

MA 4207 - Mathematical Logic

Course-Webpage <http://www.comp.nus.edu.sg/~fstephan/mathlogicug.html>
Homework

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Homework 3.1

Let $f(n)$ be the maximum number of negation symbols in a well-formed formula which does not contain any subformula of the form $(\neg(\neg\alpha))$ and which contains at most n atoms. Here $(\neg(A_1 \vee (\neg(A_2 \vee (\neg A_1))))))$ has 3 atoms and n is 3, as repeated atoms are counted again. Determine the value $f(n)$ in dependence of n .

Homework 3.2

Prove by induction that a well-formed formula of length n contains less than $n/3$ connectives and at most $(n + 3)/4$ atoms.

Homework 3.3

Use the truth-table method to prove that the following formulas are equivalent:

- $((\neg A_1) \vee (\neg A_2))$;
- $(\neg(A_1 \wedge A_2))$;
- $((A_1 \vee A_2) \leftrightarrow (A_1 \oplus A_2))$.

Homework 3.4

Use the truth-table method to check whether the following statement is correct:

$$\{(A_1 \vee A_2), (A_2 \vee A_3), (A_1 \vee A_3)\} \models ((A_1 \wedge A_2) \vee A_3).$$

Homework 3.5

Use the truth-table method to check whether the following statement is correct:

$$\{(A_1 \rightarrow A_2), (A_2 \rightarrow A_3), (A_3 \rightarrow A_4)\} \models ((A_1 \rightarrow A_3) \wedge (A_2 \rightarrow A_4))$$

Homework 3.6

Use the truth-table method to check whether the following statement is correct:

$$\{(A_1 \rightarrow A_2), (A_2 \rightarrow A_3), (A_3 \rightarrow A_4)\} \models (A_4 \rightarrow A_1)$$

Homework 3.7

List out the truth-table for the formula $((A_1 \oplus A_2) \wedge (\neg A_3)) \oplus (A_1 \vee A_3)$.

Homework 3.8

List out the truth-table for the formula $((A_1 \oplus A_3) \vee (((A_1 \oplus A_2) \wedge (A_2 \oplus A_3))))$.

Homework 3.9

Consider the following formulas:

$$\begin{aligned}\phi_1 &= (((A_1 \vee A_2) \vee A_3) \wedge ((A_4 \vee A_5) \vee A_6)); \\ \phi_2 &= (((A_1 \vee A_2) \wedge (A_3 \vee A_4)) \wedge (A_5 \vee A_6)); \\ \phi_3 &= (((((A_1 \oplus A_2) \oplus A_3) \oplus A_4) \oplus A_5) \oplus A_6).\end{aligned}$$

There are $2^6 = 64$ ways to assign the truth-values to the sentence symbols (or atoms) A_1, \dots, A_6 . Determine for each of the formulas ϕ_1, ϕ_2, ϕ_3 , how many of these assignments make the formula true and how many of these assignments make the formula false.

Homework 3.10

For the formulas from Homework 3.9, is the statement

$$\{\phi_1, \phi_2, \phi_3\} \models (((((A_1 \wedge A_2) \wedge A_3) \wedge A_4) \wedge A_5) \wedge A_6)$$

true or false? Prove your answer.

Homework 3.11

For the formulas from Homework 3.9, is the statement

$$\{\phi_1, \phi_2, \phi_3\} \models (((((A_1 \vee A_2) \vee A_3) \vee A_4) \vee A_5) \vee A_6)$$

true or false? Prove your answer.

Homework 3.12

Using the connectives $\vee, \wedge, \rightarrow, \leftrightarrow, \oplus, \neg$, construct a formula using atoms A_1, A_2, A_3, A_4 which says that at least two and at most three of these atoms are true.

Homework 3.13

Using the connectives $\vee, \wedge, \rightarrow$, construct a formula using atoms $A_1, A_2, A_3, A_4, A_5, A_6$ which says that either all six atoms are false or all six atoms are true.

Homework 3.14

Use the truth-table method to prove the associativity of \leftrightarrow , that is, prove that $(A_1 \leftrightarrow (A_2 \leftrightarrow A_3))$ and $((A_1 \leftrightarrow A_2) \leftrightarrow A_3)$ are the same. Furthermore, check whether there is a truth-assignment ν with $\bar{\nu}((A_1 \leftrightarrow (A_2 \leftrightarrow A_3))) = 1$ and $\nu(A_1) \neq \nu(A_2)$.

Homework 3.15

Use the truth-table method to check whether $(A_1 \oplus (A_2 \oplus A_3))$ and $(A_1 \leftrightarrow (A_2 \leftrightarrow A_3))$ are equivalent.

Homework 3.16

Make the truth-tables of \wedge, \oplus and \neg for $\{0, u, 1\}$ -valued logic where the value u stands for an unknown value of 0 and 1 and where the output u is taken iff one cannot derive from the inputs what the output is. Note that two inputs u need not to represent the same of 0 and 1.

Homework 3.17

Make the truth-tables of $\rightarrow, \leftrightarrow$ and \vee for the $\{0, u, 1\}$ -valued logic from 3.16.