Introduction to Information Retrieval

http://informationretrieval.org

IIR 21: Link Analysis

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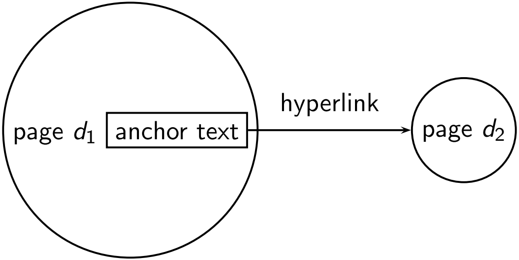
Overview

1. Anchor text
2. PageRank
3. HITS

Outline

The web as a directed graph



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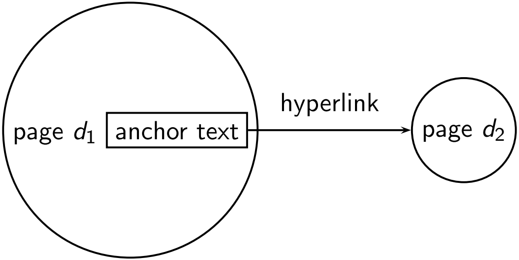

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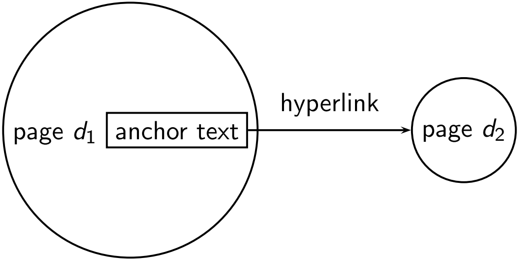

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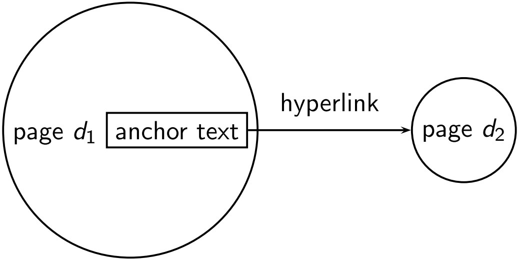

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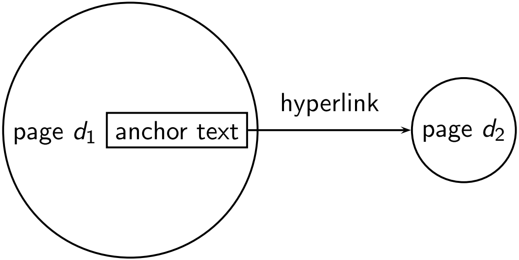

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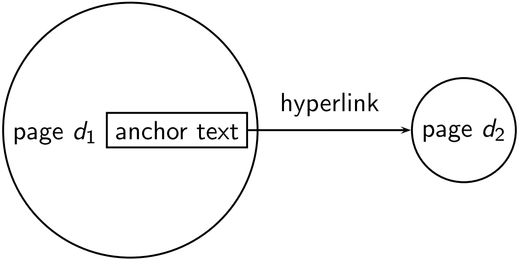

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- Examples for hyperlinks that violate these two assumptions?

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Anchor text containing *IBM* pointing to www.ibm.com

www.nytimes.com: "IBM acquires Webify"

www.slashdot.org: "New IBM optical chip"

www.stanford.edu: "IBM faculty award recipients"

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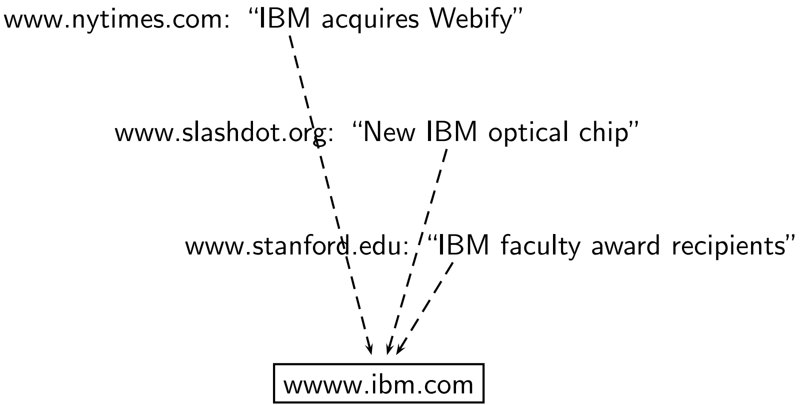

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- Any "live" Google bombs?

Google bomb

- "who is a failure" on Google

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Cocitation similarity on Google: similar pages

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- Recall: Citation in scientific literature = hyperlink on the web

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- Simple link popularity (= number of in-links) is easy to spam. Why?

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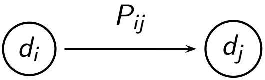
- For $1 \leq i, j \leq N$, the matrix entry P_{ij} tells us the probability of j being the next page, given we are currently on page i.



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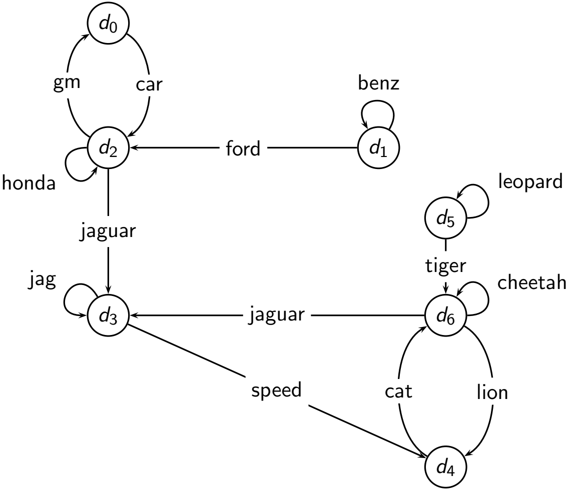
- Markov chains are abstractions of random walks.

Example web graph



Link matrix for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0	0	1	0	0	0	0
d_1	0	1	1	0	0	0	0
d_2	1	0	1	1	0	0	0
d_3	0	0	0	1	1	0	0
d_4	0	0	0	0	0	0	1
d_5	0	0	0	0	0	1	1
d_6	0	0	0	1	1	0	1

Transition probability matrix P for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

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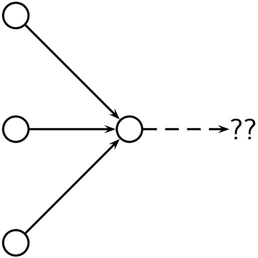
- First a special case: The web graph must not contain dead ends.

Dead ends



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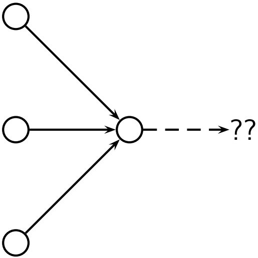

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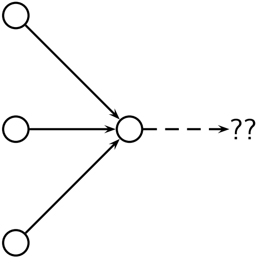

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- If there are dead ends, long-term visit rates are not well-defined (or non-sensical).

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- 10% is a parameter.

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- More generally, we require that the Markov chain be ergodic.

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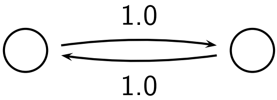
- A non-ergodic Markov chain:



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- This is the steady-state probability distribution.
- Over a long time period, we visit each state in proportion to this rate.
- It doesn't matter where we start.

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(	0.05	0.01	0.0	...	0.2	...	0.01	0.05	0.03	)
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(	0	0	0	...	1	...	0	0	0	)
	1	2	3	...	i	...	N-2	N-1	N	

- More generally: the random walk is on page i with probability x_i.
- Example:

(	0.05	0.01	0.0	...	0.2	...	0.01	0.05	0.03	)
	1	2	3	...	i	...	N-2	N-1	N	

- $\sum x_i = 1$

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- So from $\vec{x}$, our next state is distributed as $\vec{x}P$.

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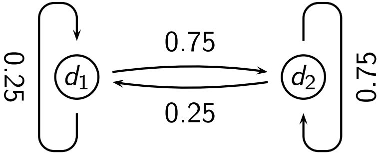
- π_i is the long-term visit rate (or PageRank) of page i.
- So we can think of PageRank as a very long vector – one entry per page.

Steady state example

- What is the steady state in this example?



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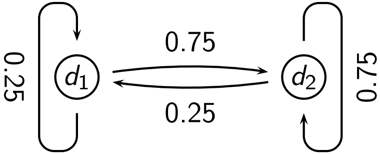

- Solution: $\vec{\pi} = (\pi_1 \ \pi_2) = (0.25 \ 0.75)$

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- Transition probability matrices always have largest eigenvalue 1.

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- This is called the power method.

Power method: Example

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- Convergence in one iteration!

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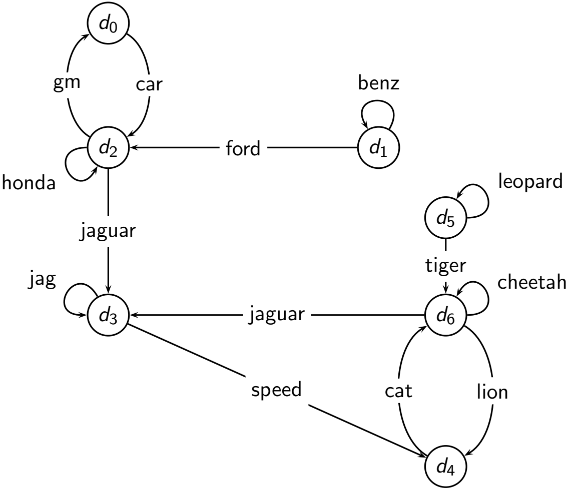
- In practice: rank according to weighted combination of many factors, including raw text match, anchor text match, PageRank and many other factors

Web graph example



Transition (probability) matrix

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

Transition matrix with teleporting

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.02	0.02	0.88	0.02	0.02	0.02	0.02
d_1	0.02	0.45	0.45	0.02	0.02	0.02	0.02
d_2	0.31	0.02	0.31	0.31	0.02	0.02	0.02
d_3	0.02	0.02	0.02	0.45	0.45	0.02	0.02
d_4	0.02	0.02	0.02	0.02	0.02	0.02	0.88
d_5	0.02	0.02	0.02	0.02	0.02	0.45	0.45
d_6	0.02	0.02	0.02	0.31	0.31	0.02	0.31

Power method vectors $\vec{x}P^k$

	$\vec{x}$	$\vec{x}P^1$	$\vec{x}P^2$	$\vec{x}P^3$	$\vec{x}P^4$	$\vec{x}P^5$	$\vec{x}P^6$	$\vec{x}P^7$	$\vec{x}P^8$	$\vec{x}P^9$	$\vec{x}P^{10}$	$\vec{x}P^{11}$	$\vec{x}P^{12}$	$\vec{x}P^{13}$
d_0	0.14	0.06	0.09	0.07	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
d_1	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_2	0.14	0.25	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.12	0.12	0.11	0.11	0.11
d_3	0.14	0.16	0.23	0.24	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25
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  - However, variants of a page's PageRank are still an essential part of ranking.
  - Adressing link spam is difficult and crucial.

Outline

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- Most approaches to search (including PageRank ranking) don't make the distinction between these two very different types of relevance.

Hubs and authorities

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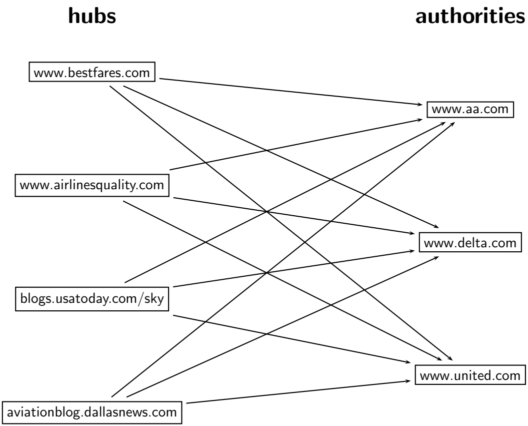
- Circular definition – we will turn this into an iterative computation.

Example for hubs and authorities



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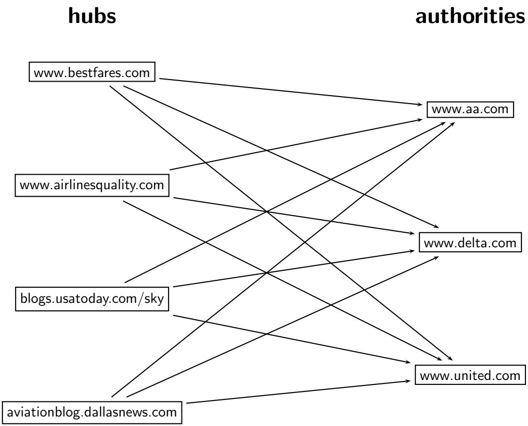

Definition clear?

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- Finally, compute hubs and authorities for this (small) web graph

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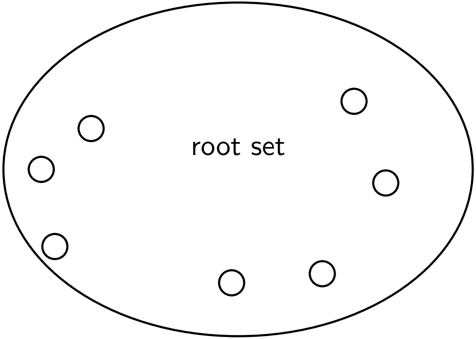

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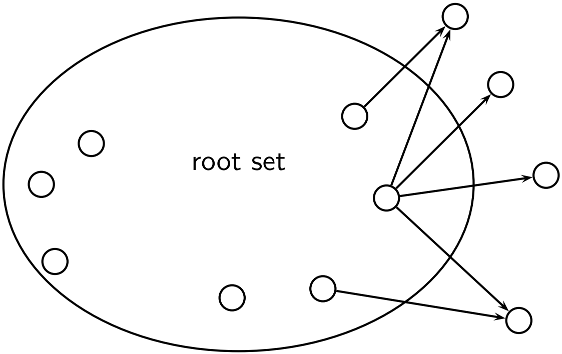

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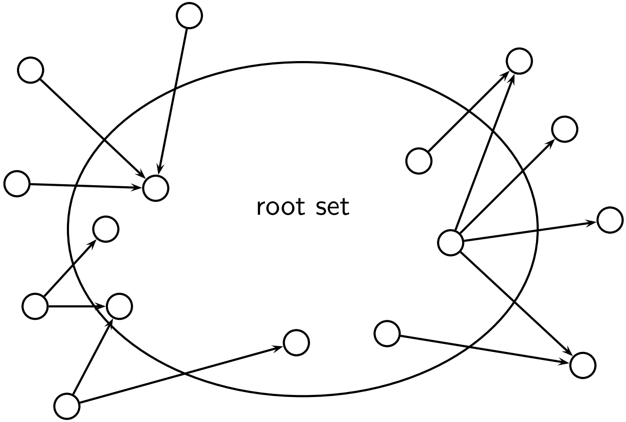

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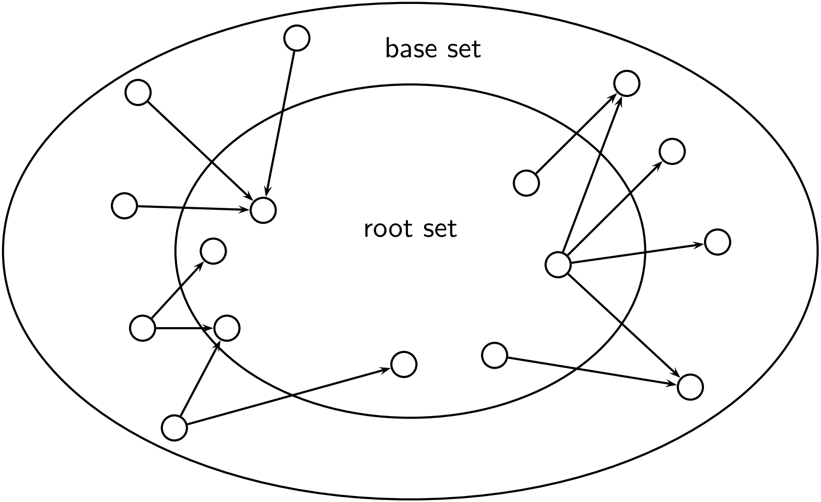

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  - This assumes that our inverted index supports search for links (in addition to terms).

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Iterative update

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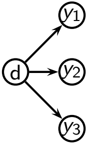

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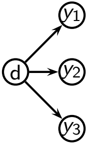

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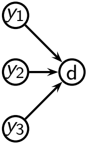

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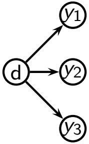

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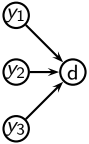

- Iterate these two steps until convergence

Details

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  - To prevent the $a()$ and $h()$ values from getting too big, can scale down after each iteration

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  - Scaling factor doesn't really matter.
  - We care about the relative (as opposed to absolute) values of the scores.
- In most cases, the algorithm converges after a few iterations.

Japan elementary schools

Hubs

- schools
- LINK Page-13
- “ú–{‚ÌŠw□Z
- □a‰„□¬Šw□Zƒz□[ƒ□ƒy□[ƒW
- 100 Schools Home Pages (English)
- K-12 from Japan 10/...rnet and Education)
- http://www...iglobe.ne.jp/~IKESAN
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- □ÒŠ—’¬—§□ÒŠ—“Œ□¬Šw□Z
- Koulutus ja oppilaitokset
- TOYODA HOMEPAGE
- Education
- Cay's Homepage(Japanese)
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- UNIVERSITY
- ‰J—³□¬Šw□Z DRAGON97-TOP
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Authorities

- The American School in Japan
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- KEIMEI GAKUEN Home Page (Japanese)
- Shiranuma Home Page
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Japan elementary schools

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Japan elementary schools

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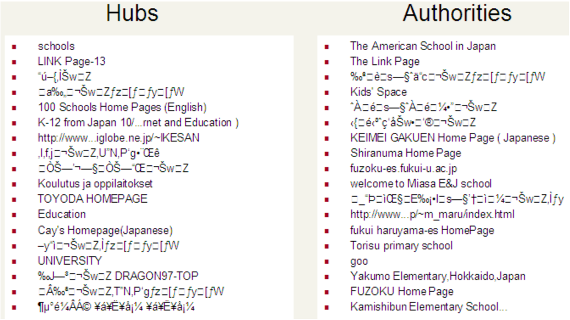

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- Danger: topic drift – the pages found by following links may not be related to the original query.

Proof of convergence

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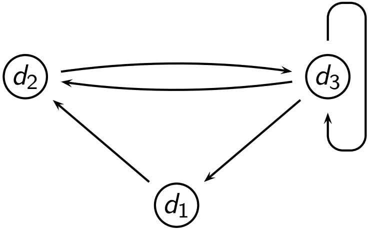

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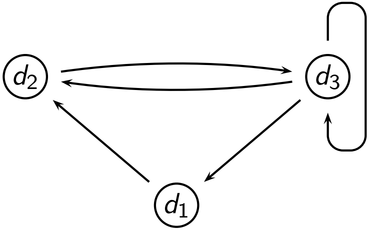

	d_1	d_2	d_3
d_1	0	1	0
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HITS as eigenvector problem

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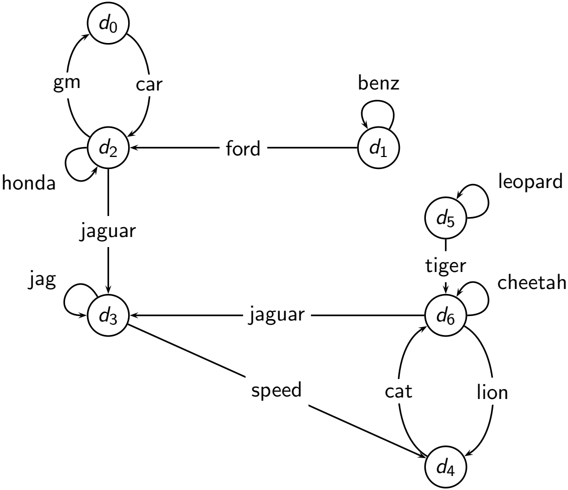
- So the HITS algorithm is actually a special case of the power method and hub and authority scores are eigenvector values.
- HITS and PageRank both formalize link analysis as eigenvector problems.

Example web graph



Raw matrix H for HITS

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0	0	1	0	0	0	0
d_1	0	1	1	0	0	0	0
d_2	1	0	1	2	0	0	0
d_3	0	0	0	1	1	0	0
d_4	0	0	0	0	0	0	1
d_5	0	0	0	0	0	1	1
d_6	0	0	0	2	1	0	1

Hub vectors $h_0, \vec{h}_i = \frac{1}{d_i} H \cdot \vec{a}_i, i \geq 1$

	$\vec{h}_0$	$\vec{h}_1$	$\vec{h}_2$	$\vec{h}_3$	$\vec{h}_4$	$\vec{h}_5$
d_0	0.14	0.06	0.04	0.04	0.03	0.03
d_1	0.14	0.08	0.05	0.04	0.04	0.04
d_2	0.14	0.28	0.32	0.33	0.33	0.33
d_3	0.14	0.14	0.17	0.18	0.18	0.18
d_4	0.14	0.06	0.04	0.04	0.04	0.04
d_5	0.14	0.08	0.05	0.04	0.04	0.04
d_6	0.14	0.30	0.33	0.34	0.35	0.35

Authority vectors $\vec{a}_i = \frac{1}{c_i} H^T \cdot \vec{h}_{i-1}, i \geq 1$

	$\vec{a}_1$	$\vec{a}_2$	$\vec{a}_3$	$\vec{a}_4$	$\vec{a}_5$	$\vec{a}_6$	$\vec{a}_7$
d_0	0.06	0.09	0.10	0.10	0.10	0.10	0.10
d_1	0.06	0.03	0.01	0.01	0.01	0.01	0.01
d_2	0.19	0.14	0.13	0.12	0.12	0.12	0.12
d_3	0.31	0.43	0.46	0.46	0.46	0.47	0.47
d_4	0.13	0.14	0.16	0.16	0.16	0.16	0.16
d_5	0.06	0.03	0.02	0.01	0.01	0.01	0.01
d_6	0.19	0.14	0.13	0.13	0.13	0.13	0.13

Top-ranked pages

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- Pages with highest PageRank: d_6

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- Pages with highest authority score: d_3

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- The actual difference between PageRank ranking and HITS ranking is therefore not as large as one might expect.

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- Google's official description of PageRank: *PageRank reflects our view of the importance of web pages by considering more than 500 million variables and 2 billion terms. Pages that we believe are important pages receive a higher PageRank and are more likely to appear at the top of the search results.*