GENERALISATIONS OF A RESULT BY GUL'KO ON SPACES OF CONTINUOUS FUNCTIONS

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Abstract. For a Tychonov space X we define $C_p(X)$ to be the linear subspace of $\mathbb{R}^X$ consisting of all real-valued continuous functions on X. Let βX be the Čech-Stone compactifaction of X and let $X^* = \beta X \setminus X$ be the remainder of X. For $u \in X^*$, we denote $X_u = X \cup \{u\} \subseteq \beta X$.

For the countable discrete space ω, elements of ω^* can be identified with free ultrafilters on ω. In 1990 Gul'ko proved in [3] for $u, v \in \omega^*$ that $C_p(\omega_u)$ and $C_p(\omega_u)$ are linearly homeomorphic if and only if ω_u and ω_v are homeomorphic. In [1] this result was generalized for finite sums of spaces ω_u as follows: For $n, m \geq 1$ and $\{u_1, \ldots, u_n, v_1, \ldots, v_m\} \subseteq \omega^*$, let $X = \bigoplus_{i=1}^n \omega_{u_i}$ and $Y = \bigoplus_{i=1}^m \omega_{v_i}$. Then $C_p(X)$ and $C_p(Y)$ are linearly homeomorphic if and only X and Y are homeomorphic, in particular $n = m$.

Gul'ko's result does not hold for all spaces X. For example in [2] it was shown that for each ordinal space α, where $\omega < \alpha < \omega_1$ is a limit ordinal, there are $u, v \in \alpha^*$ such that $C_p(\alpha_u)$ and $C_p(\alpha_v)$ are linearly homeomorphic but α_u and α_v are not homeomorphic.

References

[1] J. Baars, *On the l_p-equivalence of ultrafilters*, Bull. Pol. Acad. Sci. Math. Vol 70 No 1. (2022), 63–82.
[2] J. Baars and J. van Mill, *Function spaces and points in Čech-Stone remainders*, To appear in Topology Appl.
[3] S.P. Gul'ko, *Spaces of continuous functions on ordinals and ultrafilters*, Math. Notes, Vol. 47,4 (1990), 329–334.