes. Below the list of the homeworks for the tutorials from 24.10.2006 onwards. It is not mandatory to hand in homework; but it is recommended to solve the questions by yourself before going to the tutorials. Homework can be corrected on request.

Exercise 13.6. Let A be some set and let $a_0 a_1 \ldots a_{n-1} \, R \, b_0 b_1 \ldots b_{m-1} \Leftrightarrow n < m$ and there is a function $f : n \to m$ such that $b_{f(i)} = a_i$ and $(i < j \Rightarrow f(i) < f(j))$ for all $i, j \in n$ where $a_0 a_1 \ldots a_{n-1}, b_0 b_1 \ldots b_{m-1} \in A^*$. Show that R is well-founded.

Let R be such that $x \, R \, y$ iff there is a z with $x \in z \wedge z \in y$. Show that R is well-founded.

Let $(x, y) \, R \, (v, w)$ iff either $x = v \wedge y \in w$ or $y = w \wedge v \in x$. Is R well-founded?

Is the relation R given as $x \, R \, y \Leftrightarrow x \cap y = x \cup y$ well-founded?

Exercise 13.12. Construct by transfinite recursion a function on ordinals which tells whether an ordinal is even or odd. More formally, construct a function F such that $F(\alpha) = 0$ if α is even, $F(\alpha) = 1$ if α is odd. Limit ordinals should always be even; the successor of an even ordinal is odd and the successor of an odd ordinal is even.

Exercise 13.13. Is it possible to define a function F on all sets such that $F(X) = n$ iff n is the maximal number such that there are $Y_0, Y_1, \ldots, Y_n$ with $Y_{m+1} = S(Y_m)$ for all $m \in n$ and $X = Y_n$? If so, construct the corresponding function F by transfinite recursion.