11.1* Substructures. Recall that for a logical language $L$ and an $L$-structure $A$, the language $LA$ is the language of all formulas which use besides constants from $L$ also constants $c_a$ for each $a$ in the domain of $A$. The diagramme $DA$ is the set of all true $LA$-formulas in $A$ which do not contain any free or bound variable. The elementary diagramme $D_{el}A$ is the set of all true $LA$-sentences in $A$. Assume that the domain of $A$ is a subset of the domain of $B$. Now $A$ is a substructure of $B$ iff $B \models DA$ and $A$ is an elementary substructure of $B$ iff $B \models D_{el}A$.

So assume that $A$ is a 2-dimensional sub space of a given 3-dimensional vector space $B$ over the finite field $\mathbb{F}_3$ with 3 elements. Is $A$ a substructure or an elementary substructure of $B$? Note that the scalar multiplication with 0 is the function mapping all vectors to the zero vector, the scalar multiplication with 1 is the identity mapping and the scalar multiplication with 2 is the mapping $x \mapsto x + x$.

11.2* Categoricity. Assume that $L$ contains infinitely many constants $c_0, c_1, \ldots$ and that $X = \{c_i \neq c_j : i, j \in \mathbb{N} \land i \neq j\}$. Is $T$ be the theory of all sentences logically implied by $X$? Is $T$ $\aleph_0$-categorical? Is $T$ $\aleph_1$-categorical? Justify both answers.

11.3* Decidability. Let $L$ be the logical language with one unary function symbol $f$, let $\beta$ be $\forall x [f(f(x)) = x]$, let $\gamma$ be $\forall x, y [x = f(x) \land y = f(y) \rightarrow x = y]$ and let $T = \{\alpha : \alpha$ is a sentence and $\{\beta, \gamma\} \models \alpha\}$. Show that $T$ is decidable.

11.4 Groups. Make a finitely axiomatisable theory $T$ such that (a) every model of $T$ is a group, (b) $T$ has both finite and infinite models and (c) $T$ is decidable.

11.5 Boolean Basis. Let $L$ be a logical language with the extra symbols $<$ and $P$ and consider the theory $T$ of all sentences implied by the set $Y$ consisting of $\forall x \forall y [x < y \lor x = y \lor y < x]$, $\forall x [\neg x < x]$, $\forall x \forall y \forall z [x < y \land y < z \rightarrow x < z]$, $\forall x \exists y \exists z [y < x \land x < z]$, $\forall x \forall y \exists z [x < y \rightarrow x < z \land z < y]$, $\forall x \forall y [P y \land x < y \rightarrow Px]$. Determine a finite set $X$ of sentences which is a Boolean basis for $T$. That is, $X$ has to satisfy that given any two structures $\mathcal{A}$ and $\mathcal{B}$ of $T$, either $\mathcal{A}$ and $\mathcal{B}$ are elementary equivalent or there is a sentence $\alpha$ in $X$ such that exactly one of $\mathcal{A}$ and $\mathcal{B}$ makes $\alpha$ true.