Exercise 7.4. Let $X$ be finite. Prove that the set of all functions from $X$ to $X$ is finite.

Exercise 7.10. Prove that $V_\omega$ satisfies the following property: if $x \in V_\omega$ and $y \subseteq x$ or $y \in x$, then $y \in V_\omega$. Show that $\mathbb{N}$ does not satisfy this property, but that some proper infinite subset of $V_\omega$ does.

Exercise 7.11. Determine all $x_0 \in V$ which satisfy that there are no $x_1, x_2, x_3, x_4 \in V$ with $x_1 \in x_0, x_2 \in x_1, x_3 \in x_2, x_4 \in x_3$. The set $\{\emptyset\}$ is such an $x_0$, although $x_1 = \{\emptyset\}$ and $x_2 = \emptyset$ exist, $x_3$ and $x_4$ do not exist. The set $\{\emptyset, \{\emptyset\}\}$ does not qualify.

Exercise 8.9*. Let $D = \{f : \mathbb{N} \to \mathbb{N} | \forall n (f(S(n)) \leq f(n))\}$ be the set of all decreasing functions. Show that $D$ is countable.

Exercise 8.12. Let $\mathbb{A}$ be the set of algebraic real numbers, that is, the set of all $r \in \mathbb{R}$ for which there are $n \in \mathbb{N}$ and $z_0, z_1, \ldots, z_n \in \mathbb{Z}$ such that $z_n \neq 0$ and $z_0 + z_1r + z_2r^2 + \ldots + z_nr^n = 0$. Note that such a polynomial of degree $n$ can have up to $n$ places $r$ which are mapped to 0. Show that $\mathbb{A}$ is countable by giving a one-to-one mapping from $A$ into $\mathbb{N}$.

Exercise 8.14. Call a set $A$ hereditarily at most countable iff for every $B \in \mathcal{T}(A)$ it holds that $B$ is at most countable. For example, $\mathbb{N}$ and $V_\omega$ are hereditarily at most countable. Now assume that $X, Y$ are hereditarily at most countable. Show that the following sets are hereditarily at most countable as well: $X \cup Y, X \cap Y, X - Y$ and $\{X\}$. Furthermore, show that whenever $\mathcal{T}(Z)$ is at most countable then $Z$ is hereditarily at most countable.