Theory of Computation Additional Exercises

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Additional Exercises A–B

Each student can write up one of these exercises in the Forum to get 2 additional marks; please check before writing that no other student already wrote it up in the Forum. Maximum marks for Exercises is 10.

Exercise A.
Provide a regular expression for all words which have either the subword $\mathbf{110}$ or the subword $\mathbf{120}$ over the ternary alphabet $\{\mathbf{0}, \mathbf{1}, \mathbf{2}\}$.

Exercise B.
Use structural induction to define $\mathbf{L}^{\mathbf{mi}} = \{\mathbf{w}^{\mathbf{mi}} : \mathbf{w} \in \mathbf{L}\}$ for all regular sets $\mathbf{L}$. So first define this for all sets containing words up to length $\mathbf{1}$ and then explain, how one goes on with concatenation, union, Kleene star and Kleene plus. Sets consisting of words of length at least $\mathbf{2}$ can be obtained from the base cases this way.

Additional Exercises C–E

Exercise C.
Provide a DFA accepting those binary numbers which are multiples of $\mathbf{3}$: $\mathbf{11}, \mathbf{110}, \mathbf{1001}, \mathbf{1100}, \mathbf{1111}, \mathbf{10010}, \mathbf{10101}$, $\mathbf{11000}, \mathbf{11011}$ and so on.

Exercise D.
Construct over the ternary alphabet $\{\mathbf{0}, \mathbf{1}, \mathbf{2}\}$ a DFA which accepts all words which contain each of the digits an odd number of times.

Exercise E.
Provide a non-deterministic finite automaton of thirteen states which accepts all decimal numbers which are not a multiple of $\mathbf{210}$; for this note that $\mathbf{0}$ is also a multiple of $\mathbf{210}$ and should be rejected. The numbers accepted should not having leading zeroes.

Additional Exercises F–H

Exercise F.

Construct a context-free grammar in Chomsky Normal Form for the language $\{\mathbf{0^n 1^m 2^n : n, m \in \mathbb{N}}\}$.

Exercise G.

Construct a context-free grammar in Greibach Normal Form for the language $\{\mathbf{0^n 1^m 2^n : n, m \in \mathbb{N}}\}$.

Exercise H.

Use the grammar from F and the Cocke Kasami Younger algorithm to check whether $\mathbf{F}$ generates the following words: $\mathbf{00122}$ and $\mathbf{00112}$.

Additional Exercises I–K

Exercise I.

Recall that a language $\mathbf{L}$ satisfies the weakest form of the Pumping Lemma iff there is a constant $\mathbf{k}$ such that all words of length at least $\mathbf{k}$ in $\mathbf{L}$ can be split into parts $\mathbf{xyz}$ with $\mathbf{y} \neq \varepsilon$ and $\{\mathbf{x}\} \cdot \{\mathbf{y}\}^* \cdot \{\mathbf{z}\} \subseteq \mathbf{L}$. Which of the following choices for $\mathbf{L}$ satisfy this pumping lemma:

1. $\mathbf{L} = \{\mathbf{0^n 1^m 2^n} : \mathbf{n}, \mathbf{m} \in \mathbb{N}\}$;
2. $\mathbf{L} = \{\mathbf{0^n 1^m 0^n} : \mathbf{n}, \mathbf{m} \in \mathbb{N}\}$;
3. $\mathbf{L} = \{\mathbf{0^n 1^m 2^k} : \mathbf{n} + \mathbf{k} \neq \mathbf{m}\}$?

Exercise J.

Which of the languages in Exercise I have a linear grammar?

Exercise K.

Use Ogden's Lemma to prove that the language $\{\mathbf{0^n 1^m 2^k} : \mathbf{n} \neq \mathbf{m} \wedge \mathbf{n} \neq \mathbf{k} \wedge \mathbf{m} \neq \mathbf{k}\}$ is not context-free.

Additional Exercises L–O

Let $\mathbf{h}, \mathbf{k}$ be homomorphisms with $\mathbf{h(a) = a}$, $\mathbf{h(b)} = \varepsilon$, $\mathbf{k(a)} = \varepsilon$ and $\mathbf{k(b) = b}$ for $\mathbf{a} \in \{\mathbf{0, 1, 2, 3, 4}\}$ and $\mathbf{b} \in \{\mathbf{5, 6, 7, 8, 9}\}$. Prove the following statements for $\mathbf{L} \subseteq \{\mathbf{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}\}^*$.

Exercise L. If $\mathbf{k(L), h(L)}$ are both regular then $\mathbf{L}$ is regular.

Exercise M. If $\mathbf{k(L)}$ is deterministic context-free and $\mathbf{h(L)}$ regular then $\mathbf{L}$ is deterministic context-free.

Exercise N. There is an $\mathbf{L}$ for which $\mathbf{h(L), k(L)}$ are both context-free but $\mathbf{L}$ is not.

Exercise O. If $\mathbf{k(L), h(L)}$ are context-sensitive languages not containing ε, so is $\mathbf{L}$.

Additional Exercises P–S

Do for $\mathbf{L} = \{\mathbf{w} \in \{\mathbf{000}, \mathbf{111}, \mathbf{222}\}^+ : \mathbf{w}$ is a palindrome$\}$ the following exercises.

Exercise P. Which is the least constant $\mathbf{k}$ such that every word $\mathbf{w} \in \mathbf{L}$ can be split into three parts $\mathbf{x}, \mathbf{y}, \mathbf{z}$ with $\mathbf{w} = \mathbf{xyz}$ and $\mathbf{1} \leq |\mathbf{y}| \leq \mathbf{k}$ and $\mathbf{xy^*z} \subseteq \mathbf{L}$. Give the answer ∞ if there is no such constant $\mathbf{k}$. Prove the answer.

Exercise Q. Provide a context-free grammar in Chomsky Normal Form for $\mathbf{L}$.

Exercise R. Provide a context-free grammar in Greibach Normal Form for $\mathbf{L}$.

Exercise S. Provide a PDA accepting by state for $\mathbf{L}$. Can this PDA be made deterministic? Give a short reason for the answer.

Additional Exercises T–W

Prove that the following functions are primitive recursive.

Exercise T. Function $\mathbf{f(n) = n^2}$.

Exercise U. Function $\mathbf{g(n) = n^n}$.

Exercise V. Function $\mathbf{h(m, n) = \binom{m+n}{m}}$.

Exercise W. Function $\mathbf{k(m, n) = m! + n!}$.

Additional Exercises X-Z

Exercise X. Is it decidable to check whether a polynomial with integer coefficients and one input variable $\mathbf{x}$ takes on some input the value $\mathbf{1024}$? For example, for input function $\mathbf{f(x) = x^2 + 1}$, one wants to check whether there is an integer $\mathbf{x}$ with $\mathbf{f(x) = 1024}$. Prove the answer.

Exercise Y. Let $\varphi_\mathbf{0}, \varphi_\mathbf{1}, \ldots$ be an acceptable numbering of all partial-recursive functions. Is $\mathbf{L} = \{\mathbf{e} : \varphi_\mathbf{e}(\mathbf{x})$ is undefined for some $\mathbf{x}\}$ recursively enumerable? Prove the answer using Rice's Theorem.

Exercise Z. Let $\varphi_\mathbf{0}, \varphi_\mathbf{1}, \ldots$ be an acceptable numbering of all partial-recursive functions. Is $\mathbf{H} = \{\mathbf{e} :$ the domain of $\varphi_\mathbf{e}$ is the range of a primitive recursive function$\}$ (a) decidable, (b) recursively enumerable and undecidable, (c) not recursively enumerable? Prove the answer using Rice's Theorem.