Selftest for MA 3205 – Set Theory

Matriculation Number: ______________________. Marking: Each question 1 mark.

Question 1. List all elements of the set $(\{a,b,c\} \cup \{c,d,e\}) \Delta (\{a,b,e\} \cap \{c,d,e\})$:

____.

Question 2. Determine the cardinality of $\mathcal{P}(\mathcal{P}(\mathcal{P}(\{\{\mathbb{N}\}\}))) \cap \mathcal{P}(\mathcal{P}(\mathcal{P}(\{\{\{\{\mathbb{N}\}\}\}\})))$:

☐ 0, ☐ 1, ☐ 2, ☐ 4, ☐ 8, ☐ $\aleph_0$, ☐ $\aleph_1$, ☐ $2^{\aleph_0}$.

Question 3. Tick the three properties true for every inductive set A:

☐ $\emptyset \in A$;
☐ $A \in \{A\}$;
☐ $\exists B \in A\, (|B| = \aleph_2)$;
☐ $\forall x \in A\, ((x \cup \{x\}) \in A)$;
☐ $\forall x \in A\, \forall y \in x\, (\{y\} \in A)$.

Question 4. Tick the four statements true for all ordinals α and β:

☐ $\alpha \subseteq \beta \vee \beta \in \alpha$;
☐ $|\alpha| \cdot |\beta| = \max\{|\alpha|, |\beta|\}$;
☐ $\alpha \cup \beta$ is transitive;
☐ $\alpha \cap \beta$ is well-ordered;
☐ $\alpha \times \beta$ has two elements equal to $\emptyset$;
☐ $\exists \gamma, \delta \in \alpha\, (\gamma \in \delta)$;
☐ $\forall \gamma, \delta \in \alpha\, (\gamma \in \delta \vee \delta \in \gamma \vee \gamma = \delta)$.

Question 5. Let $f(0) = 2, f(1) = 2, f(2) = 3, f(3) = 3$. Determine the image and preimage of $\{1,2\}$: The image is {___________,___________} and the preimage is {___________,___________}.

Question 6. What is the code for the natural number 3? The codes for 0, 1 and 2 are given as an example.

$0 = \emptyset$; $1 = \{\emptyset\}$; $2 = \{\emptyset, \{\emptyset\}\}$;
$3 =$ {___________, ___________, ______________________}.

Question 7. List all elements of the set $\mathcal{TC}(\{\{1,3\},5\})$ using that natural numbers have smaller numbers as alements, list the large sets in the second line:

___________, ___________, ___________, ___________, ___________, ___________,
______________________ and _________________________________.

Question 8. Put the strings 0011,001122,1122,00,22 into Kleene-Brouwer Ordering:

___________< ___________< ___________< ___________< ___________.

Question 9. In the following, let A_α be a set of cardinality $\aleph_\alpha$ for all α. Determine the cardinal of the set $A_{\omega\cdot 2+3} \cup (A_\omega \times A_\omega \times A_{\omega+5}) \,\Delta\, A_{\omega+3} \,\Delta\, A_{\omega\cdot 3+3}$: ______________________.

Question 10. Give a definition when a function $f : \mathbb{N} \to \mathbb{N}$ dominates a set G of functions $g : \mathbb{N} \to \mathbb{N}$:

f domintates G iff for ___________ $g \in G$, $\exists$___________$\forall$___________(______________________).

Question 11. Tick those two sets of the following sets which are countable.

☐ $\mathcal{P}(\mathcal{P}(\emptyset))$, ☐ V_{20}, ☐ V_ω, ☐ $V_{\omega+1}$, ☐ $\mathbb{R}$, ☐ $\mathbb{Z}$.

Question 12. Are the linearly ordered sets $(\mathbb{Q}, <)$ and $(\mathbb{R}, <)$ order isomorphic? Tick that answer where the statement and the reason are correct:

☐ Yes, because both sets are dense linear orders;
☐ Yes, because there is an order-preserving mapping from $\mathbb{Q}$ to $\mathbb{R}$;
☐ No, because the cardinality of $\mathbb{Q}$ is strictly below that of $\mathbb{R}$;
☐ No, because $\mathbb{Q}$ is complete and $\mathbb{R}$ not.

Question 13. Tick four cardinals known to be different from $2^{\aleph_1}$:

☐ 256, ☐ $\aleph_0$, ☐ $\aleph_1$, ☐ $\aleph_2$, ☐ $\aleph_\omega$, ☐ $\aleph_{\omega+5}$, ☐ $2^{\aleph_1}$.

Question 14. By the Theorem of Hessenberg, the sets $\mathbb{N}$ and $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ have the same cardinality. Complete the proof for this.

Define on $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ the following ordering: $(a, b, c) \sqsubset (d, e, f) \Leftrightarrow a + b + c < d + e + f$ or (______________________ $=$ ______________________ and $(a, b, c) <_{lex} (d, e, f)$). Then before every element in $(d, e, f) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ there are only finitely many elements, for example the elements before $(0, 0, 2)$ are ___________, ___________, ___________ and ___________. Furthermore, $(\mathbb{N} \times \mathbb{N} \times \mathbb{N}, \sqsubset)$ is _________________________________. Now both sets are ______________________, well-ordered and have that any initial segment is ______________________. Hence both sets are order-isomorphic to the first countable ordinal ___________ and thus isomorphic to each other.

Question 15. Give the Cantor Normal Form of the following two ordinals:

$\omega^{\omega^2} + \omega^{\omega+2} + \omega^{\omega^2} + \omega^{\omega+2} + \omega^3 + \omega^7 + \omega^9$: ____;

$\omega^{\omega^2} + \omega^{\omega_1} + \omega^{\omega_2} + \omega^{\omega_1} + \omega^{13} + \omega^{17} + \omega^2$: ____.