MA 5219 - Logic and Foundations of Mathematics 1
Homework due in Week 4, Tuesday.

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Office hours Thursday 14.00-15.00h

Hand in each starred homework; 1 mark per homework (if it is correct), up to 10
marks in total for homework.

4.1* Proof systems. Consider the following rules.

\[
\begin{align*}
\emptyset & \vdash A \\
X \vdash A & \cup Y \vdash A & X, A \vdash B \\
X \vdash A & \rightarrow B & X \vdash A \rightarrow B \\
X \vdash A & \rightarrow \bot & X \vdash \neg A \\
X \vdash \neg (A \rightarrow \neg B) & & X \vdash A, B \rightarrow \neg B \\
X \vdash A & \rightarrow \neg A & X \vdash \neg A \\
X \vdash \neg B & \rightarrow A & X, A \vdash B \\
X \vdash \neg B & \rightarrow A & X, A \vdash B \text{ and } X, \neg A \vdash B \
\end{align*}
\]

Derive the following rules from the above rules.

\[
\begin{align*}
X \vdash A & \rightarrow B, \neg A \rightarrow B \\
X \vdash A & \rightarrow B, B \rightarrow C \\
X \vdash B & \rightarrow A \rightarrow C \\
\emptyset & \vdash A \rightarrow B \\
X \vdash A & \rightarrow B \\
X \vdash (A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C \
\end{align*}
\]

4.2 Models. Consider a set $A$ with an operation $\circ$ and let the lower case letters be
variables in the models:

- $\forall x, y, z[(x \circ y) \circ z = x \circ (y \circ z)]$ and $\exists a, b \forall x, y[a \circ x = a \land y \circ b = b]$;
- $\forall x, y, z[(x \circ y) \circ z = x \circ (y \circ z)]$ and $\exists a \forall x, y[e \circ x = e \land y \circ e = e]$.

Is there a model $(A, \circ)$ which satisfies one set of axioms but not the other one?