MA 3205 – Set Theory – Homework due Week 11

Frank Stephan, fstephan@comp.nus.edu.sg, 6516-2759, Room S14#04-13.

Homework. The homework follows the lecture notes. You have to hand in at least three starred homeworks throughout the semester. Further homework can be checked on request. Homework to be marked should be handed in after the lecture on Tuesday of the week when the homework is due.

Exercise 14.4. For any ordinal α, consider the successor function S restricted to α, that is, consider the set

$$S\!\restriction\alpha = \{\{\beta, \{\beta, S(\beta)\}\} \mid \beta \in \alpha\}.$$

Determine $\rho(S\!\restriction\alpha)$ for $\alpha = 42, 1905, 2004, \omega, \omega+1, \omega+131501, \omega^2+\omega\cdot 2+1, \omega^{17}+\omega^4$.

Exercise 14.6. V_ω has been defined twice. Let A be the version of V_ω as defined in Definition 7.5, that is, let A consist of all hereditarily finite sets. Let $B = \bigcup\{V_n \mid n < \omega\} = \{x \in V \mid \rho(x) < \omega\}$ be the version defined here. Show that both definitions coincide, that is, show $A \subseteq B \wedge B \subseteq A$.

Show that B contains $\emptyset$, is closed under unions of two sets and is closed under the operation forming $\{v\}$ from v. Thus, by Theorem 7.9, $A \subseteq B$.

Show by induction that all members of V_n with $n < \omega$ are hereditarily finite. Thus $B \subseteq A$.

Exercise 15.6*. Which of the following statements are true and which are false?

1. The addition $\oplus$ is commutative.
2. There are ordinals α, β such that $\alpha+\beta$ and $\beta+\alpha$ both differ from $\alpha \oplus \beta$.
3. There are ordinals α, β such that $\alpha+\beta > \alpha \oplus \beta$.
4. There are ordinals α, β such that $\alpha < \beta$ and $\alpha \oplus \gamma \neq \beta$ for all ordinals γ.
5. There are ordinals α, β such that $\alpha < \beta$ and $\alpha + \gamma \neq \beta$ for all ordinals γ.