Mathematisches Forschungsinstitut Oberwolfach

T a g u n g s b e r i c h t 5/1996

Rekursionstheorie

28.01.-03.02.1996

Die Tagung fand unter der Leitung von Klaus Ambos-Spies (Heidelberg), Gerald E. Sacks (Cambridge) und Robert I. Soare (Chicago) statt. Unter anderen wurden folgende Themen behandelt: der Halbverband der aufzählbaren Turing Grade sowie der n-r.a. Grade, die Struktur aller Turing Grade, der Verband der aufzählbaren Mengen und seine Automorphismen, höhere Rekursionstheorie, rekursive Modelltheorie, berechenbare algebraische Strukturen, Π^0_1 Klassen sowie Beziehungen der Berechenbarkeitstheorie zu verwandten Gebieten (z.B. Beweis- und Lerntheorie). Die interessanten Beiträge wurden lebhaft diskutiert. Außerdem gedachte die Tagung der Wissenschaftler Stephen C. Kleene (1909-1994) und Robin O. Gandy (1919-1995) in zwei besonderen Veranstaltungen.

Vortragsauszüge

Marat Arslanov (Kazan)

Degree structures and their elementary theories

We consider (Turing) degrees of unsolvability containing finite Boolean combinations of recursively enumerable (r.e.) sets. Let D_n denote the set of n-r.e. degrees (degrees of Boolean combinations of n r.e. sets) partial ordered by the relation $\leq_T$ of Turing reducibility. We prove the following main theorem.

Theorem If the predicates "$\mathbf{x}$ is r.e." and "$\mathbf{x}$ is r.e. in $\mathbf{y}$" are definable in the degree structures $\{\mathbf{D}_n, \leq_T\}$ and $\{\mathbf{D}_m, \leq_T\}$, $2 \leq n, m,\ n \neq m$, then $\mathbf{D}_n$ and $\mathbf{D}_m$ are not elementarily equivalent.

Douglas Cenzer (Gainesville)

Index sets for Π^0_1 Classes

A Π^0_1 class is an effectively closed set of reals. We study properties of these classes determined by cardinality, measure and category as well as by the complexity of the members of a class P. Given an effective enumeration $\{P_e : e < \omega\}$ of the Π^0_1 classes, the index set I for a certain property (such as having positive measure) is the set of indices e such that P_e has the property. For example, the index set of binary Π^0_1 classes of positive measure is Σ^0_2 complete. Various notions of boundedness (including a new notion of "almost bounded" classes) are discussed and classified. For example, the index set of the recursively bounded classes is Σ^0_3 complete and the index set of the recursively bounded classes which have infinitely many recursive members is Π^0_4 complete. Consideration of the Cantor-Bendixson derivative leads to index sets in the transfinite levels of the hyperarithmetic hierarchy.

(*Joint work with Jeffrey Remmel*)

Peter Cholak (Notre Dame)

Automorphisms of computably enumerable sets

We discuss the computably enumerable sets with the following conjecture in mind:

Conjecture (Slaman and Woodin)
The set $\{\langle e, i\rangle : W_e \simeq W_i\}$ is Σ^1_1-complete.

In addition we discuss our recent result:

Theorem (Cholak and Downey)
There are automorphic computably enumerable sets which are not Δ_3-automorphic.

(*Joint work with Rod Downey*)

Chi Tat Chong (Singapore)

Π_2-Blocking and the jump operator

Let P^- denote the Peano axioms without induction. Let $I\Sigma_n$ denote the Σ_n-induction scheme, and $B\Sigma_n$ the Σ_n-bounding scheme. We show that in the absence of $I\Sigma_2$, any model of $P^- + B\Sigma_2$ contains only three degrees: $\mathbf{0}'$, $\mathbf{0}^{1.5}$ and $\mathbf{0}''$ to be the jump of r.e. degrees. Furthermore, using the technique of Π_2 blocking, we identify those models in which every incomplete r.e. degree is low. There is a link between the work reported here and that of Maass on the existence of high α-r.e. sets in higher recursion theory.

(*Joint work with Yue Yang*)

Decheng Ding (Nanjing)

Some properties of d-r.e. degrees

The talk presented recent results on the d-r.e. and n-r.e. degrees and focussed on the embeddability problem of finite lattices into the d-r.e. and n-r.e. degrees.

Rod Downey (Wellington)

The contiguous degrees are definable

It is proven that $\mathbf{a}$ is contiguous iff $\mathbf{a}$ is locally distributive in $\mathbf{R}$, i.e., iff

$$(\forall \mathbf{a}_1, \mathbf{a}_2, \mathbf{b}) \quad [\mathbf{a}_1 \cup \mathbf{a}_2 = \mathbf{a} \ \wedge \ \mathbf{b} \leq \mathbf{a} \ \Rightarrow \\ (\exists \mathbf{b}_1, \mathbf{b}_2)\,[\mathbf{b}_1 \leq \mathbf{a}_1 \ \wedge \ \mathbf{b}_2 \leq \mathbf{a}_2 \ \wedge \ \mathbf{b}_1 \cup \mathbf{b}_2 = \mathbf{b}]\,]$$

(*Joint work with Steffen Lempp*)

Sergei S. Goncharov (Novosibirsk)

Recursive and decidable models

I would like to discuss the problem on existence of recursive and decidable models. First of all there is Morley problem on decidability for countable models of Ehrenfeucht theories with recursive types. I will give a counterexample to this hypothesis and would like to present some related problems.

In the second part of my talk I would like to present some new results about autostability and Turing spectrum of relations in recursive models. These results were proved by me together with R. Shore, A. Nerode, B. Khussainov and P. Cholak.

Edward R. Griffor (Uppsala)

Proof Theoretic Large Cardinals

Recursion theoretic hierarchies have been given for the first recursively Mahlo ordinal all of which are non-positive seen as inductive definitions. We present an extension of Martin-Loef type theory which has the proof theoretic strength of KPM (Kripke-Platek set theory with a Mahlo axiom). The definition is positive and involves an inductive generation of universe operators or quantifiers.

As a consequence we have a constructive explanation of the ordinal notation system used in M. Rathjen's ordinal analysis of KPM.

(*Joint work with M. Rathjen*)

Marcia Groszek (Hanover)

Π^1_0 classes and minimal degrees

We construct a recursive infinite binary tree T such that every path through T Turing computes a real of minimal degree. This shows that

$$\mathrm{WKL} \vdash \text{“There is a minimal degree”},$$

answering a question of Harney Friedman and Stephen Simpson.

Valentina Harizanov (Washington)

Nowhere simple sets

R. Shore proved that every computably enumerable (c.e.) set can be split into two (disjoint) nowhere simple sets. Nowhere simple sets were further studied by D. Miller and J. Remmel, and we generalize some of their results. We characterize c.e. sets which can be split into two (non)effectively nowhere simple sets, and c.e. sets which can be split into two c.e. non-nowhere simple sets. We show that every c.e. set is either the disjoint union of two effectively nowhere simple sets or two noneffectively nowhere simple sets. We characterize c.e. sets whose every nontrivial splitting is into nowhere simple sets, and c.e. sets whose every nontrivial splitting is into effectively nowhere simple sets.

R. Shore proved that for every effectively nowhere simple set A, the lattice $\mathcal{L}^*(A)$ is effectively isomorphic to ε^*, and that there is a nowhere simple set A such that $\mathcal{L}^*(A)$ is not effectively isomorphic to ε^*. We prove that every nonzero c.e. Turing degree contains a noneffectively nowhere simple set A with the lattice $\mathcal{L}^*(A)$ effectively isomorphic to ε^*.

Eberhard Herrmann (Berlin)

Finite endintervals in factor structures of the lattice of enumerable sets

Equivalence relations on r.e. sets like $=^*$ induce a factor structure. Finite endintervals in the factor structure of $=^*$ are always finite Boolean algebras and are generated by intersections of maximal sets. Similar results hold for many natural equivalence relations but it is also possible to find an equivalence relation which has finite endintervals different from any finite Boolean algebra.

Peter Hinman (Ann Arbor)

Iterated relative recursive enumerability

The n-r.e. and n-REA degrees form two intertwined hierarchies over the r.e. degrees. We study the combined hierarchies over the r.e. degrees defined by: A is $(m_0, \ldots, m_{k-1}, n)$-REA iff A is n-r.e.$[B]$ for some B which is $(m_0, \ldots, m_{k-1})$-REA and $B \leq_T A$. A sample result is that there exist $(1, 2)$-REA degrees which are not $(n, 1)$-REA for any n. We discuss many open questions.

Julia Knight (Notre Dame)

Results (and questions) related to the Ash-Nerode theorem

Let $\mathcal{A}$ be a recursive structure, and let R be a relation on $\mathcal{A}$. Consider isomorphisms f from $\mathcal{A}$ onto recursive structures. Ash and Nerode gave conditions under which $f(R)$ can be made not recursive. Harizanov showed that under these conditions, $f(R)$ can be given arbitrary r.e. degree. Barker gave conditions under which $f(R)$ can be made not Δ^0_α. Under these conditions, $f(R)$ can be made equivalent modulo a complete Δ^0_α set to an arbitrary Σ^0_α set C (more general sets D can be substituted for the complete Δ^0_α set). The setting can be varied, considering a pair of relations R and S, or a family of relations $(R_\beta)_{\beta<\alpha}$. It is possible to make $f(R) \equiv_T C$ (where C is r.e.) by coding C into $f(R)$ and making $f \leq_T C$, but making $f(R) \equiv_T C$ and $f(S) \equiv_T C'$ (where C, C' are r.e. of different degree) requires something different. Shlapentokh has some intriguing results on pairs in specific algebraic settings. Coding a family of sets $(C_\beta)_{\beta<\alpha}$ (each REA relative to those below), raises some questions of technology.

Mashiro Kumabe (Chiba City)

A fixed point free minimal degree

Let Φ_n be n-th reduction procedure for some fixed recursive enumeration of all such operators. A function f is fixed point free if for each n, $\Phi_n \neq \Phi_{f(n)}$. We show that there is a fixed point free minimal degree.

Martin Kummer (Karlsruhe)

On the complexity of random strings

We show that the set R of Kolmogorov random strings is truth-table complete. This improves the previously known Turing completeness of R and shows how the halting problem can be encoded into the distribution of random strings rather than using the time complexity of non-random strings. As an application we obtain that Post's simple set is truth-table complete in every Kolmogorov numbering. We also show that the truth-table completeness of R cannot be generalized to size-complexity with respect to arbitrary acceptable numberings. In addition we note that R is not frequency computable.

Steffen Lempp (Madison)

How on Earth did we cook up L_{20}?

We exhibit a finite lattice L_{20} – as drawn in Figure 1 – without critical triple that cannot be embedded into the enumerable Turing degrees. Our method promises to lead to a full characterization of the finite lattices embeddable into the enumerable Turing degrees, and ultimately to a decision procedure for the two-quantifier theory of this degree structure.

(*Joint work with Manuel Lerman*)

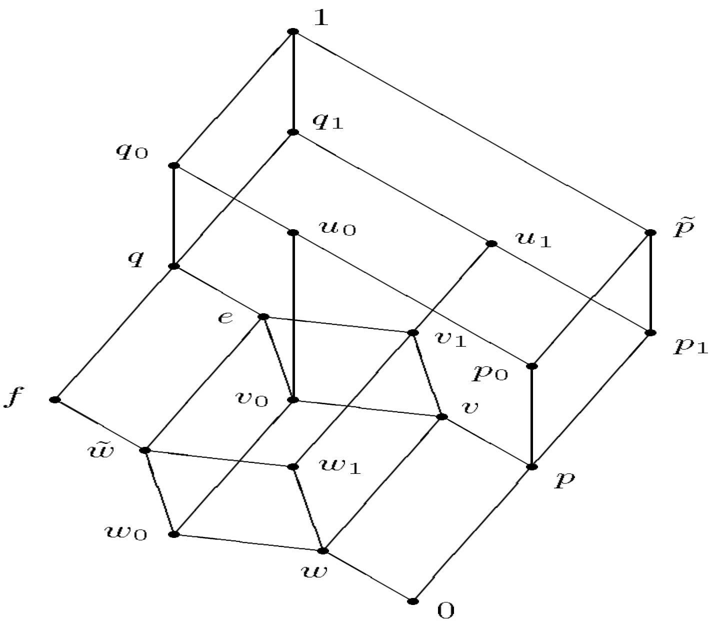

Figure 1: Lattice L_{20}

Manuel Lerman (Storrs)

The embedding problem for the enumerable degrees

We describe recent progress towards the characterization of the finite partial lattices which can be embedded into the enumerable degrees. We present a finite lattice with no critical triples which cannot be embedded and discuss the extension of our proof to finite ranked lattices. We also describe the remaining obstructions towards obtaining a (non-effective) necessary and sufficient condition for embeddability and to effectivizing this condition.

(*Joint work with Steffen Lempp*)

ANDREI S. MOROZOV (Novosibirsk)

Groups of computable automorphisms of recursive structures

This is a brief survey on results on computable automorphisms of recursive structures. It contains general results on computable permutations, recursive automorphism groups of recursive models, most of which were obtained partially by the author in last ten years.

YIANNIS N. MOSCHOVAKIS (Los Angeles)

Concurrent recursion

I discussed the problem which arises when we try to solve fixed point equations of the form

$$(1) \qquad p(n) = f(n, p), \quad (p : M \to W)$$

when $W = \Pi(D) = Power(D)\backslash\{\varphi\}$, for a domain D. These come up in the attempt to model faithfully programming languages which allow for full (functional) recursion and also have non-deterministic and concurrent constructs, including fair merge. A solution was proposed for the simpler case $M = \Phi$ (parameterless procedures, no value-passing) when (1) reduces to

$$(2) \qquad x = f(x), \quad (x \in \Pi(D))$$

[see Information and Computation '91], and the relevant notion of recursion was justified by showing that (in the proper formation) it satisfies the same identities as least fixed point recursion. In this work, the construction is extended to the more complex (1), and the same justification is given, bared on a complete and decidable axiomatization of functional recursion. [See Proceedings of ICLMPS in Florence, 1995]

ANDRÉ NIES (Chicago)

Definability in the enumerable degrees I

We show that any relation on the recursively enumerable degrees, which is invariant under the second jump and for which the corresponding relation on the indices is arithmetical, is first-order definable in the partial ordering of the r.e. degrees. In particular, for any given r.e. degree $\mathbf{a}$, the classes $\{\mathbf{b} : \mathbf{b}'' \geq \mathbf{a}''\}$

and $\{\mathbf{b} : \mathbf{b}'' = \mathbf{a}''\}$ are definable. It follows that the low_n and high_n degrees are definable for $n \geq 2$. Moreover, the definability of the high_1 degrees is shown too.

(*Joint work with Richard Shore and Theodore Slaman*)

Dag Norman (Oslo)

Categories of domains with totality; connections to higher recursion theory

Higher recursion theory provides tools for investigating structures of the form $L_\kappa(HC)$ for certain ordinals κ.

We sharpen those tools via representation theorems for higher recursion theories using domains with totality and via implementing ideas of Π^1_2-logic for every domain with totality.

At the same time we show that there are connections between higher recursion theory and quite different branches of logic.

Wolfram Pohlers (Münster)

Proof theory vs. recursion theory

We give an outline of methods and results in proof theory and indicate how these are linked to generalized recursion theory.

Gerald E. Sacks (Cambridge)

Forcing and type-omitting in recursion theory

A technique for simultaneously applying forcing to local requirements and type-omitting to global requirements is used to obtain further results on the minimal upper bound problem for hyperdegrees.

Helmut Schwichtenberg (München)

Finite notations for infinite terms

Kleene has observed that for his system $\mathcal{O}$ of ordinal notations some standard functions like addition are in fact primitive recursive; this is a consequence of the Kleene recursion theorem for primitive recursive functions. The author has adapted this method to a notation system for infinitary terms, and used it to prove a trade-off result, which essentially says that any detour through higher type levels in the definition of a $<\varepsilon_0$-recursive functional can be eliminated at the price of rising the order types of the recursive well-orderings used by some powers of omega.

Here we present a different proof of this result, by adapting a method of Buchholz. The basic idea is to introduce a primitive recursive notation system for well-founded ω-derivations in the same way as one usually introcduces an ordinal notation system as a system of terms generated from constants for particular ordinals by function symbols corresponding to certain ordinal functions. A less-than relation between ordinal notations is then derived from the properties of the denoted ordinals and ordinal functions. Instead of ordinals Buchholz considers well-founded derivations in the standard system $\mathbf{Z}^\infty$ of ω-arithmetic, with an unrestricted ω-rule. Each derivation in Peano-arithmetic $\mathbf{Z}$ can be viewed as a notation for a particular $\mathbf{Z}^\infty$-derivation; so $\mathbf{Z}$-derivations play the role of constants here. The role of ordinal functions is taken over by the standard operators for cut elimination in infinitary derivations, as introduced by Schütte, Tait and Mints. Buchholz then introduces a system $\mathbf{Z}^*$ of *finite* terms built from derivations in $\mathbf{Z}$ by these function symbols. Each term (or notation) $h \in \mathbf{Z}^*$ then clearly denotes an infinitary derivation $[\![h]\!] \in \mathbf{Z}^\infty$.

The main technical achievement of Buchholz' paper is that without explicit use of indices for (primitve) recursive functions in a coding of infinitary derivations he obtains rather elegantly defined *primitive* recursive functions $\cdot[\cdot]$ and $o[\cdot]$ such that $h[n]$ denotes the n-th premise of the infinitary derivation $[\![h]\!]$ denoted by h, and $o(h)$ gives an ordinal bound for $[\![h]\!]$. It is quite clear that one can give *recursive* definitions of these functions, since the corresponding operators on infinitary derivations $\mathbf{Z}^\infty$ are defined by transfinite recursion. The trick to obtain primitve recursive such functions is the usual one, namely to introduce some delay operators, in the present case the repetition rule introduced by Mints.

Here we adapt Buchholz' approach to the somewhat more flexible setting of infinitary (typed) terms. This allows applications in recursion theory as well as in proof theory, since by the well-known Curry-Howard correspondence any natural (infinitary) deduction can be viewed as an (infinitary) term with formulas as types. However, in the talk we restrict ourselves to just one application of the method in recursion theory, namely the trade-off theorem mentioned above.

Victor Selivanov (Novosibirsk)

Finite hierarchies of regular ω-languages

By applying descriptive set theory to the Wagner's fine structure of regular ω-languages we obtain quite different proofs of his results. This approach leads to several new results, e.g. an easy description of the Wagner hierarchy in terms of Boolean operations, and easy invariants for all levels.

Alexandra Shlapentokh (Greenville)

Weakly computable fields and rings

Let F be a countable field. Then a weak presentation of F is an isomorphism of F onto a field whose elements are natural numbers such that the field operations of F are translated by functions which have total recursive extensions. We show that if F is infinitely generated then it has a weak presentation of arbitrary Turing degree and arbitrarily high enumeration degree.

Joseph R. Shoenfield (Durham)

Recursive functions of finite type

This talk is a brief description of the theory of recursive functions of finite type objects. It discusses the definitions and principal results of Kleene and the main advances in the subject which have been made since.

Richard A. Shore (Ithaca)

Definability in the enumerable degrees II

We sketch the proof of the following theorem which is one of the main technical results needed for the definability of the jump classes as described in the talk by André Nies:

Given any $0 <_T A \leq_T U$ with U promptly simple, there are r.e. sets E_0, E_1, F_0, F_1, B, P, Q, R and G_i (for $i \in \omega$) with $R = \oplus G_i, F_0 = \oplus G_{2i}, F_1 = \oplus G_{2i+1}$ and $B = G_i^{[0]}$ (for each $i \in \omega$) such that $P, Q \leq_T U$, all the other sets constructed are recursive in A and

$(T) : G_i \oplus P \geq_T Q$.

$(D): G_i \not\geq_T G_j$ for $i \neq j$.

(M) : If B is recursive in an r. e. W which is recursive in R and $W \oplus P \geq_T Q$, then there is a $j \in \omega$ such that $G_j \leq_T W$.

(K) : $R \oplus P$ is low.

(Y) : For each $i \in \omega$, $(G_{2i} \oplus E_1) \wedge F_1 \equiv_T G_{2i+1}$ and $(G_{2i+1} \oplus E_0) \wedge F_0 \equiv_T G_{2i+2}$.

(*Joint work with André Nies and Theodore Slaman*)

RICHARD A. SHORE (Ithaca)

There is no degree invariant half-jump

We prove that there is no degree invariant solution to Post's problem that always gives an intermediate degree. In fact, assuming definable determinacy, if W is any definable operator on degrees such that $\mathbf{a} < W(\mathbf{a}) < \mathbf{a}'$ on a cone then W is low_2 or high_2 on a cone of degrees, i. e. there is a degree $\mathbf{c}$ such that $W(\mathbf{a})'' = \mathbf{a}''$ for every $\mathbf{a} \geq \mathbf{c}$ or $W(\mathbf{a})'' = \mathbf{a}'''$ for every $\mathbf{a} \geq \mathbf{c}$.

(*Joint work with Rod Downey*)

STEPHEN G. SIMPSON (University Park)

Reverse mathematics: recent results

1. $ATR_0 \leftrightarrow$ König duality theorem for countable graphs.
   ($\leftarrow$ is due to Aharoni-Magidor-Shore, JCT, 1988.
   $\rightarrow$ is due to Simpson, JSL, 1994.)

2. $\Pi_1^1 - CA_0 \leftrightarrow$ for every separable Banach space X
   and any countable set $C \subseteq X^*$,
   the weak* closed linear span of C exists.
   (Humphreys & Simpson, TAMS, 1996, to appear.)

3. We survey the role of weak-weak König's Lemma in measure theory. Yu & Simpson (1989) have shown that WWKL is equivalent to countable additivity of Lebesgue measure on open sets.
   Brown-Giusto-Simpson (1995, in preparation) have shown that WWKL is equivalent to the Vitali covering theorem.

Theodore A. Slaman (Chicago)

On a Question of Łoś

Suppose that P is a countable partial ordering. We say that P^* is a *constrained η-extension* of P, if P^* is a dense linear order without endpoints, the domain of P^* is equal to the domain of P, and for all x and y in the domain of P, $x >_P y$ implies $x >_{P^*} y$.

J. Łoś posed the following problem: Find a necessary and sufficient condition for a given countable partial ordering to have a constrained η-extension.

Whether P has a constrained η-extension is a $\Sigma_1^1(P)$ property. Further, by the Kleene Basis Theorem for sets which are $\Sigma_1^1(P)$, if P has a constrained η-extension, then it has such an extension which is recursive in $\mathcal{O}^P$, where $\mathcal{O}^P$ is the complete $\Pi_1^1(P)$ set. Thus, Łoś's question about P is naturally reduced to $\mathcal{O}^P$. We give a logician's solution to Łoś's question, by showing that it has no simpler answer.

We show that any question about $\mathcal{O}^X$ can be reduced to an instance of Łoś's question for a partial order which is recursive in X, and so the question of whether a partially ordered set has an η-extension is a $\mathbf{\Sigma}_1^1$-complete question. Consequently, the upper bound provided by the Kleene Basis Theorem is optimal. We also exhibit a natural $\mathbf{\Pi}_1^1$-rank on those countable partial orders without constrained η-extensions, and observe that this rank can be used to uniformly construct a constrained η-extension of P, whenever there is such an extension.

(*Joint work with W. Hugh Woodin*)

Robert I. Soare (Chicago)

Definable properties and information content of c.e. sets

We announce and explain recent results on the computably enumerable (c.e.) sets, especially their *definability* properties (as sets in the spirit of Cantor), their *automorphisms* (in the spirit of Felix Klein's *Erlanger Programm*), their *dynamic* properties, expressed in terms of how *quickly* elements enter them relative to elements entering other sets, and the *Martin Invariance Conjecture* on their Turing degrees, i.e., their *information content* with respect to relative computability (Turing reducibility).

(*Joint work with Leo Harrington*)

Frank Stephan (Heidelberg)

Learning with Queries and Oracles

Inductive inference considers two types of queries: Queries to a teacher about the function to be learned and queries to a non-recursive oracle. This talk combines these two types — it considers three basic models of queries to a teacher (QEX[Succ], QEX[<] and QEX[+]) together with membership queries to some oracle.

The results for each of these three models of query-inference are the same: If an oracle is omniscient for query-inference then it is already omniscient for EX. There is an oracle of trivial EX-degree, which allows nontrivial query-inference. Furthermore, queries to a teacher can not overcome differences between oracles and the query-inference degrees are a proper refinement of the EX-degrees.

Stanley S. Wainer (Leeds)

The hierarchy of terminating recursive programs over $\mathcal{N}$

This talk concerned the Kleene hierarchy $REC(\alpha)$, $\alpha < \omega_1^{CK}$, of total recursive functionals $F_e : N^N \to N^N$ defined by programs e whose (well-founded) computation trees have height $\|e\| \leq \alpha$. The Hardy-functionals H_α play a natural and crucial role as dominating functionals, and provide a canonical link with proof theory via ω-role, and a general framework for measuring complexity. Thus for chosen well-orderings of type α (α an epsilon-number) $REC(< \alpha) = PROVREC(TI(< \alpha)) = ELEMENTARY(H_{<\alpha})$.

(*Joint work with M. Fairtlough, related to similar results of A. Weiermann*)

Stephan Wehner (Burnaby)

Computable Enumeration and the Problem of Repetition

A class $\mathcal{C}$ of computably enumerable (c.e.) sets is called *computably enumerable* if the members are uniformly computably enumerable, i.e. if there is a computable function f such that $\mathcal{C} = \{W_{f(n)} : n \in \omega\}$, where W denotes a Gödelnumbering of the c.e. sets as usual. Pour-El and Putnam introduced the notion k-c.e. A c.e. class $\mathcal{C}$ is called *k-c.e.* for $k \in \omega$, if there is a computable function f such that $\mathcal{C} = \{W_{f(n)} : n \in \omega\}$ and $|\{n : W_{f(n)} = C\}| \leq k$ for every $C \in \mathcal{C}$. A class is called *ω-c.e.* if there is a computable function f such that $\mathcal{C} = \{W_{f(n)} : x \in \omega\}$ and $|\{n : W_{f(n)} = C\}| < \omega$ for every $C \in \mathcal{C}$. Pour-El and Putnam showed that there are $(k+1)$-c.e. classes which are not k-c.e., and that there are ω-c.e. classes which are not k-c.e. for any finite k. Furthermore there exist c.e. classes which are not ω-c.e.

It is an open problem of computability theory to characterize the k-c.e. classes. The talk discusses some results from the speakers recent Ph.D.-thesis of the same title.

1. Using a computably enumerable listing $\mathcal{C}^{(e)}$ of all computably enumerable classes the index sets

$$\mathrm{CE}_k := \{e : \mathcal{C}^{(e)} \text{ is } k\text{-c.e. }\}$$

are defined, where $k \in \omega \cup \{\omega\}$. CE_k is Σ^0_5-complete for $k \in \omega$, $k > 0$ and CE_ω is Σ^0_6-complete.

2. A characterization is given for the 1-c.e. classes of cofinite sets which satisfy $\mathcal{P}_n$. A class satisfies $\mathcal{P}_n$ ($n \in \omega$) if the complement of every member contains at most n numbers. A class $\mathcal{C}$ satisfying $\mathcal{P}_n$ is 1-c.e. if and only if every subclass $\mathcal{C}'$ such that $|\mathcal{C} - \mathcal{C}'| \leq 2^{(n-1)}$ is c.e. This implies that for finite k the notions k-c.e. are equivalent for c.e. classes satisfying $\mathcal{P}_n$. On the other hand there are c.e. classes $\mathcal{C}$ which are not ω-c.e. but every subclass $\mathcal{C}'$ such that $|\mathcal{C} - \mathcal{C}'| < 2^{(n-1)}$ is c.e.

For classes of cofinite sets which do not satisfy $\mathcal{P}_n$ the characterization problem seems as difficult as in general: There is a c.e. class of cofinite sets which is not 1-c.e. but has a “dense” 1-c.e. subclass. Here “dense” means that every finite set which has an extension in the class has infinitely many extensions in the subclass. This shows that the following necessary property (G) of 1-c.e. classes is not also sufficient:

> to have a 1-c.e. subclass which contains for every finite set the same number of extensions.

3. The corresponding necessary property of ω-c.e. classes *is* also sufficient: A class is ω-c.e. if and only if it has an ω-c.e. subclass which contains for every finite set the same number of extensions.

No such statement is possible for the notions k-c.e. with finite k, for there is a c.e. class which is not k-c.e. for any $k \in \omega$ but satisfies (G).

Xiaoding Yi (Leeds)

The quotient semilattice of the recursively enumerable degrees modulo the cappable degrees

For a given 1st order structure S, studying its algebraic properties is usually the first step to understand the structure. For the structure R of the recursively enumerable (r.e.) degrees, virtually all the major algebraic results about R can be viewed as embedding, extension/nonextension of embedding results for R in a certain language. An important and unexpected result in 80's is an algebraic decomposition of R as the disjoint union of two sets of degrees – M, the set of r.e. degrees which are halves of minimal pairs (cappable degrees), and PS, the set of r.e. degrees which contain a promptly simple set – the former being an ideal. A major obstruction to formulate a "decision procedure of the two-quantifier theory of $(R, \leq, 0, 1)$ is the interact among prompt simplicity, non-embeddability and Slaman-Soare phenomenon. A better understanding of the relation between R and M is highly needed.

Two major questions about the quotient structure R/M of the r.e. degrees modulo the cappable degrees are:

1. Is R/M dense?
2. Does R/M satisfy the Shoenfield's Conjecture?

It is known that if a countable structure satisfies the Shoenfield's Conjecture, then it has many nice properties that the structure of rationals does. One motivation for looking at R/M is the hope of finding a natural, degree-theoretic upper semi-lattice satisfying Shoenfield's Conjecture. I have proved that the Shoenfield's Conjecture fails on R/M. In the present talk, I'll discuss the proofs of the following results:

1. For a given p.s. degree a, there exist degrees b, c such that $[c] < [b] < [a]$ and for all $w \geq c$, $a \leq b \vee w$ implies $a \leq w$. (Therefore, Shoenfield's Conjecture fails on every initial interval of R/M).
2. R/M is not dense. (Therefore, Shoenfield's Conjecture fails on $(R/M, \leq, 0, 1)$.)

Bereichterstatter: Frank Stephan

Participant	Email Address
Ambos-Spies, Klaus	ambos@math.uni-heidelberg.de
Arslanov, Marat	marat.arslanov@ksu.ru
Cenzer, Douglas	cenzer@math.ufl.edu
Cholak, Peter	cholak@turing.math.nd.edu
Chong, Chi Tat	scicct@nus.sg
Cooper, Barry	s.b.cooper@leeds.ac.uk
Ding, Decheng	postmath@netra.nju.edu.cn
Downey, Rod	rod.downey@vuw.ac.nz
Fejer, Peter A.	fejer@cs.umb.edu
Goncharov, Sergei S.	gonchar@imi.nsu.nsk.su
Griffor, Edward R.	griffor@math.uu.se
Groszek, Marcia	marcia.groszek@dartmouth.edu
Harizanov, Valentina	val@math.gwu.edu
Herrmann, Eberhard	herrmann@mathematik.hu-berlin.de
Hinman, Peter	pgh@umich.edu
Jockusch, Carl	jockusch@math.uiuc.edu
Knight, Julia	——
Kučera, Antonin	kucera@kki.mff.cuni.cz
Kumabe, Mashiro	kumabe@s5.math.umn.edu
Kummer, Martin	kummer@ira.uka.de
Lempp, Steffen	lempp@math.wisc.edu
Lerman, Manuel	mlerman@math.uconn.edu
Morozov, Andrei S.	morozov@math.nsk.su
Moschovakis, Yiannis N.	ynm@math.ucla.edu
Müller, Gert Heinz	ghmueller@math.uni-heidelberg.de
Nies, André	nies@math.uchicago.edu
Norman, Dag	dnormann@math.uio.no
Odifreddi, Piergiorgio	piergior@di.unito.it
Pohlers, Wolfram	pohlers@math.uni-muenster.de
Sacks, Gerald E.	sacks@math.harvard.edu
Schwichtenberg, Helmut	schwicht@rz.mathematik.uni-muenchen.de
Selivanov, Victor	vseliv@ngpi.nsk.su
Shlapentokh, Alexandra	mashlape@ecuvax.cis.ecu.edu
Shoenfield, Joseph R.	jrs@math.duke.edu
Shore, Richard A.	shore@math.cornell.edu
Simpson, Stephen G.	simpson@math.psu.edu
Slaman, Theodore A.	ted@math.uchicago.edu
Soare, Robert I.	soare@cs.uchicago.edu
Sorbi, Andrea	sorbi@unisi.it
Stephan, Frank	fstephan@math.uni-heidelberg.de
Wainer, Stanley S.	s.s.wainer@leeds.ac.uk
Wehner, Stephan	wehner@cs.sfu.ca
Weihrauch, Klaus	klaus.weihrauch@fernuni-hagen.de
Woodin, W. Hugh	woodin@math.berkeley.edu
Yi, Xiaoding	xiaoding@math.uconn.edu