**12.1 First-Order Logic.** Which of the following sets of formulas in first-order logic are recursive (= decidable), recursively enumerable but not recursive or even not recursively enumerable? The underlying logical language is that of first-order logic with two operations $+, \cdot$ permitted on the members of the model and three constants $0, 1, 2$. Furthermore, the logical language uses equality.

(a) $A = \{ \phi : \phi$ is a tautology, that is, $\phi$ is true in all models$\}$;
(b) $B = \{ \phi : \phi$ is not true in any model$\}$;
(c) $C = \{ \phi : \phi$ is true in some but not all models$\}$.

Note that formally, one has to say that for a set $D$ of formulas, $D$ is recursive iff \{gn$(\phi) : \phi \in D$\} is recursive where gn$(\phi)$ is the Gödel number of $\phi$ in some numbering system (where the numbers need not to coincide with the members of models of $\phi$ but are just members of $\mathbb{N}$). Similarly one defines when a set $D$ of formulas is recursively enumerable.

**12.2 Isomorphy and equivalence of models.** Assume that a logic language only consists of formulas using equality, constants, variables, Boolean combinations of equalities and quantified open formulas of such Boolean combinations. Furthermore, let $X$ be the set of all $c_i \neq c_j, c_i \neq x_j$ and $x_i \neq x_j$ for all pairwise distinct $i, j$. Which of the following statements are true:

(a) Any two models of $X$ are isomorphic;
(b) Any two models of $X$ are elementary equivalent but they might not be isomorphic;
(c) There is a formula $\alpha$ which is true in some but not all models of $X$.

**12.3 Decidability.** Given the set $X$ of formulas from 12.2, can one decide whether $X \models \alpha$ for any formula $\alpha$? If one cannot decide that set, can one enumerate the set of all formulas $\alpha$ which satisfy $X \models \alpha$?

**12.4 Primitive Recursive Functions.** Consider the function $f(x) = 2^{2^x}$ and $g$ given by $g(x, y) = 1$ if $2^{f(x)}>y$ and $g(x, y) = 0$ if $2^{f(x)} \leq y$. Are the functions $f$ and $g$ primitive-recursive?