Mathematisches Forschungsinstitut Oberwolfach

T a g u n g s b e r i c h t 3/2001

Berechenbarkeitstheorie

21.01.-27.01.2001

Die Tagung fand unter der Leitung von Klaus Ambos-Spies (Heidelberg), Steffen Lempp (Madison) und Ted Slaman (Berkeley) statt. Unter anderen wurden folgende Themen behandelt: der Halbverband der berechenbar aufzählbaren Turing-Grade sowie der n-r.a. Grade, die Struktur aller Turing-Grade, der Verband der berechenbar aufzählbaren Mengen und seine Automorphismen, höhere Berechenbarkeitstheorie, rekursive Modelltheorie, berechenbare algebraische Strukturen, Π^0_1-Klassen sowie Beziehungen der Berechenbarkeitstheorie zu anderen Gebieten (z.B. der Beweistheorie und der Algebra). Die interessanten Beiträge wurden lebhaft diskutiert. Außerdem gedachte die Tagung der Wissenschaftler Joseph Shoenfield and Linda Jean Richter.

Vortragsauszüge

Douglas Cenzer (University of Florida, Gainesville)

Structure and Applications of Π^0_1-Classes

We discuss some new results on the structure of the lattice of Π^0_1 classes as well as new applications.

An important general project is to compare and contrast the lattice $\mathcal{L}(\Pi^0_1)$ of Π^0_1 classes with the lattice $\mathcal{E}$ of computably enumerable sets. Together with A. Nies, we have characterized the finite lattices which can be isomorphic to initial segments of the lattice $\mathcal{L}^*(\Pi^0_1)$ of Π^0_1 classes, modulo finite difference. We have shown that in general, the theory of the initial segment $\mathcal{L}(P)$ may be decidable even though the lattice is not a Boolean algebra. This work led to the notion of a *minimal extension* Q of P, where $Q - P$ is not a Π^0_1 class and there is no Π^0_1 class properly between P and Q, modulo finite difference. F. Riazati and I showed that, for example, if P has a single limit point $x \leq_T 0'$, then P admits a minimal extension. A. Nies and I showed that if the Π^0_1 class P is itself decidable (that is, the set of infinite branches through a computable tree T with no dead ends) and the initial segment $\mathcal{L}(P)$ is not a Boolean algebra, then the theory of $\mathcal{L}(P)$ is undecidable. Recently we answered two open questions from the Boulder volume on "Computablity Theory and Its Applications". First, we show that the lattice $\mathcal{S}(P)$ of supersets of a non-clopen Π^0_1 class is not isomorphic to $\mathcal{L}(\Pi^0_1)$. Second, we show that the set of Π^0_1 classes P such that $\mathcal{L}(P)$ is isomorphic to the Boolean algebra of finite or cofinite sets, is definable in $\mathcal{L}(\Pi^0_1)$.

The restriction of the Medvedev lattice $\leq_m$ of degrees of difficulties to Π^0_1 classes gives rise to many open questions. Together with P. Hinman, we have shown that there is no minimal Π^0_1 class in this lattice. We have also shown that, for any class P of positive measure, there is a class $Q >_M P$.

Together with J. Remmel, we have analyzed the stable marriage problem of Gale and Shapley and shown that the solutions of a computable stable marriage may represent an arbitrary Π^0_1 class. This leads to versions of the theorem which are equivalent, for example, to König's Lemma in a certain natural subsystem of second-order arithmetic.

Peter Cholak (Notre Dame)

Extension Theorem and Automorphisms of the Computably Enumerable Sets

We discuss several algebraic extension theorems and use them to show the following theorem:

Theorem: If A and $\widehat{A}$ are automorphic via Ψ then they are automorphic via Λ where $\Lambda|\mathcal{L}^*(A) = \Psi|\mathcal{L}^*(A)$ and $\Lambda|\mathcal{E}^*(A)$ is Δ^0_3.

Rod Downey (Wellington)

Randomness and Reducibilities

We study computably enumerable reals (i.e. their left cut is computably enumerable) in terms of their relative randomness. We begin by revising the various notions of randomness such as Martin-Löf, Chaitin, Schnorr etc. Then we introduce Solovay reducibility; a c.e. real α is Solovay reducible to a c.e. real β iff there is a partial computable function f and a constant c such that, for all rationals $q < \beta$

$$c(\beta - q) > \alpha - f(q)\downarrow .$$

We look at a recent collection of results of the author with Hirschfeldt, LaForte, and Nies concerning the structure of the Solovay degrees, and new methods of calibrating randomness, which overcome various weaknesses in the notion of Solovay reducibility. For instance, the Solovay degrees are proven to be a dense USL with join arithmetical addition. It is shown that the complete degree, that of Chaitin's Ω cannot be split, yet all other c.e. Solovay degree split over all lesser ones. Similar results are obtained for a new reducibility called rH reducibility, which captures precisely, relative initial segment complexity. This material is taken from the following publications:

(1) R. Downey, D. Hirschfeldt, and A. Nies, *Computability, Randomness and Density,* submitted, extended abstract appears in STACS'01.

(2) R. Downey, D. Hirschfeldt, and G. LaForte, *Randomness and Reducibility,* submitted.

(3) R. Downey, *Computability-Theoretical Aspects of Reals and Randomness,* to appear.

Harvey Friedman (Columbus)

Boolean Relation Theory

Boolean relation theory is the study of the Boolean relations that hold between one dimensional sets and their images under multivariate functions. The two basic theorems that initiated the subject are the thin set theorem and the complementation theorem. The thin set theorem asserts that every multivariate function on the integers omits a value on some infinite set of integers. The complementation theorem asserts that for every strictly dominating multivariate function there exists an infinite set of integers which is sent to its complement (in fact this complementor is unique). The complementation theorem leads to theorems in discrete mathematics involving two functions and three sets which can be proved using large cardinals but not in ZFC, and which are, in fact, equivalent to the 1-consistency of Mahlo cardinals of finite order (as a scheme), over ACA. We sketch the proof of the simplest such theorem using Mahlo cardinals of finite order.

Sergei S. Goncharov (Novosibirsk)

The Theory of Computable Models

In this my talk in Oberwolfach I discuss some results and open problems in the theory of Computable models.

We assume that the reader knows some basic facts and concepts from model theory, universal algebra, and computability theory. Some knowledge of the first several chapters of the textbooks by Chang and Keisler on model theory [2], by C. Ash and J. Knight [1], Handbook [4] and Ershov and Goncharov [3] on theory of computable models will suffice to follow the paper.

We can define constructive and strongly constructive models. In the literature there is an equivalent terminology for constructive models and constructivizations that does not refer to numberings. These are computable models and computable presentations. An model is called **computable** if the domain of the system is ω and the atomic diagram is a computable set. A model is called **decidable** if the domain of the system is ω and the full diagram is a computable set.

One of the most interesting problem in computable models theory is the existence problem of computable models for theories. Many of the problems in this approach have been opened and known for many years and, perhaps, new ideas, constructions, and concepts will be needed to solve any of these problems. These problems and open topics have arisen in the course of development of computable model theory.

The effective completeness theorem suggests several fundamental questions about computability of special models of theories: prime, saturated and homogeneous models. These problems were solved in the case of decidability of these kind of models. For a survey of results related to strongly constructive models of theories we refer the reader to [3] and [4].

Hypothesis 1 For every $n \geq 1$, there exists a countably categorical theory of Turing degree $\mathbf{0}^n$ that has a constructive model.

We formulate the next hypothesis that states that a non-arithmetic countably categorical theory with constructive models exists.

Another class of theories well-studied in model theory is the class of uncountably categorical theories. In this section we deal with models of uncountably categorical theories. Baldwin and Lachlan showed that all models of any uncountably categorical theory T can be listed into the following chain, denoted by $chain(T)$, of elementary embeddings:

$$\mathcal{A}_0 \preceq \mathcal{A}_1 \preceq \mathcal{A}_2 \preceq \ldots \mathcal{A}_\omega,$$

where $\mathcal{A}_0$ is the prime model of T, $\mathcal{A}_\omega$ is the saturated model of T, and each $\mathcal{A}_{i+1}$ is prime over $\mathcal{A}_i$.

Thus a natural open problems about constructive models of uncountably categorical theories are the following:

Problem 1 Characterize all the subsets X of $\omega \bigcup \{\omega\}$ for which there exist uncountably categorical theories T such that $SCM(T) = X$.

Problem 2 Characterize uncountably categorical theories that have constructive models.

Question 1 If an uncountably categorical theory has a computable (arithmetical) model is then the theory arithmetic?

Question 2 Does there exist an uncountably categorical theory T whose models are $\mathcal{A}_0 \preceq \mathcal{A}_1 \preceq \ldots \preceq \mathcal{A}_\omega$ such that $\mathcal{A}_0$ has a constructivization, and each $\mathcal{A}_{i+1}$, $i \in \omega$, has $\mathbf{0}^{i+1}$–constructivization but does not have $\mathbf{0}^{i}$–constructivization?

Problem 3 If T is Ehrenfeucht whose all types are arithmetic then all models of T have arithmetic presentations.

There has been extensive research in the study of computable isomorphisms of constructive models. Many researchers [3] have worked on problems and research directions discussed in this section. These are still in the center of research interest and play a significant role in creation of new ideas, theorems and concepts.

It is easy to see that the next conditions are equivalent:

(1) for any relation R in $\mathcal{A}^2$ the sets $\nu^{-1}(R)$ and $\mu^{-1}(R)$ have the same Turing degree;

(2) for any relation R in $\mathcal{A}^2$ the set $\nu^{-1}(R)$ is computable iff the set $\mu^{-1}(R)$ is computable;

(3) the numeration ν and μ are recursively equivalent (i.e. identity automorphism from (A, ν) onto (A, μ) are computable).

But for autoequivalence we have another equivalent conditions:

(1) for any relation R in $\mathcal{A}^2$ the sets $\nu^{-1}(R)$ and $\mu^{-1}(\alpha(R))$ have the same Turing degree for some automorphism of our model;

(2) for any relation R in $\mathcal{A}^2$ the set $\nu^{-1}(R)$ is computable iff the set $\mu^{-1}(\alpha(R))$ is computable for some automorphism of our model;

(3) the constructivizations ν and μ are autoequivalent.

We discussed some new results and problem in this very interesting approach of computable models theory. Some of them were connected with the new notion of test-relations and test-families.

The natural problems is connected with set-theoretical properties. What the conditions have the relation which will be maximal (hh-simple, h-simple and so on) in some computable representation of our model.

One can be interested in computability–theoretic complexity of R under different constructivizations.

Question 3 Is it true that for any $n \geq 3$ there exists a non Δ^0_n-autostable but Δ^0_{n+1}-autostable model of dimension 2?

The last part of my talk was connected with structural and non-structural theorem. It was based on our paper with J. Knight.

(1) C. J. Ash and J. F. Knight, *Computable Structures and the Hyperarithmetical Hierarchy*, Springer-Verlag, 2001.

(2) C. C. Chang and H. J. Keisler. *Model Theory*, 3rd edn., Stud. Logic Found. Math., **73**, 1990.

(3) Yu. L. Ershov and S. S. Goncharov, *Constructive Models*, Consultant Bureau, Plenum, 2000.

(4) Yu. L. Ershov, S. S. Goncharov, A. Nerode and J. Remmel, eds., V. Marek, assoc. ed., *Handbook of Recursive Mathematics*, Vol. 1–2, North-Holland, 1999.

Denis Hirschfeldt (Chicago)

Degree Spectra of Relations on Computable Structures

The study of additional relations on computable structures, which began with the work of Ash and Nerode, has proved to be connected to a wide range of issues in computable structure theory. One particularly fruitful approach to this study, initiated by Harizanov, is to look at the collection of (Turing) degrees of the images of a relation in the various computable copies of a structure, which is known as the *degree spectrum* of the relation. This talk is a survey of results about possible degree spectra of relations in both the general case and with various restrictions placed on the relation and/or the structure. Particular attention is paid to the dichotomy between structure theorems, which allow us to rule out certain kinds of degree spectra in some restricted settings, and nonstructure results, which imply that in some other seemingly restricted settings, every degree spectrum phenomenon that can happen in general can already happen in the given setting. Connections with issues such as computable dimension of structures and Δ^0_n-categoricity are also emphasized.

CARL JOCKUSCH (Urbana)

Ramsey's Theorem for Pairs and Weak König's Lemma

I discuss metamathematical analogies and possible logical implications between Weak König's Lemma and Ramsey's Theorem for pairs.

Let RCA_0 be the usual weak base theory for Reverse Mathematics, and let WKL_0 be RCA_0 + Weak König's Lemma. Let ACA_0 be RCA_0 together with the arithmetic comprehension scheme. It is well-known that WKL_0 is strictly stronger than RCA_0 and strictly weaker than ACA_0 (see [5]). L. Harrington [5] used forcing over models of second-order arithmetic to show that WKL_0 is Π^1_1-conservative over RCA_0. The forcing is similar to that used to prove the low basis theorem.

Let RT^n_k be the assertion that for any k–coloring of the n–element sets of natural numbers there is an infinite set H of natural numbers which is homogeneous (i.e. all n–element subsets of H have the same color).

It is well-known [5] that RT^n_k is equivalent to ACA_0 over RCA_0 for $n \geq 3, k \geq 2$ and that RT^2_k is equivalent to RT^2_2 over RCA_0 for $k \geq 2$. Hence I will concentrate on RT^2_2. The strength of this result is analyzed in [4] and [1]. $\mathrm{RCA}_0 + \mathrm{RT}^2_2$ is strictly stronger than RCA_0 and strictly weaker that ACA_0. It is shown in [1] using forcing over models of second-order arithmetic that $\mathrm{RCA}_0 + I\Sigma_2 + \mathrm{RT}^2_2$ is Π^1_1-conservative over $\mathrm{RCA}_0 + I\Sigma_2$, where $I\Sigma_2$ is the Σ^0_2 induction scheme. The forcing used to prove this is similar to that used to prove the "low_2 basis theorem" [1]: For any computable 2-coloring of the 2-element sets of natural numbers there is an infinite low_2 homogeneous set.

Thus the metamathematics of Weak König's Lemma and RT^2_2 are quite parallel. Also, it was shown by J. Hirst that Weak König's Lemma does not imply RT^2_2 over RCA_0. It is unknown whether RT^2_2 implies Weak König's Lemma in RCA_0. The following is a new but very weak result in the direction of a negative answer: For any set X of natural numbers and any natural numbers a and b, there is an infinite set A such that either $A \subseteq X$ and $\{a\}^A \notin DNR_2$, or $A \subseteq \overline{X}$ and $\{b\}^A \notin \mathrm{DNR}_2$. Here $\mathrm{DNR}_2 = \{f \in 2^\omega : (\forall e)[f(e) \neq \varphi_e(e)]\}$. The proof uses propositional logic as a tool, much as it was used in [2, Corollary 3.4] to show that there is no noncomputable set X such that both X and $\overline{X}$ are uniformly introreducible. However, the proof is somewhat more involved than the corresponding proof in [2].

(1) P. Cholak, C. Jockusch and T. Slaman, *The Strength of Ramsey's Theorem for Pairs*, to appear in Journal of Symbolic Logic.

(2) C. Jockusch, *Uniformly Introreducible Sets*, Journal of Symbolic Logic **33** (1968), 521–536.

(3) C. Jockusch, *Ramsey's Theorem and Recursion Theory*, Journal of Symbolic Logic **37** (1972), 268–280.

(4) D. Seetapun and T. Slaman, *On the Strength of Ramsey's Theorem*, Notre Dame Journal of Formal Logic **36** (1995), 570–582.

(5) S. Simpson, *Subsystems of Second Order Arithmetic*, Springer-Verlag, 1999.

Steffen Lempp (University of Wisconsin, Madison)

Lattice Embeddings into the Computably Enumerable Turing Degrees

The characterization of the finite lattices embeddable into the computably enumerable degrees has defied researchers for over three decades.

Due to the complexity of the problem, it appears best to try to pinpoint first the simplest case where embeddability/nonembeddability is not fully understood, namely, for partial principally decomposable lattices with all joins and only two meets. (Any partial lattice with all joins and at most one meet is embeddable.) Along these lines, Lerman, Solomon and I have recently found a much easier example of nonembeddability than the lattice L_{20} of Lerman and myself, leading us to ideas for characterizing embeddability at least for this restricted classes of partial lattices.

I will survey the techniques needed, introducing one type of strategy at a time.

Manuel Lerman (Connecticut)

Homomorphisms and Quotients of the Computably Enumerable Degrees

We investigate pseudolattice homomorphisms of degree structures. Our focus here is on finite quotients of the c.e. degrees $\mathcal{R}$. Results of Calhoun [1] imply that every finite boolean algebra is a pseudolattice quotient of $\mathcal{R}$. We generalize this result to a class of distributive lattices.

Definition. We say that the finite distributive lattice $\langle L, < \rangle$ with greatest element 1 is *bi-orderable* if there are a pair of orderings $<_d$ and $<_p$ on the set L_{MI} of meet irreducible elements $\in L - \{1\}$ which satisfy the following conditions for all $a, b \in L_{MI}$:

$$\begin{aligned} a < b &\rightarrow b <_d a \ \& \ b <_p a; \\ a \mid b &\rightarrow (a <_p b \leftrightarrow b <_d a). \end{aligned}$$

Theorem. Let $\langle L, < \rangle$ be a bi-orderable finite distributive lattice. Then there is a pseudolattice homomorphism from $\mathcal{R}$ onto $\langle L, < \rangle$.

We note that all finite linearly ordered sets are bi-orderable, but that there are finite distributive lattices which are not bi-orderable. We conjecture that the bi-orderability of $\langle L, <\rangle$ can be characterized in terms of the non-embeddability of a finite set of lattices into $\langle L, <\rangle$.

(*Joint work with Burkhard Englert* (*UCLA*) *and Kevin Wald* (*Connecticut*))

(1) W. C. Calhoun, *Incomparable Prime Ideals of Recursively Enumerable Degrees*, Annals of Pure and Applied Logic **631** (1993), 39–56.

Andrei S. Morozov (Novosibirsk)

Computable Automorphisms of Models

In this talk, I present the modern state of affairs in computable symmetries of computable objects and discuss some open problems.

Here is the list of some unsolved questions. Some of them are known to be difficult while the other ones were not seriously tested yet.

- Which natural classes of permutations related to reducibilities form a group? It is known that for at most all reducibilities except for Turing reducibility there exists a degree such that the group of all permutations whose graph reduces to it is not closed under composition. Is it true that for such reducibilities there exist degrees such that this set forms a group?

- Find a more or less transparent description of the class of groups of computable automorphisms of computable models.

- (A. Nies) Assume $\mathbf{d}$ is a Turing degree. Define $G_{\mathbf{d}} = \{f \in \mathtt{Sym}\,\omega \mid f \leq \mathbf{d}\}$. Let $\mathtt{Fin}$ be a the group of all finitary permutations and $\mathtt{A}$ be the group of all even permutations. Is it true that $G_{\mathbf{d}}/\mathtt{Fin} \hookrightarrow G_{\mathbf{s}}/\mathtt{Fin} \Leftrightarrow \mathbf{d}' \leq \mathbf{s}'$? The direction $\Rightarrow$ is already known. The same question is open for quotients modulo $\mathtt{A}$.

- Do groups of all automorphisms of the basic computably enumerable predicates $x \in W_y$ and $\{x\}(y) = z$ have computable presentations? In particular, it is known that all automorphisms of these predicates are computable.

- Describe the finitely generated subgroups of the group of all computable permutations. It is known that the co-computable enumerability of the word problem is not enough.

- Does there exist a countably categorical decidable model such that computable automorphism group of each of its decidable copies is trivial?

- Describe computable automorphism groups of decidable countably categorical models with computable sets of complete sentences. Are they always non computably presentable?

- (S.Goncharov) Assume G_0 and G_1 are full computable automorphism groups of appropriate computable models. Does $G_0 * G_1$ have the same property? I know how to embed this group into the group of all computable permutations.

- (Finite signature problem) Assume $G = \mathtt{Aut}_c M$. Is it possible to find a computable model N whose language is finite such that $G = \mathtt{Aut}_c N$? The same question is also open for usual automorphisms.

- Does there exist a computable Boolean algebra B such that all its computable families of automorphisms are finite?

- Classify the computable vector spaces by their computable automorphism groups.

André Nies (Chicago)

Model Theoretic Properties of Structures from Computability Theory

We study structures from computability theory using model theoretic concepts and tools, especially coding with first-order formulas. The questions considered are:

Which restrictions on possible automorphisms exist? Which natural subsets are (parameter-) definable? Can a copy of the natural numbers be interpreted?

We answer the latter question to the affirmative for the c.e. wtt and Turing degrees. Moreover, for the c.e. Turing degrees, we obtain an approximation to the biinterpretability conjecture with parameters. For each nonrecursive degree $\mathbf{d}$, there is a 1–1 parameter definable map form $[\mathbf{d}, \mathbf{1}]$ into a coded copy of the natural numbers.

We also address a method from effective algebra, involving ideal lattices of c.e. Boolean algebras. For instance, this method can used to show the theory of many distributive structures is complex.

Wolfram Pohlers (Münster)

Proof Theory and Generalized Recursion Theory

We give a survey on the methods of generalized recursion theory which are needed in the ordinal analysis of theories. While so called “predicative theories” need nearly no recursion theoretic background we claim that for increasingly stronger impredicative theories recursion theoretic background is (practically) indispensable. This claim is substantiated on the examples of theories for inductive definitions, set theoretic reflection and the set theoretic stability.

Alexandra Shlapentokh (Greenville)

Diophantine Undecidability of Function Fields of Positive Characteristic

Let F be a function field of characteristic $p > 2$, finitely generated over a field C, where C is algebraic over a finite field and has an extension of degree p. We show that Hilbert's Tenth Problem is undecidable over F.

Stephen G. Simpson (Pennsylvania State University, University Park)

Medvedev Degrees, Muchnik Degrees, Subsystems of Z_2, and Reverse Mathematics

Foundations of mathematics (f.o.m.) is the study of the most basic concepts and logical structure of mathematics as a whole. An important f.o.m. research program is reverse mathematics, where one discovers which subsystems of second order arithmetic are necessary and sufficient to prove specific theorems in core mathematical areas such as analysis, algebra, geometry, and countable combinatorics. One of the most important subsystems for reverse mathematics is $\mathsf{WKL_0}$, consisting of $\mathsf{RCA_0}$ plus Weak König's Lemma.

Let $\mathcal{P}$ be the set of nonempty Π^0_1 subsets of 2^ω. Forcing with $\mathcal{P}$ (ordered by inclusion) is known as Jockusch/Soare forcing. I have used iterated Jockusch/Soare forcing to obtain an ω-model M of $\mathsf{WKL_0}$ with the following property: for all $X, Y \in M$, X is definable over M from Y if and only if $X \leq_T Y$. The proof is based on a homogeneity argument involving the Recursion Theorem, and a factorization lemma. I discuss the foundational significance of M and its hyperarithmetical analog.

For $P, Q \in \mathcal{P}$ one says that P is Muchnik reducible to Q ($P \leq_w Q$) if for all $Y \in Q$ there exists $X \in P$ such that $X \leq_T Y$. One says that P is Medvedev reducible to Q ($P \leq_M Q$) if there exists a recursive functional $\Phi : Q \to P$. I introduce the countable distributive lattices $\mathcal{P}_w$ ($\mathcal{P}_M$) consisting of the Muchnik (Medvedev) degrees of members of $\mathcal{P}$. I have shown that $P \in \mathcal{P}$ is Muchnik (Medvedev) complete if and only if P is degree isomorphic (recursively homeomorphic) to the set of complete extensions of PA.

Structural aspects of $\mathcal{P}_w$ and $\mathcal{P}_M$ present a rich problem area for recursion theorists. Stephen Binns and I have shown that every countable distributive lattice is lattice-embeddable below any nonzero degree in $\mathcal{P}_w$. We also have partial results in this direction for $\mathcal{P}_M$.

The lattices $\mathcal{P}_w$ and $\mathcal{P}_M$ are in some respects similar to the upper semilattice of Turing degrees of recursively enumerable subsets of ω. However, $\mathcal{P}_w$ and $\mathcal{P}_M$ are much better in that they contain specific, known, natural examples of degrees

$\neq 0, 1$. Such examples are especially relevant for f.o.m. In $\mathcal{P}_w$ there is the maximum Muchnik degree of Π^0_1 subsets of 2^ω of positive measure. This is related to the reverse mathematics of measure theory. In $\mathcal{P}_M$ there are the Medvedev degrees of $\{X : X$ is k-valued DNR$\}$, $k \geq 3$. This is related to the reverse mathematics of graph coloring.

For references see `http://www.math.psu.edu/simpson/talks/obwf0101/`.

THEODORE A. SLAMAN (University of California, Berkeley)

Aspects of the Turing Jump

The *Turing Jump* is the function which maps a set $X \subseteq \mathbb{N}$ to X', the halting problem relative to X. Fixing a recursive enumeration of all Turing machines,

$$X' = \{e : \text{The } e\text{th Turing machine with oracle } X \text{ halts.}\}$$

We will discuss two aspects of the jump and its iterations. First, we will show that they are implicitly characterized by general properties of relative definability. Second, we will present the Shore and Slaman [1] theorem that the function $x \mapsto x'$ is first order definable in the partial order of the Turing degrees.

(1) R. Shore, T. Slaman, *Defining the Turing Jump*, Mathematical Research Letters, **6** (1999), 711–722.

REED SOLOMON (University of Wisconsin, Madison)

Computable Algebra

This talk surveys some recent work in computable algebra. First, we look at relations on computable structures, contrasting sets of Archimedean representatives in ordered groups (which can be noncomputable in every computable presentation of the group (Solomon)) with bases in torsion free abelian groups (which always have a computable copy in some computable presentation of the group (Dobritsa)). Second, we examine degrees and jump degrees of structures, focusing on rank one torsion free abelian groups (which always have second jump degree (Coles, Downey, and Slaman)). Third, we consider examples of computable categoricity and computable dimension, specifically trees (trees of infinite height have computable dimension ω (Miller)) and ordered abelian groups (which are computably categorical if they have finite rank and have computable dimension ω otherwise (Goncharov, Lempp, and Solomon)). Also, we look at conditions determining when Boolean algebras and linear orders are Δ^0_2 categorical or relatively Δ^0_2 categorical (McCoy). Finally, we look at effectivizing theorems, considering several results about linear extensions of partial orders (Downey, Hirschfeldt, Lempp, and Solomon) which answer two open questions of Rosenstein.

STANLEY S. WAINER (Leeds)

Provable Recursiveness and Complexity

This talk describes some joint work with my students G. Ostrin and N. Cagman, on the proof theory of low subrecursive classes, between Grzegorczyk's E2 and E3. The basis is "A new recursion-theoretic characterization of the polytime functions" by Bellantoni and Cook (1992), in which it is shown that a natural two-sorted reinterpretation of the primitive recursion schemes characterizes polynomially bounded computation. We show that if Peano Arithmetic is instead formulated in this two-sorted fashion, with quantification allowed only over one sort ("safe" variables) and induction allowed only over the other ("normal" variables), then the provably recursive functions are exactly the E3 (elementary) functions. The provably recursive functions of the n-quantifier inductive fragments of this theory turn out to be closely related to the levels of Ritchie's induction corresponding to the E2 (linear space) functions. This work is (clearly) related to other results of Buss, Bellantoni, Leivant, Beckmann and Weiermann, and others too. In addition it illustrates nicely the use of classical ordinal analysis techniques even at this low level. The difference with classical PA is that the separation of induction from quantification means that the bounding functions are now Slow Growing rather than Fast Growing. Below ϵ_0 the Slow Growing functions only give elementary bounds — hence the results.

KLAUS WEIHRAUCH (Hagen)

Computable Analysis

Computable analysis is the theory of those functions on the real numbers and related sets which can be computed by digital computers. There are various partly non-equivalent mathematical approaches to computable analysis. In the talk the representation approach (TTE), which generalizes a definition of computable real functions by Grzegorczyk and Lacombe, is used as a framework. It merges concepts of approximation from analysis with concepts of machine models and discrete computation. Basic ideas of TTE are explained and the present state of computable analysis is illustrated by some recent results: Kolmogorov superposition, Riemann mapping theorem, the Pour-El/Richards paradox and computable solution operators of the wave equation, solution operators of the Schrödinger equation, computational complexity of on-line computation, admissible representations of weak limit spaces.

For details of TTE and a discussion of other models see K.Weihrauch: *Computable analysis*, Springer, 2000. References and more information can be obtained via `http://www.informatik.fernuni-hagen.de/cca/`.

Yang Yue (Singapore)

Σ_3-Induction and Slaman Density Theorem

Let PA^- denote the first order Peano axioms minus the induction scheme and $I\Sigma_n$ ($B\Sigma_n$, resp.) denote the induction (collection, resp.) scheme for Σ^0_n formulas. People has been interested in finding a theorem in computability theory, which requires $I\Sigma_3$. We show that over the base theory $PA^- + B\Sigma_3$, the Slaman Density Theorem, saying that the branching degrees are dense in computably enumerable degrees, is equivalent to $I\Sigma_3$.

(*Joint work with S. B. Cooper*)

Jessica Young (M.I.T.)

A prime model theorem

Barwise compactness is used to prove the following question posed by Goncharov: If a theory T is decidable and has only countably many countable models, then does T have a decidable homogeneous model? The answer is yes: in fact, if a theory has less than $2^{\aleph_0}$ many models, then it has a prime model whose elementary diagram is computable in T.

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