Text Processing on the Web

Week 10

Partitional and Hierarchical Text Clustering

Edited from source slides from the Stanford textbook site

Recap and Outline

- TC as different from standard machine learning
  - High dimensionality
  - Feature selection / weighting
  - Dataset skew / # of examples
- **Clustering**
  - Partitional Text Clustering
  - Hierarchical Text Clustering
  - Evaluation Methods

"The Curse of Dimensionality"

- Dealing with high dimensionality is difficult
  - While clustering looks intuitive in 2 dimensions, many of our applications involve 10,000 or more dimensions…
  - High-dimensional spaces look different: the probability of random points being close drops quickly as the dimensionality grows.
  - One way to look at it: in large-dimension spaces, random sparse vectors are almost all almost perpendicular.

  Why?

What is clustering?

Clustering: the process of grouping a set of objects into classes of similar objects

– Most common form of *unsupervised learning* (no class labels)

Why cluster?

- Whole corpus analysis/navigation – Enabling better UIs
- For improving recall in search applications
- For better navigation of search results - Effective “user recall” will be higher
- For speeding up vector space retrieval - Faster search

Issues for clustering

- Representation for clustering
  - Document representation
    - Vector space? Normalization?
  - Similarity/distance metric
- Cluster properties
  - Number of clusters?
    - Given or need to figure out?
    - Avoid "trivial" clusters - too large or small (In search UIs, if a cluster is too large, then for navigation purposes you've wasted a user click without narrowing the set of docs)
  - Hard or soft assignments?
- Clustering algorithm properties
  - Completely data driven? Interactive or takes user data?

What makes docs "related"?

- Ideal: semantic similarity.
- Practical: statistical similarity
  - We will use cosine similarity.
  - Docs as vectors
  - For many algorithms, easier to think in terms of a *distance* (rather than <u>similarity</u>) between docs.

Partitional Clustering

Partitioning Algorithms

- Partitioning method: Construct a partition of ***n*** documents into a set of ***K*** clusters
- Given: a set of documents and the number ***K***
- Find: a partition of ***K*** clusters that optimizes the chosen partitioning criterion
  - Globally optimal: exhaustively enumerate all partitions
  - Effective heuristic methods: *K*-means and *K*-medoids algorithms

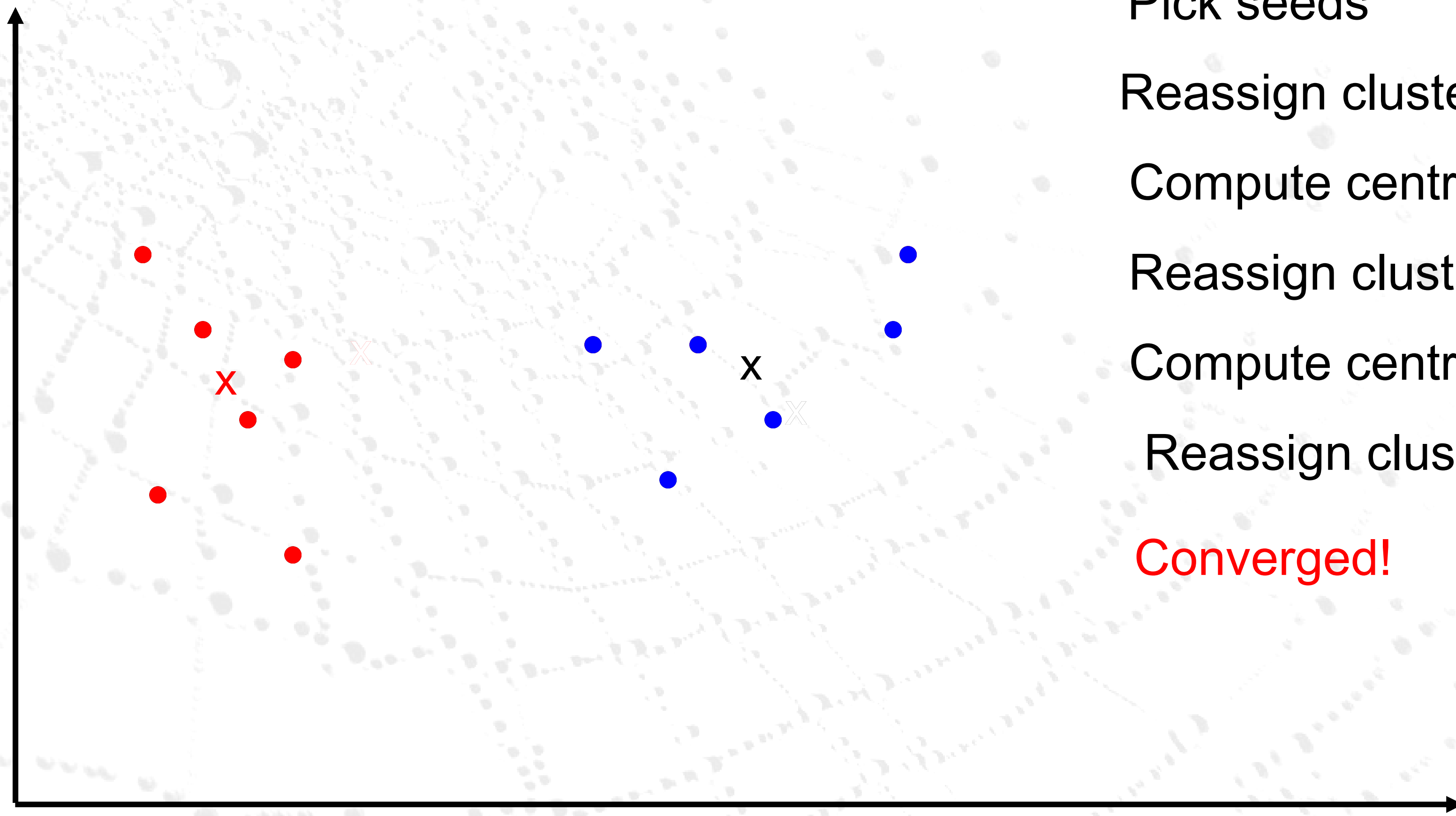
K-Means

- Assumes documents are real-valued vectors.
- Clusters based on *centroids* (aka the *center of gravity* or mean) of points in a cluster, *c*:

$$\vec{\mu}(\mathrm{c}) = \frac{1}{|c|}\sum_{\vec{x}\in c}\vec{x}$$

- Reassignment of instances to clusters is based on distance to the current cluster centroids.
  - (Or one can equivalently phrase it in terms of similarities)

K Means Example (*K*=2)



Pick seeds

Reassign clusters

Compute centroids

Reassign clusters

Compute centroids

Reassign clusters

Converged!

Time Complexity

- Computing distance between two docs is O*(m)* where *m* is the dimensionality of the vectors.
- Reassigning clusters: O*(Kn)* distance computations, or O*(Knm).*
- Computing centroids: Each doc gets added once to some centroid: O*(nm).*
- Assume these two steps are each done once for *I* iterations: O*(IKnm).*

Efficiency: Medoid As Cluster Representative

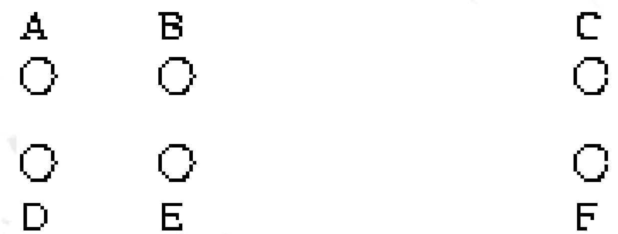
- The centroid does not have to be a document.
- Medoid: A cluster representative that is one of the documents, the document closest to the centroid
- One reason this is useful
  - Consider the representative of a large cluster (>1000 documents)
  - The centroid of this cluster will be a dense vector
  - The medoid of this cluster will be a sparse vector

- Compare: mean/centroid vs. median/medoid
- How does this relate to the curse of dimensionality?

Seed Choice

- Results can vary based on seed selection.
- Some seeds can result in poor convergence rate, or convergence to sub-optimal clusterings.
  - Select good seeds using a heuristic (e.g., doc least similar to any existing mean)
  - Try out multiple starting points
  - Initialize with the results of another method

Example showing sensitivity to seeds



Select B and E as centroids: Converge to {A,B,C} and {D,E,F}

Select D and F, converge to {A,B,D,E} {C,F}

How Many Clusters?

- Number of clusters *K* is given
  - Partition *n* docs into predetermined number of clusters
- Finding the “right” number of clusters is part of the problem
  - Given docs, partition into an “appropriate” number of subsets.
  - E.g., for query results - ideal value of *K* not known up front - though UI may impose limits.

K not specified in advance

- Grade clustering versus a metric.
- Metric must have at least two parts:
  Total Benefit - Total Cost

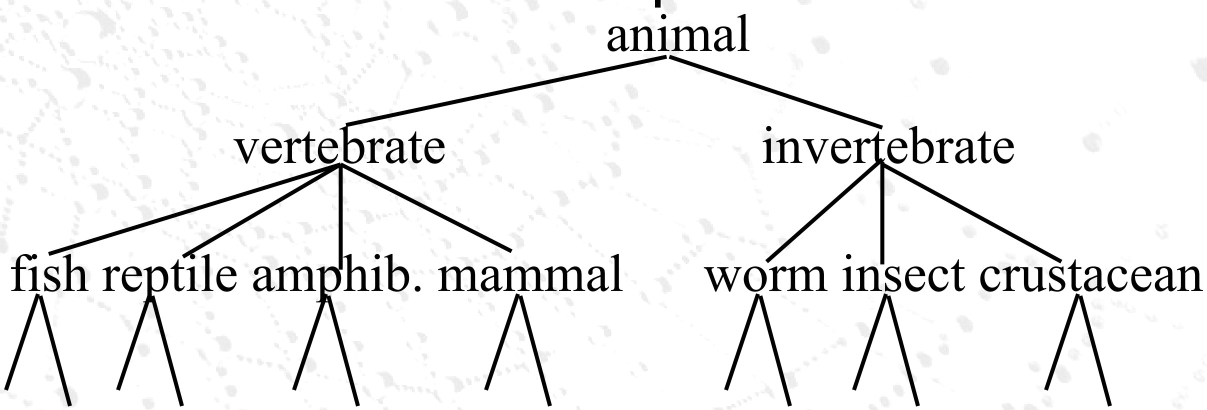
- Benefit (of a doc) = cosine sim to its centroid
- Cost (constant cost c) in creating a new cluster

What happens if one of these criterion is missing?

Hierarchical Clustering

Hierarchical Clustering

- Build a tree-based hierarchical taxonomy (*dendrogram*) from a set of unlabeled examples.



- One option to produce a hierarchical clustering is to recursively apply partitional clustering.
- What are other ways?

Hierarchical Agglomerative Clustering (HAC)

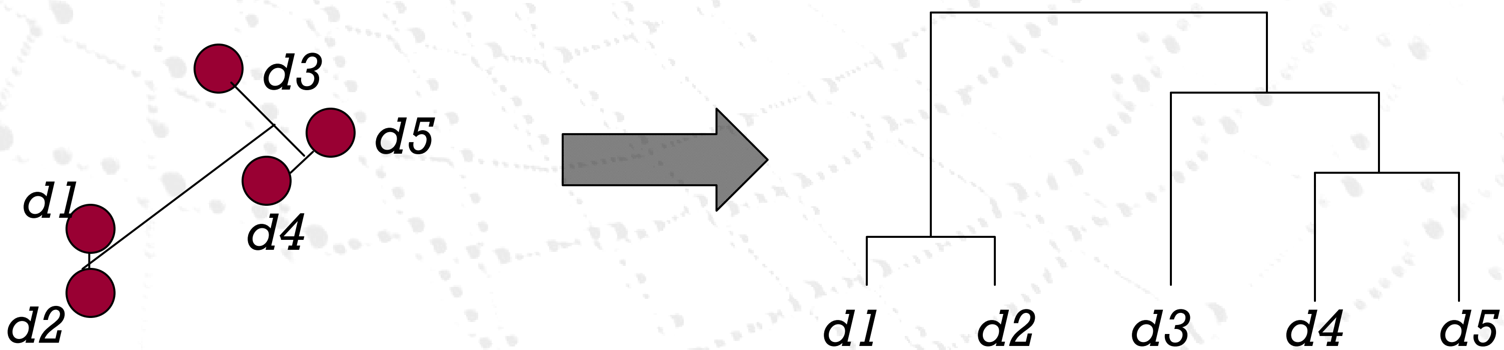
- Agglomerative (bottom-up):
  - Start with each document being a single cluster.
  - Eventually all documents belong to the same cluster.

- Divisive (top-down):
  - Start with all documents belong to the same cluster.
  - Eventually each node forms a cluster on its own.

- Does not require the number of clusters *k* in advance
- Merging/splitting history yields the binary hierarchy
- Assumes a binary symmetric distance function.
- Needs a termination/readout condition - why?
  - The final state in both agglomerative and divisive is no use.

Dendrogram: Document Example

- As clusters *agglomerate*, docs likely to fall into a hierarchy of “topics” or concepts.



Bisecting K-means

Almost identical to X-means as in Nomoto and Matsumoto's summarization approach. How is it different?

- Divisive hierarchical clustering method using K-means

For I=1 to k-1 do {
    Pick a leaf cluster C to split
    For J=1 to ITER do {
        Use K-means to split C into two sub-clusters, C_1 and C_2
        Choose the best of the above splits and make it permanent}
      }
}

- Steinbach *et al.* suggest HAC is better than k-means but Bisecting K-means is better than HAC for their text experiments

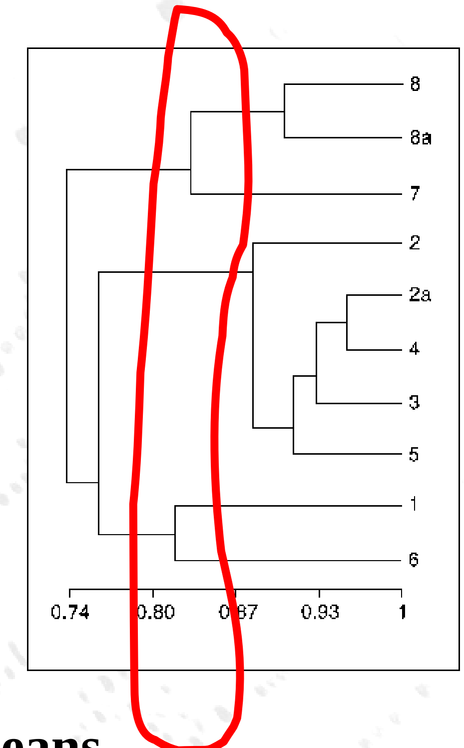
Complexity

- In the first iteration, all HAC methods need to compute similarity of all pairs of n individual instances which is $O(n^2)$.
- In each of the subsequent $n-2$ merging iterations, it must compute the distance between the most recently created cluster and all other existing clusters.
  - Since we can just store unchanged similarities
- In order to maintain an overall $O(n^2)$ performance, computing similarity to each other cluster must be done in constant time.
  - Else $O(n^2 \log n)$ or $O(n^3)$ if done naively

Buckshot Algorithm

- Another way to an efficient implementation:
  - Cluster a sample, then assign the entire set
- Buckshot combines HAC and K-Means clustering.
- First randomly take a sample of instances of size $\sqrt{n}$
- Run group-average HAC on this sample, which takes only O(n) time.
- Use the results of HAC as initial seeds for K-means.
- Overall algorithm is O(n) and avoids problems of bad seed selection. **Uses HAC to bootstrap K-means**

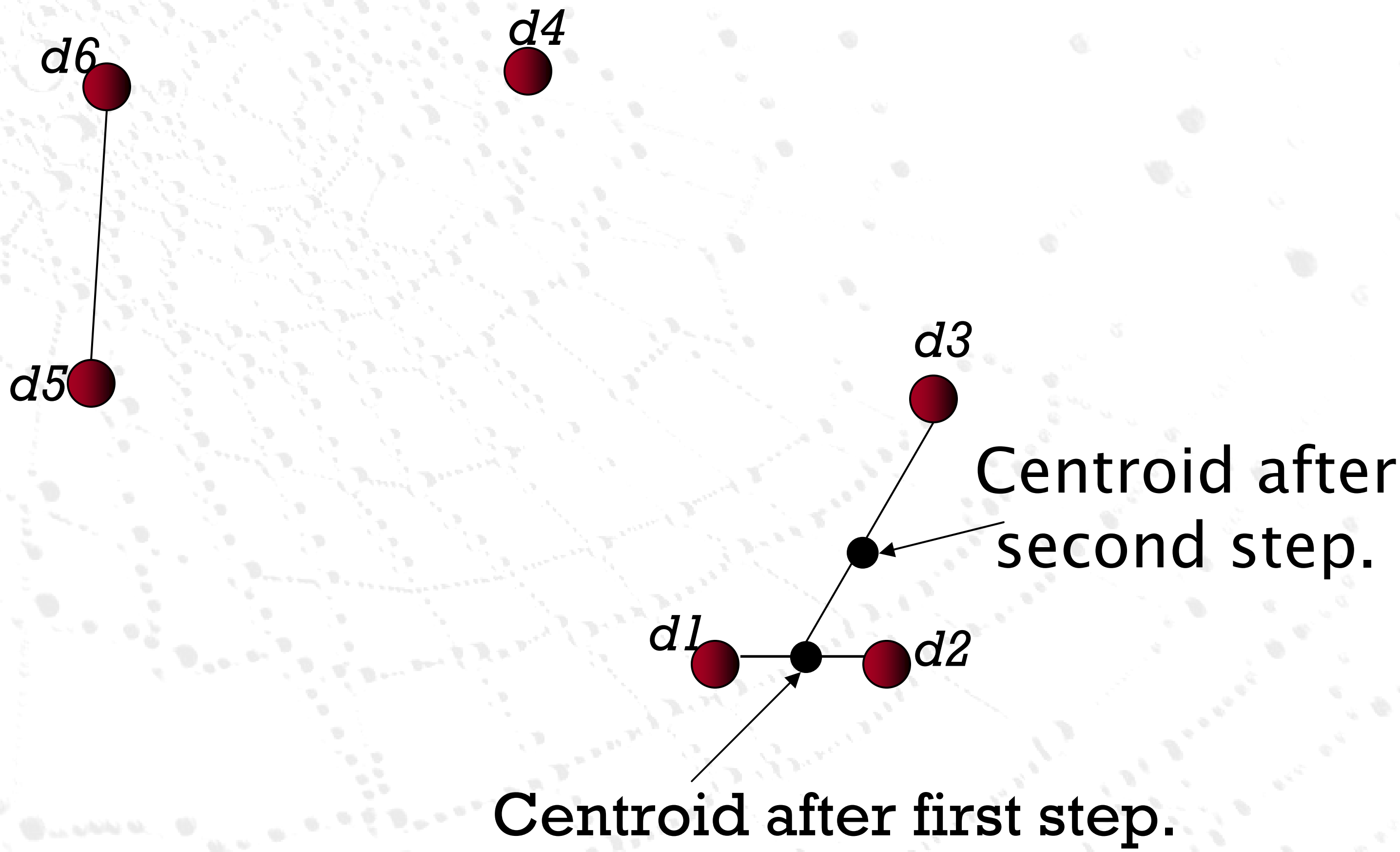
Cut where You have k clusters



Cluster representative

- We want a notion of a representative point in a cluster
- Representative should be some sort of "typical" or central point in the cluster, e.g.,
  - point inducing smallest radii to docs in cluster
  - smallest squared distances, etc.
  - point that is the "average" of all docs in the cluster
    - Centroid or center of gravity

Example: *n*=6, *k*=3, closest pair of centroids



Outliers in centroid computation

- Can ignore outliers when computing centroid.
- What is an outlier?
  – Lots of statistical definitions, e.g.
  – *moment* of point to centroid > M × some cluster *moment*.

Say 10.

Common similarity functions

Many variants to define closest pair of clusters

- "Center of gravity"
  - Clusters whose centroids (centers of gravity) are the most cosine-similar
- Average-link
  - Average cosine between pairs of elements
- Single-link
  - Similarity of the most similar (single-link)
- Complete-link
  - Similarity of the "furthest" points, the least similar

Single vs. Complete Link

- Use max sim pairs:

$$sim(c_i, c_j) = \max_{x \in c_i, y \in c_j} sim(x, y)$$

- Can result in long and thin clusters due to chaining effect.
  - When is it appropriate?

- Use min. sim of pairs:

$$sim(c_i, c_j) = \min_{x \in c_i, y \in c_j} sim(x, y)$$

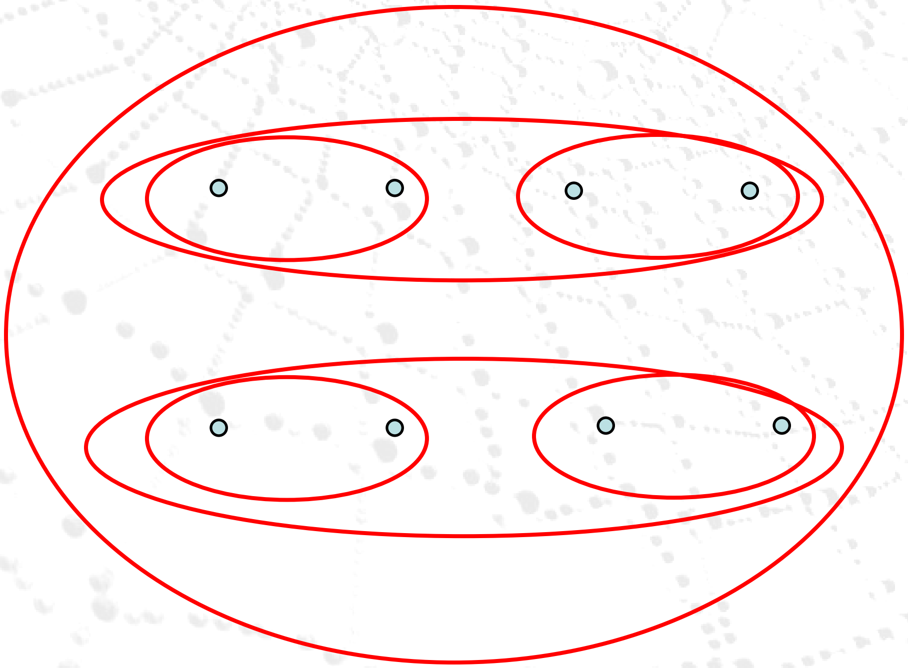
- Makes “tighter,” spherical clusters that are typically preferable.

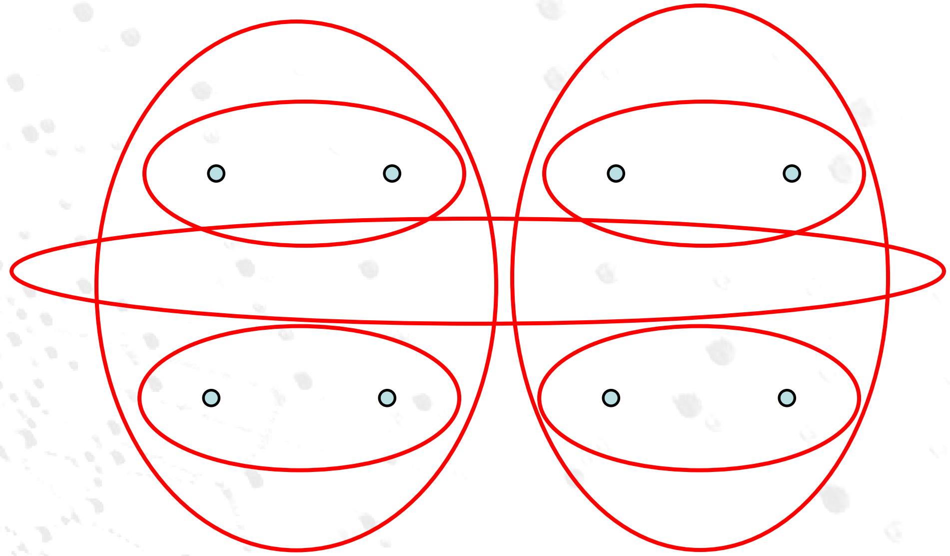
- After merging c_i and c_j, the similarity of the resulting cluster to another cluster, c_k, is:

$$sim((c_i \cup c_j), c_k) = \max(sim(c_i, c_k), sim(c_j, c_k))$$

$$sim((c_i \cup c_j), c_k) = \min(sim(c_i, c_k), sim(c_j, c_k))$$

Complete Link!





Single Link!

Group(wise) Average

- Use average similarity across all pairs within the merged cluster to measure the similarity of two clusters.

$$sim(c_i, c_j) = \frac{1}{\left|c_i \cup c_j\right|\left(\left|c_i \cup c_j\right| - 1\right)} \sum_{\vec{x} \in (c_i \cup c_j)} \sum_{\vec{y} \in (c_i \cup c_j) : \vec{y} \neq \vec{x}} sim(\vec{x}, \vec{y})$$

- Compromise between single and complete link.

- Two options:
  - Averaged across all ordered pairs in the merged cluster
  - Averaged over all pairs *between* the two original clusters

- Some previous work has used one of these options; some the other. No clear difference in efficacy

Computing Group Average Similarity

- Assume cosine similarity and normalized vectors with unit length.
- Always maintain sum of vectors in each cluster.

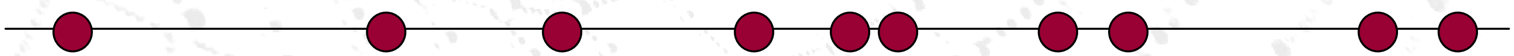
$$\vec{s}(c_j) = \sum_{\vec{x} \in c_j} \vec{x}$$

- Compute similarity of clusters in constant time:

$$sim(c_i, c_j) = \frac{(\vec{s}(c_i) + \vec{s}(c_j)) \bullet (\vec{s}(c_i) + \vec{s}(c_j)) - (|c_i| + |c_j|)}{(|c_i| + |c_j|)(|c_i| + |c_j| - 1)}$$

Quick Question

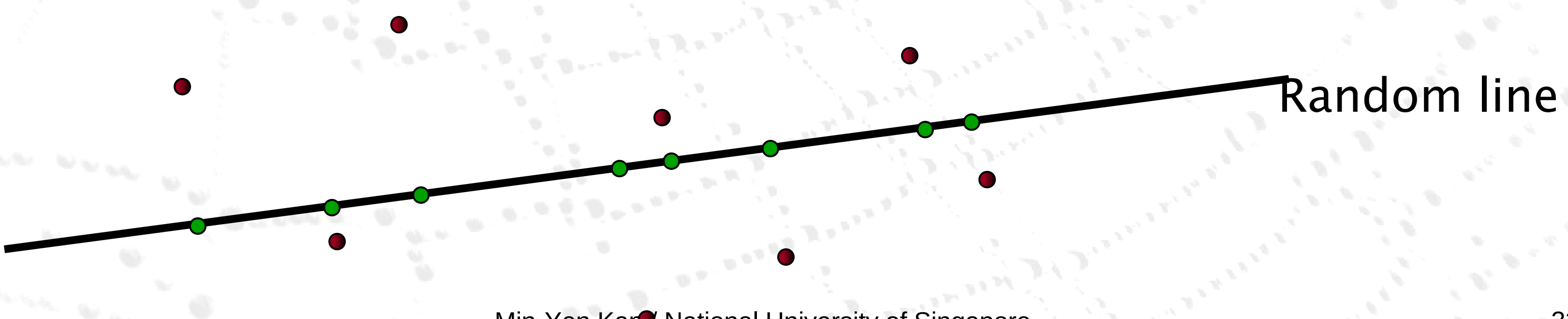
- Consider agglomerative clustering on *n* points on a line. Explain how you could avoid n^3 distance computations - how many will your scheme use?



This idea is actually employed in topical (text) segmentation!

Efficiency by approximation

- In standard algorithm, must find closest pair of centroids at each step
- Approximation: instead, find nearly closest pair
  - use some data structure that makes this approximation easier to maintain
  - simplistic example: maintain closest pair based on distances in projection on a random line



Multi-lingual docs

- E.g., Canadian government docs.
- Every doc in English and equivalent French
  - Must cluster by concepts rather than language
- Simplest: pad docs in one language with dictionary equivalents in the other
  - thus each doc has a representation in both languages
- Axes are terms in both languages

Feature selection

Which terms to use as axes for vector space?
Discussed previously last week

- Better is to use highest weight *mid-frequency* words – the most discriminating terms
- Pseudo-linguistic heuristics, e.g.,
  - drop stop-words
  - stemming/lemmatization
  - use only nouns/noun phrases
- Good clustering should figure out some of these

Labeling

- After clustering algorithm finds clusters - how can they be useful to the end user?
- Need pithy label for each cluster
  - In search results, say “Animal” or “Car” in the ***jaguar*** example.
  - In topic trees (Yahoo), need navigational cues.
    - Often done by hand, a posteriori.

How to Label Clusters

Actually a summarization task!

- Show titles of typical documents
  - Titles are easy to scan
  - Authors create them for quick scanning!
  - But you can only show a few titles which may not fully represent cluster
- Show words/phrases prominent in cluster
  - More likely to fully represent cluster
  - Use distinguishing words/phrases
    - Differential labeling, like diversity in summarization
  - But harder to scan

Labeling

- Common heuristics - list 5-10 most frequent terms in the centroid vector.
  - Drop stop-words; stem.
- Differential labeling by frequent terms
  - Within a collection “Computers”, clusters all have the word ***computer*** as frequent term.
  - Discriminant analysis of centroids.
- Perhaps better: distinctive noun phrase
  - Such work also goes by the name *keyphrase extraction*

Clustering Evaluation

Partitional vs. Hierarchical

Internal vs. External

Evaluation of clustering

- Most measures focus on computational efficiency
  - Time and space
- For application of clustering to search:
  - Measure retrieval effectiveness

What Is A Good Clustering?

- Internal criterion: A good clustering will produce high quality clusters in which:
  - the intra-class (that is, intra-cluster) similarity is high
  - the inter-class similarity is low
  - The measured quality of a clustering depends on both the document representation and the similarity measure used
- Similar to benefit in computing number of clusters – what wasn't considered?

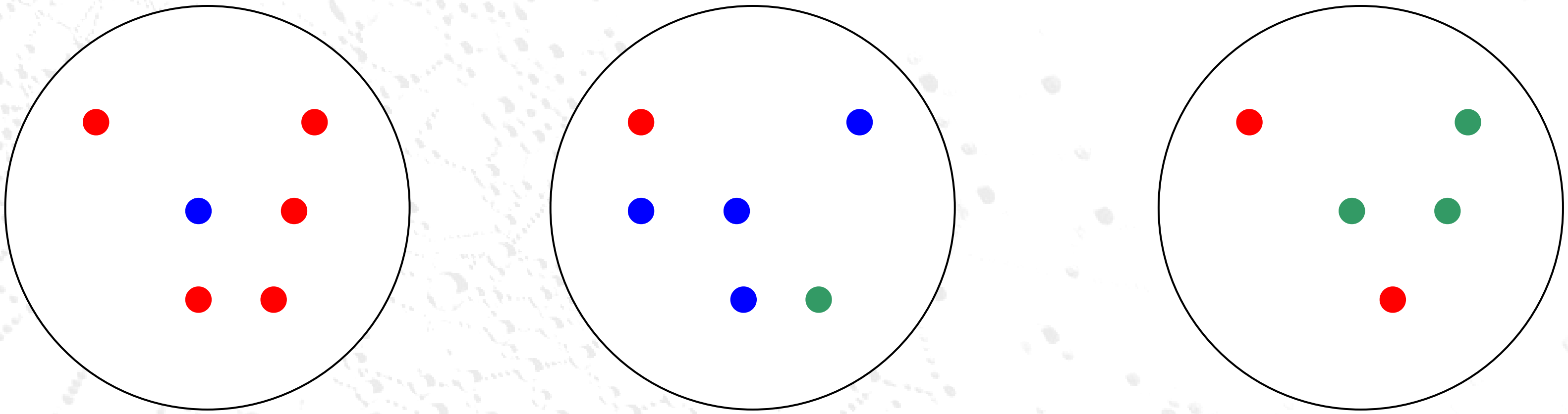
Cluster Quality Evaluation

- Simple measure: purity, the ratio between the dominant class in the cluster π_i and the size of cluster ω_i

$$Purity\ (\omega_i) = \frac{1}{n_i} \max_j (n_{ij}) \quad j \in C$$

- Others are entropy of classes in clusters (or mutual information between classes and clusters)

Purity example



Cluster I Cluster II Cluster III

Cluster I: Purity = 1/6 (max(5, 1, 0)) = 5/6

Cluster II: Purity = 1/6 (max(1, 4, 1)) = 4/6

Cluster III: Purity = 1/5 (max(2, 0, 3)) = 3/5

Rand Index

Number of points	Same Cluster in clustering	Different Clusters in clustering
Same class in ground truth	A	C
Different classes in ground truth	B	D

Rand index: symmetric version

Number of points	Same Cluster in clustering	Different Clusters in clustering
Same class in ground truth	A	C
Different classes in ground truth	B	D

$$RI = \frac{A + D}{A + B + C + D}$$

Compare with standard Precision and Recall. What's different?

$$P = \frac{A}{A + B}$$

$$R = \frac{A}{A + C}$$

Hierarchical Evaluation: User inspection

- Induce a set of clusters or a navigation tree
- Have subject matter experts evaluate the results
  - Subjective, may have more than one good tree
- Often combined with search results clustering
- Not clear how reproducible across tests.
- Expensive / time-consuming

Extrinsic evaluation

- Anything - including clustering - is only as good as the economic utility it provides
- For clustering: net economic gain produced by an approach (vs. another approach)
- Strive for a concrete optimization problem
- Examples
  - recommendation systems
  - clock time for interactive search

Resources

- Scatter/Gather: A Cluster-based Approach to Browsing Large Document Collections (1992)
  - Cutting, Karger, Pedersen, Tukey
  - http://citeseer.ist.psu.edu/cutting92scattergather.html
- Data Clustering: A Review (1999)
  - Jain/Murty/Flynn
  - http://citeseer.ist.psu.edu/jain99data.html
- A Comparison of Document Clustering Techniques
  - Michael Steinbach, George Karypis and Vipin Kumar. TextMining Workshop. KDD. 2000.
- Initialization of iterative refinement clustering algorithms. (1998)
  - Fayyad, Reina, and Bradley
  - http://citeseer.ist.psu.edu/fayyad98initialization.html
- Scaling Clustering Algorithms to Large Databases (1998)
  - Bradley, Fayyad, and Reina
  - http://citeseer.ist.psu.edu/bradley98scaling.html