MA 4207 - Mathematical Logic
Homework due in Week 12, Monday 6 April 2015
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Homework 12.1
Let $\alpha, \beta, \gamma$ be any well-formed formulas. Which of the following formulas are valid,
independent of what values for $\alpha, \beta, \gamma$ are chosen? If yes, give a formal proof, if not,
find a counter example by choosing the right values for $\alpha, \beta, \gamma$.

1. $\forall x [\alpha \rightarrow \beta] \rightarrow \forall x [\neg \beta] \rightarrow \forall x [\neg \alpha]$;
2. $\forall x \forall y [\alpha \rightarrow \beta] \rightarrow \forall x \forall y [\beta \rightarrow \gamma] \rightarrow \forall x \forall y [\gamma \rightarrow \alpha]$;
3. $\forall x [\alpha \rightarrow \beta] \rightarrow \forall x [\neg \beta \rightarrow \neg \alpha]$;
4. $\forall x [\alpha \rightarrow \beta] \rightarrow \forall x [\alpha \rightarrow \neg \beta] \rightarrow \forall x [\neg \alpha]$.

Homework 12.2
Assume that the logical language contains the predicate symbols $P$ and $Q$. Make formal proofs for the following facts. You can use the Deduction and the Generalisation Theorems and use axioms of the first group in order to deal with connectives other than $\neg$ and $\rightarrow$.

1. $\{Py\} \vdash \forall x [x = y \rightarrow Px]$;
2. $\{\forall x [x = y \rightarrow Px]\} \vdash Py$;
3. $\{\forall x [Px], \forall x [Qx]\} \vdash \forall x [Px \land Qx]$.

Homework 12.3
Is there a set $S$ of formulas such that $S$ has finite models with arbitrary many elements but no infinite model?

Homework 12.4
Is the set

$$\{\forall x \forall y \exists z [x \circ z = y], \exists x \forall y \forall z [z \circ x \neq y], \forall x \forall y \forall z [x \circ (y \circ z) = (x \circ y) \circ z]\}$$

a consistent set of formulas? In other words, is there a structure $(A, \circ)$ such that $\circ$ is associative and one can transform any given $x$ into another given $y$ by multiplication from the right with a suitable element $z$: $x \circ z = y$; however, this does not work with respect to multiplication from the left.