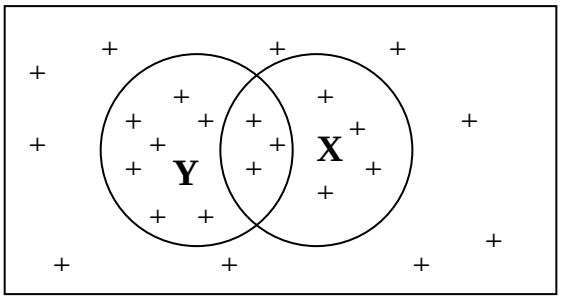
CS3243 Foundations of Artificial Intelligence (2005/2006 Semester 2)
Tutorial 9

1. Based on the following Venn diagram, complete the joint probability distribution in the table on its right.



	X	**¬ X**
Y	$\frac{3}{24}$	
¬Y		

Based on the joint probability distribution, find the following: $P(X)$, $P(Y)$, $P(\neg X)$, $P(\neg Y)$, $P(X|Y)$, $P(Y|X)$, $P(X|\neg Y)$, $P(\neg Y|X)$, $P(\neg X|Y)$, $P(Y|\neg X)$, $P(\neg X|\neg Y)$, $P(\neg Y|\neg X)$. Substituting the values of these conditional probabilities, verify the following:

$$P(X \mid Y) = 1 - P(\neg X \mid Y)$$

$$P(X \mid \neg Y) = 1 - P(\neg X \mid \neg Y)$$

$$P(\neg Y \mid X) = \frac{P(X \mid \neg Y)P(\neg Y)}{P(X \mid \neg Y)P(\neg Y) + P(X \mid Y)P(Y)}$$

2. Assume that 2% of the population in a country carry a particular virus. A test kit developed by a pharmaceutical firm is able to detect the presence of the virus from a patient's blood sample. The firm claims that the test kit has a high accuracy of detection in terms of the following conditional probabilities obtained from their quality control testing:

   P(the kit shows positive | the patient is a carrier) = 0.998
   P(the kit shows negative | the patient is not a carrier) = 0.996

   If a patient is tested to be positive using this kit, what is the likelihood of a false positive (i.e., that he actually is not a carrier but the kit shows positive)?

3. (Question 13.1 from the textbook) Show from first principles that $P(a \mid b \wedge a) = 1$.

4. (Question 13.7 from the textbook) Show that the three forms of independence below:
   (a) $P(a \mid b) = P(a)$
   (b) $P(b \mid a) = P(b)$
   (c) $P(a \wedge b) = P(a)P(b)$
   are equivalent.