Text Processing on the Web

Week 5

Link Analysis Ranking

The material for these slides are borrowed heavily from the precursor of this course by Tat-Seng Chua
as well as slides from the accompanying recommended texts Baldi et al. and Manning et al.

Recap

- Synonymy and Polysemy affect all standard IR models – not just limited to VSM
- We want to instead model latent topics
  - SVD factors the term-document matrix into orthogonal eigenvectors (“topics”), automatically ranked by salience (“eigenvalue magnitude”).
  - LSA does SVD and then drops low order topics to create approximation
  - pLSA does this by taking the unigram LM and injecting a latent variable, k (for k topics)

Outline

- The classics:
  - Page Rank
  - Hubs and Authorities
- Adaptations to the Models
  - Topic Sensitive PageRank
  - SALSA

Citation Networks

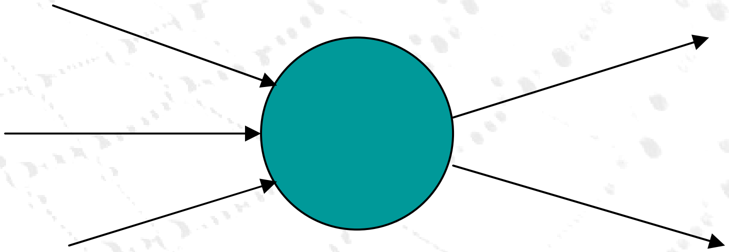
- Pioneered by Garfield 1972 to answer questions on impact

- Introduced Impact Factor
  - C = citations to articles in a journal
  - N = total number of articles in a journal

  - Impact Factor = C/N
    (Normalized in-degree of a journal)

Query-independent ordering

- How does this translate to the web?
  - Have a graph, not a DAG
- Using link counts as simple measures of prestige
  - number of inlinks (3)



Algorithm

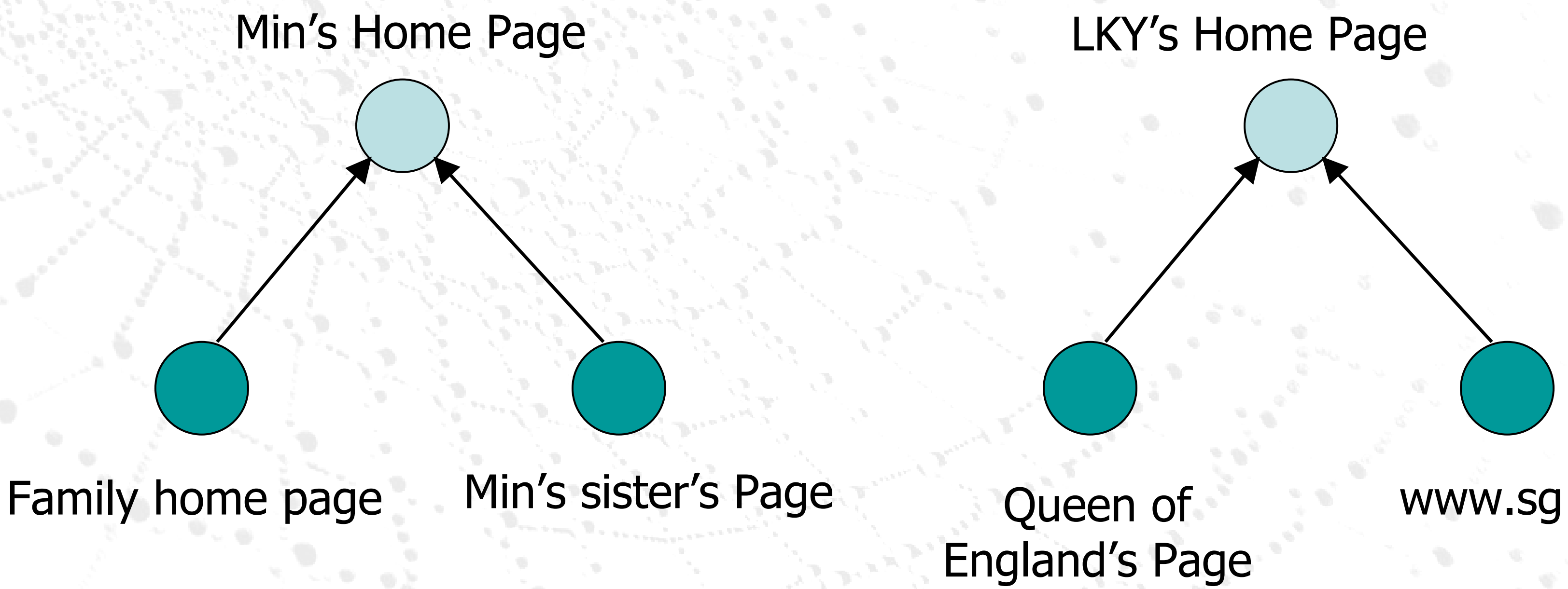
1. Retrieve all pages meeting the text query (say ***venture capital***), perhaps by using Boolean model

2. Order these by link popularity

Exercise: How do you spam each of the following heuristics so your page gets a high score?

- score = # in-links

Link Counts



Linked by 2
Unimportant pages

Linked by 2
Important Pages

Definition of PageRank

- The importance of a page is given by the importance of the pages that link to it.

$$x_i = \sum_{j \in B_i} \frac{1}{N_j} x_j$$

importance of page *i*

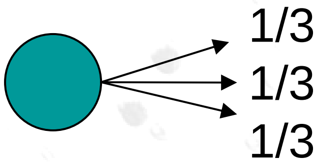
importance of page *j*

pages *j* that link to page *i*

number of outlinks from page *j*

Pagerank scoring

- Imagine a browser doing a random walk on web pages:



  - Start at a random page
  - At each step, follow one of the *n* links on that page, each with 1/*n* probability
- Do this repeatedly. Use the “long-term visit rate” as the page’s score

Markov chains

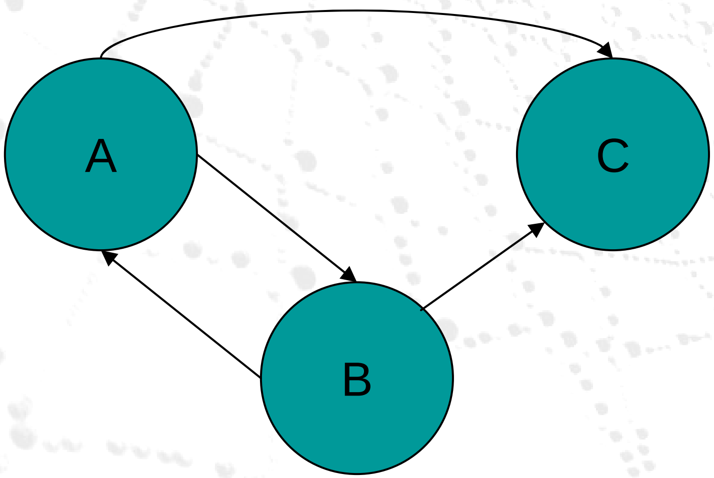
A Markov chain consists of n <u>states</u>, plus an $n \times n$ <u>transition probability matrix</u> A.

- At each step, we are in exactly one of the states.
- For $1 \leq i,k \leq n$, the matrix entry A_{ik} tells us the probability of k being the next state, given we are currently in state i.
- **Memorylessness property**: The next state depends only at the current state (first order MC)

$A_{ik} > 0$ is OK.

Markov chains

- Clearly, for all i, $\sum_{k=1}^{n} A_{ik} = 1.$
- Markov chains are abstractions of random walks

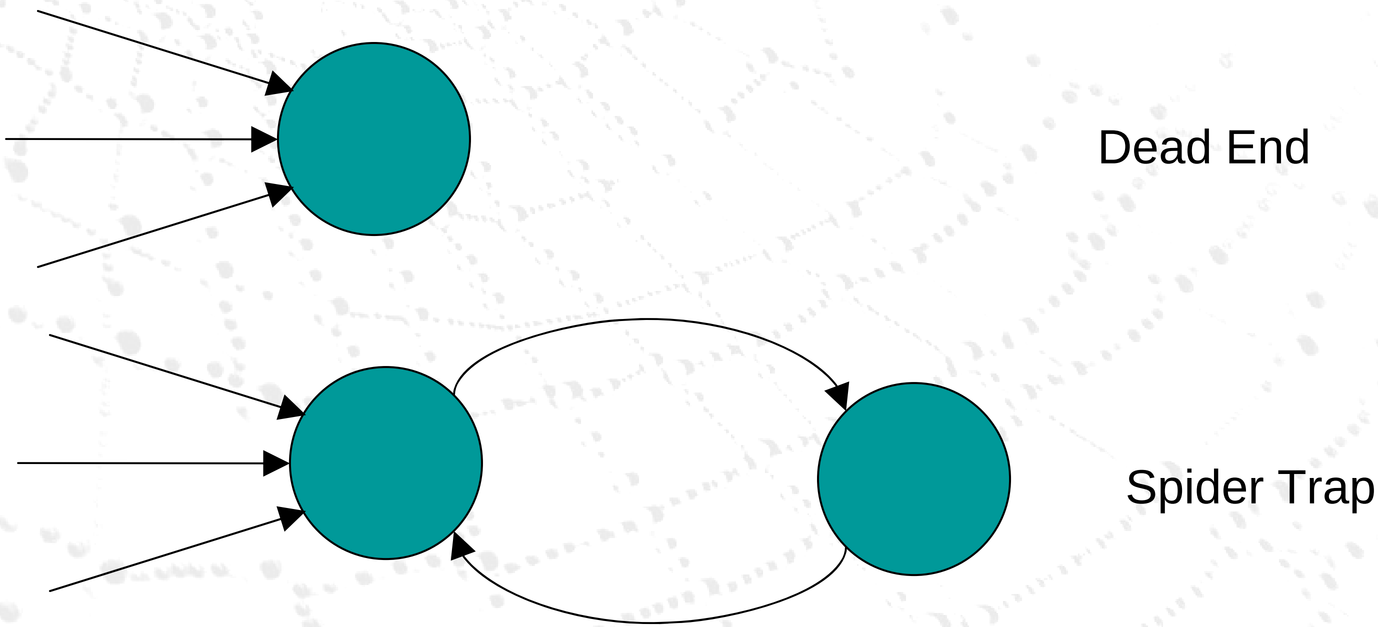


Try this: Calculate the matrix A_{ik} using 1/n possibility

A_{ik}:	**A**	**B**	**C**
A			
B			
C			

Not quite enough

- The web is full of dead ends.
  - What sites have dead ends?
  - Our random walk can get stuck.



Teleporting

- At each step, with probability 10%, teleport to a random web page
- With remaining probability (90%), follow a random link on the page
  - If a dead-end, stay put in this case

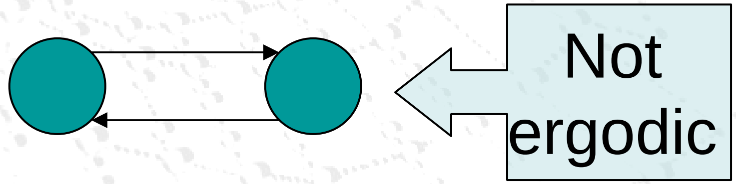
Teleport!

Follow!

$$\vec{rank} = (1-a)A \times \vec{rank} + \alpha \left[\frac{1}{N}\right] N \times 1$$

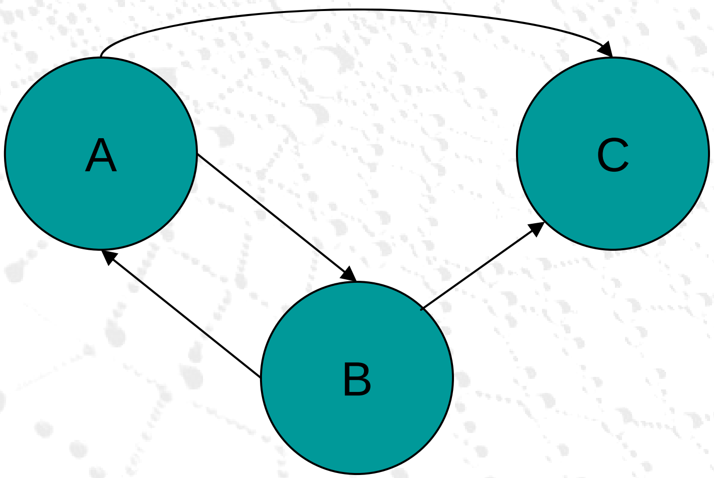
Ergodic Markov chains

- A Markov chain is ergodic if
  - you have a path from any state to any other
  - you can be in any state at every time step, with non-zero probability



  - With teleportation, our Markov chain is ergodic

Markov chains (2^{nd} Try)



Try this: Calculate the matrix A_{ik} using a 10% chance of teleportation

A_{ik}:

	A	**B**	**C**
A			
B			
C			

Probability vectors

- A probability (row) vector **x** = $(x_1, \ldots x_n)$ tells us where the walk is at any point
- E.g., (000…1…000) means we're in state *i*.

  1 i n

More generally, the vector **x** = $(x_1, \ldots x_n)$ means the walk is in state *i* with probability x_i.

$$\sum_{i=1}^{n} x_i = 1.$$

Change in probability vector

- If the probability vector is $\mathbf{x} = (x_1, \ldots x_n)$ at this step, what is it at the next step?
- Recall that row *i* of the transition prob. Matrix **A** tells us where we go next from state *i*.
- So from **x**, our next state is distributed as **xA**.

Pagerank algorithm

- Regardless of where we start, we eventually reach the steady state a
  - Start with any distribution (say x=(10…0))
  - After one step, we're at xA
  - After two steps at xA2 , then xA3 and so on.
  - "Eventually" means for "large" k, xAk = a
- Algorithm: multiply x by increasing powers of A until the product looks stable

Steady State

- For any ergodic Markov chain, there is a unique long-term visit rate for each state
  - Over a long period, we’ll visit each state in proportion to this rate
  - It doesn’t matter where we start

Eigenvector formulation

- The flow equations can be written

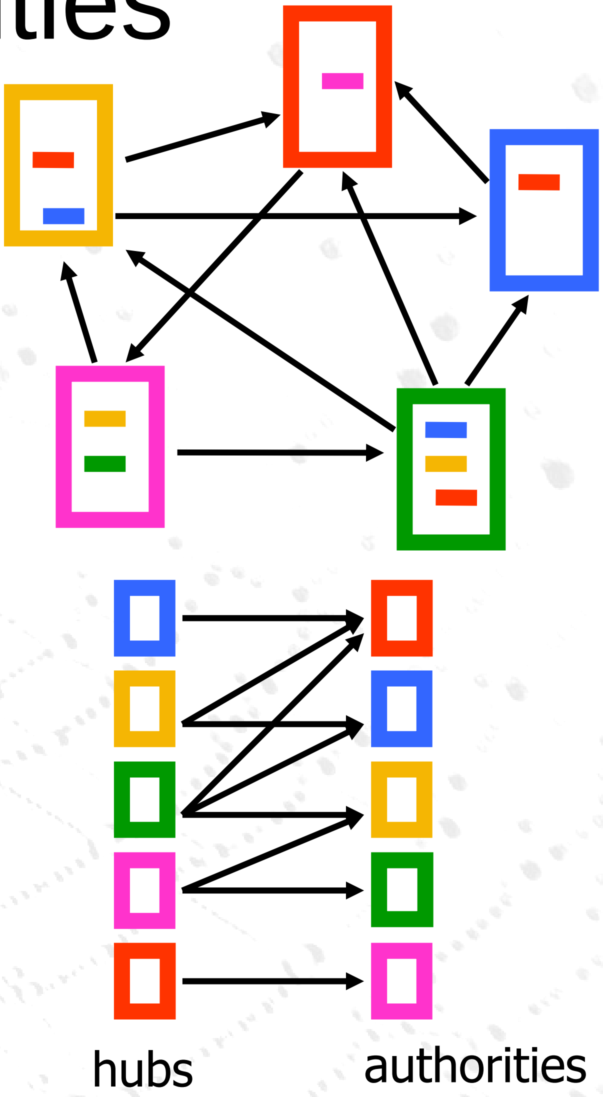
$$\mathbf{r} = \mathbf{A}\mathbf{r}$$

- So the rank vector is an eigenvector of the adjacency matrix
  - In fact, it's the first or principal eigenvector, with corresponding eigenvalue 1

Pagerank summary

- Pre-processing:
  - Given graph of links, build matrix **A**
  - From it compute **a**
  - The pagerank a_i is a scaled number between 0 and 1
- Query processing:
  - Retrieve pages meeting query
  - Rank them by their pagerank
  - Order is query-*independent*

Hubs and Authorities



- Authority is not necessarily transferred directly between authorities
- Pages have double identity
  - hub identity
  - authority identity
- Good hubs point to good authorities
- Good authorities are pointed by good hubs

High-level scheme

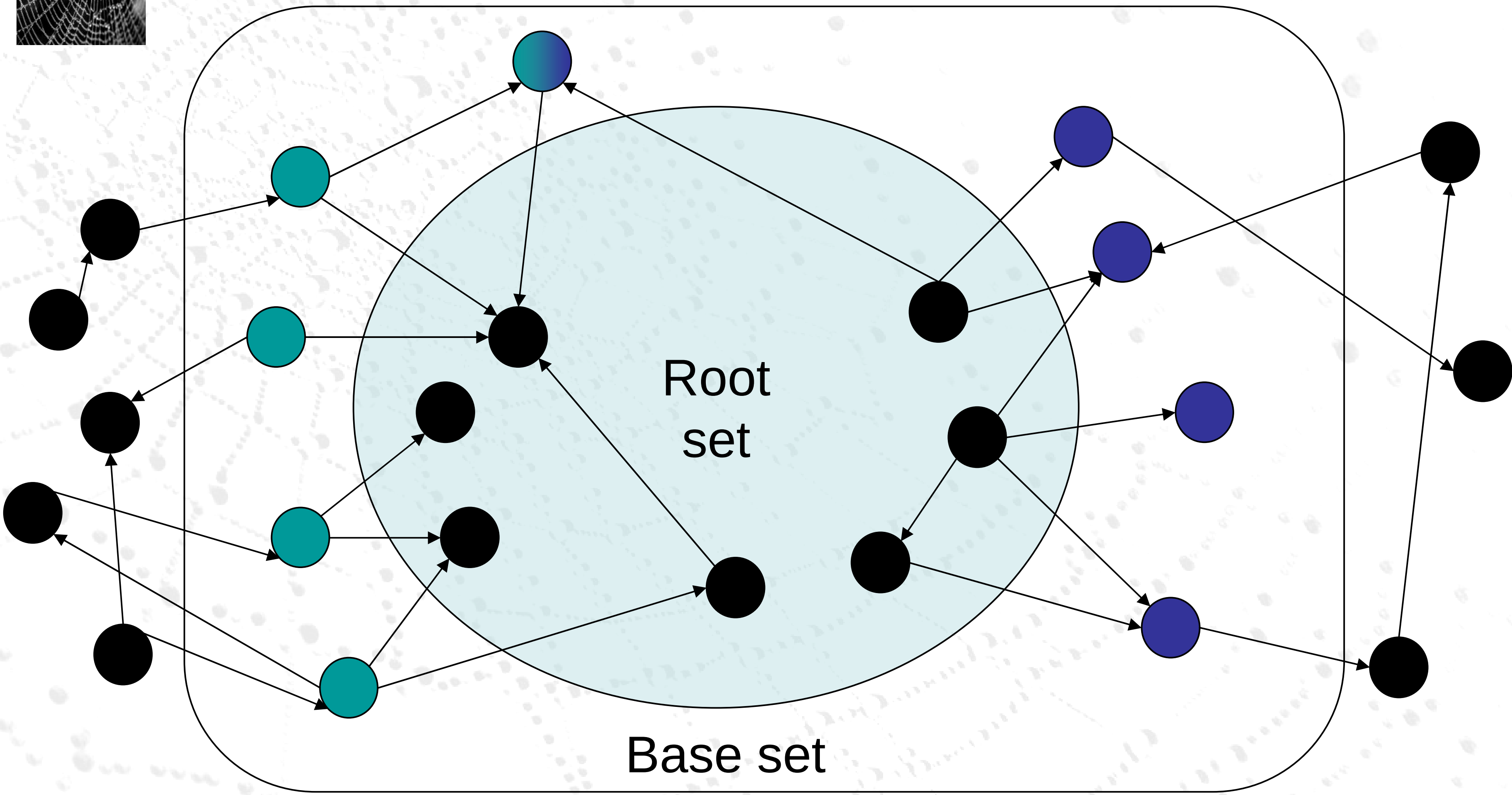
- Extract from the web a base set of pages that *could* be good hubs or authorities.

- From these, identify a small set of top hub and authority pages

  → iterative algorithm

Base set

1. Given text query (say **university**), use a text index to get all pages containing **university**.
   – Call this the root set of pages
2. Add in any page that either:
   – **points** to a page in the root set, or
   – **is pointed** to by a page in the root set
3. Call this the base set



Root
set
Base set

Assembling the base set

- Root set typically 200-1000 nodes.
- Base set may have up to 5000 nodes.
- How do you find the base set nodes?
  - Follow out-links by parsing root set pages.
  - Get in-links (and out-links) from a *connectivity server.*

Distilling hubs and authorities

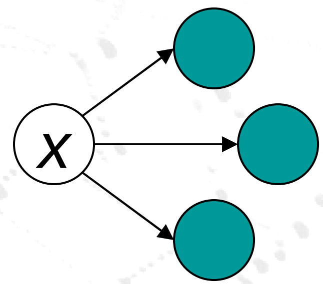
1. Compute, for each page *x* in the base set, a hub score $h(x)$ and an authority score $a(x)$.
2. Initialize: for all *x*, $h(x) \leftarrow 1$; $a(x) \leftarrow 1$;
3. Iteratively update all $h(x)$, $a(x)$;
4. After iterations:
   - highest $h()$ scores are hubs
   - highest $a()$ scores are authorities

Key

Iterative update

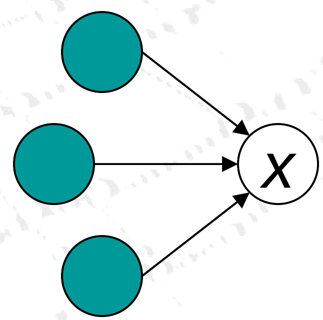
- Repeat the following updates, for all *x*:

$$h(x) \leftarrow \sum_{x \mapsto y} a(y)$$



$$h_t = A a_{t-1}$$

$$a(x) \leftarrow \sum_{y \mapsto x} h(y)$$



$$a_t = A^T h_{t-1}$$

HITS and eigenvectors

- The HITS algorithm is a power-method eigenvector computation
  - in vector terms $a_t = A^T h_{t-1}$ and $h_t = A a_{t-1}$
  - so $a_t = A^T A a_{t-1}$ and $h_t = A A^T h_{t-1}$
  - The authority weight vector a is the eigenvector of $A^T A$ and the hub weight vector h is the eigenvector of $A A^T$
  - Why do we need normalization?
- The vectors a and h are singular vectors of the matrix A

Singular Value Decomposition

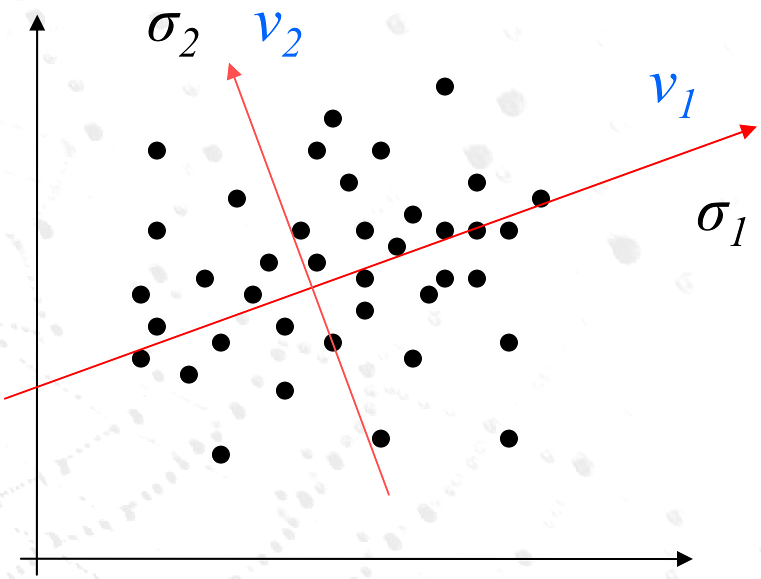
$$A = U \quad \Sigma \quad V^T = \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \cdots & \vec{u}_r \end{bmatrix} \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_r \end{bmatrix} \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vdots \\ \vec{v}_r \end{bmatrix}$$

$$[n \times r] \; [r \times r] \; [r \times n]$$

- **r** : rank of matrix A
- $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r$: singular values (square roots of eigenvalues AA^T, A^TA)
- $\vec{u}_1, \vec{u}_2, \cdots, \vec{u}_r$: left singular vectors (eigenvectors of AA^T)
- $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_r$: right singular vectors (eigenvectors of A^TA)
- $$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T + \cdots + \sigma_r \vec{u}_r \vec{v}_r^T$$

Singular Value Decomposition

- Linear trend **v** in matrix A:
  - the tendency of the row vectors of A to align with vector **v**
  - strength of the linear trend: A**v**
- SVD discovers the linear trends in the data
- **u_i , v_i** : the i-th strongest linear trends
- σ_i : the strength of the i-th strongest linear trend



- HITS discovers the strongest linear trend in the authority space

How many iterations?

- Relative values of scores will converge after a few iterations
- We only require the relative order of the *h()* and *a()* scores - not their absolute values
- In practice, ~5 iterations needed

Things to think about

- Use *only* link analysis after base set assembled
  - iterative scoring is query-independent
- Iterative computation after text index retrieval - significant overhead

Things to think about

- A pagerank score is a global score. Can there be a fusion between H&A (which are query sensitive) and pagerank?

- How does the selection of the base set influence computation of H & As?
- Can we embed the computation of H & A during the standard VS retrieval algorithm?
- How can you update PageRank without recomputing the whole thing from scratch?
- What's the eigenvector relationship between HITS' authority and PageRank?

Advanced link structure methods

Topic-Sensitive PageRank

- Basic idea:
  1. Identify topic that might be interesting for the user (e.g. via classification of the query, eval. of context, ...)
  2. Use pre-calculated, topic-sensitive PageRank

- Topic specific PageRank $rank_{jd}$:
- Now: Topics c_1, ..., c_n,
  - They used 16 top-level categories from the ODP
- Topic dependent weighting $(1/|T_i|)$
- Advantage: Can be calculated in advance

Offline PageRank Vector Computation

- Play around with Teleportation Rate

$$\vec{rank} = (1-a)A \times \vec{rank} + \alpha\left[\frac{1}{N}\right]N \times 1$$

- Don't jump to a random page; jump to a topic page!

$$v_{ij} = \begin{cases} 1/|T_j| & i \in T_j \\ 0 & i \notin T_j \end{cases}$$

T_j=set of pages relevant to a topic

Run-time TSPageRank (cont.)

- Question: Which one to select during run time?
- Idea: Classification of query q given by the user
- Extension: Consider context q' of query q
  - e.g. surrounding text if query was entered via highlighting

- Calculation using a unigram language model:

$$P(c_j|q') = \frac{P(c_j) \cdot P(q'|c_j)}{P(q')} \propto P(c_j) \cdot \prod_i P(q'_i|c_j)$$

Topic-Sensitive PageRank

- Weighted summation of all topic specific PageRanks for one document
  - Weights: Dependent on probability of a particular topic being relevant given the query q
  - Definition: Query-Sensitive Importance Score s_{qd}

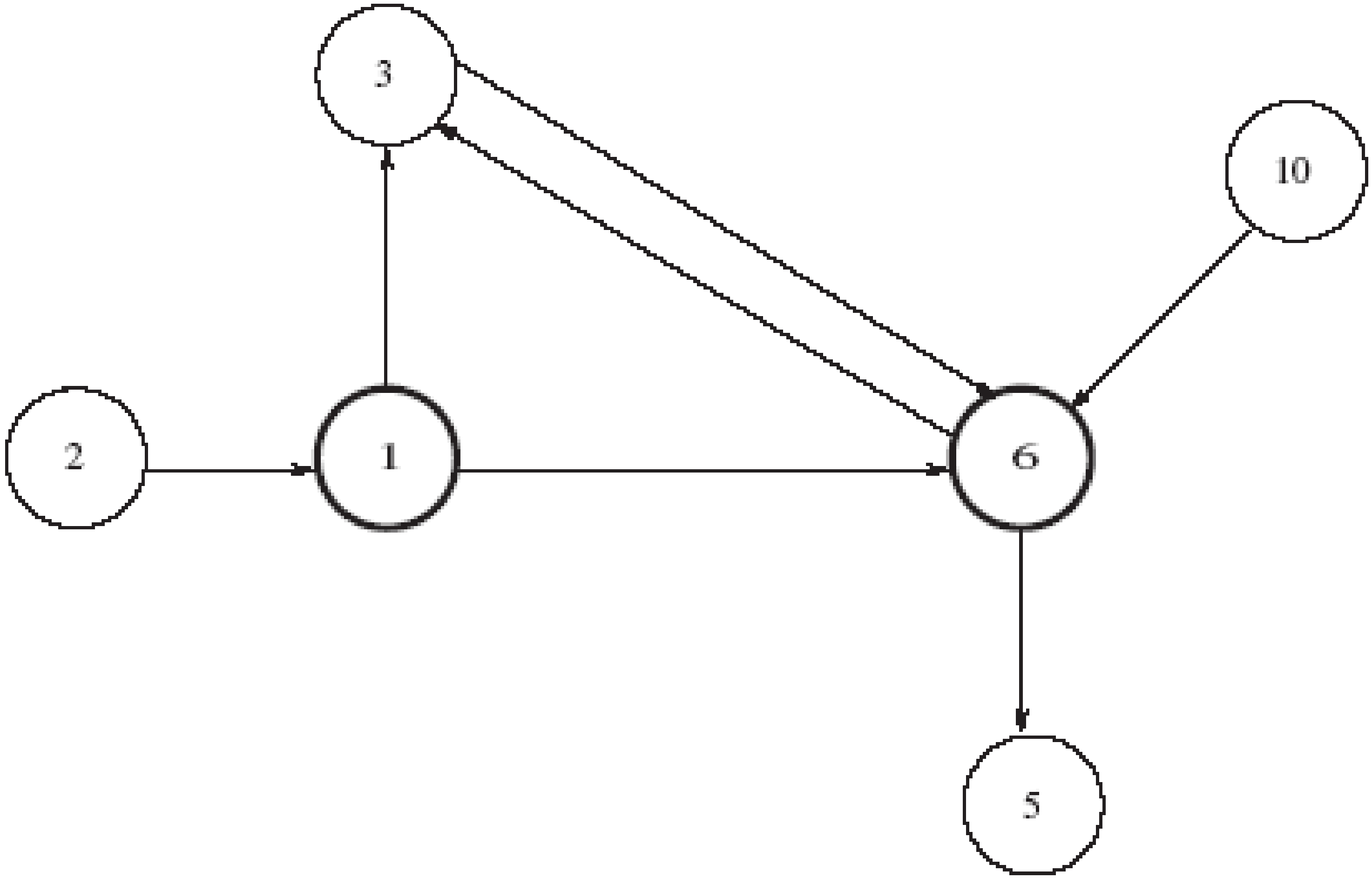
$$s_{qd} = \sum_{j} P(c_j | q') \cdot rank_{jd}$$

- Disadvantages:
  - Fixed set of topics
  - Depends on training set

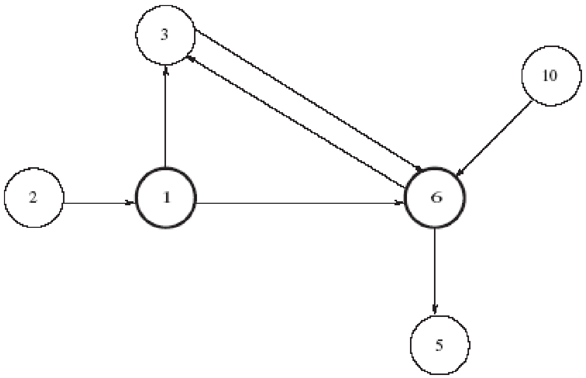
SALSA

- Similarities
  - uses authority and hub score
  - creates a neighborhood graph using authority and hub pages and links
- Differences
  - creates bipartite graph of the authority and hub pages in the neighborhood graph.
  - Each page may be located in both sets

Neighborhood Graph N

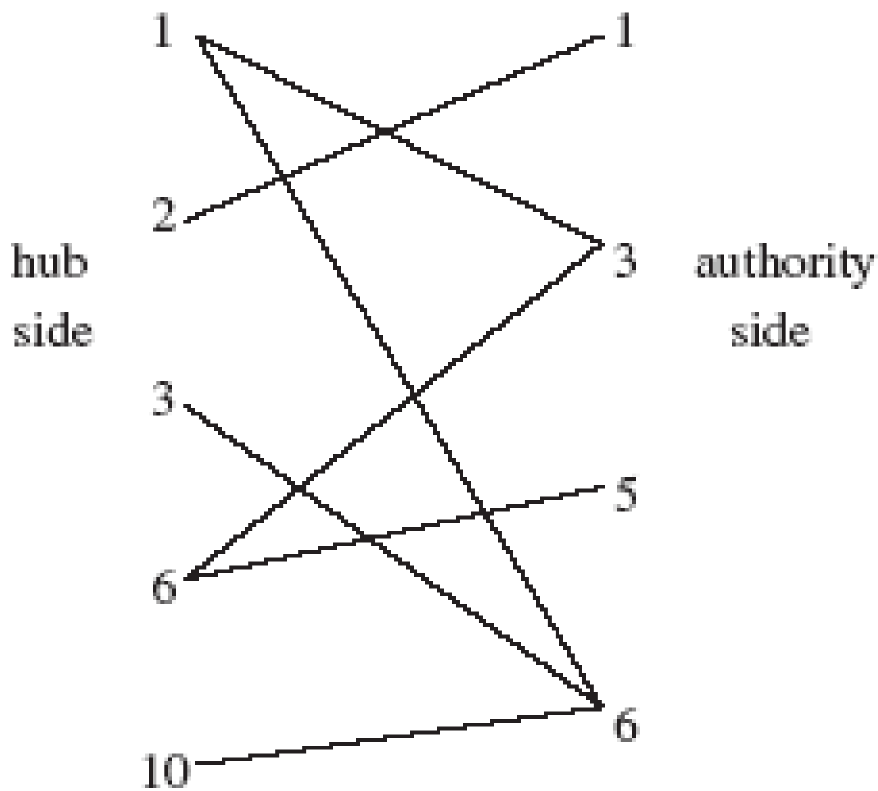


Bipartite Graph G of Neighborhood Graph N



$V_h = \{1, 2, 3, 6, 10\},$

$V_a = \{1, 3, 5, 6\}.$



Markov Chains

- Two matrices formed from bipartite graph G
- A hub Markov chain with matrix H'
  - Follow forward link, then backward

$$h_{uv} = \sum_{w:(u,w)\in E,(v,w)\in E} \frac{1}{\deg(u_h)} \frac{1}{\deg(w_a)}$$

- An authority Markov chain with matrix A'
  - Follow backward link, then forward

$$a_{uv} = \sum_{w:(w,u)\in E,(w,v)\in E} \frac{1}{\deg(v_a)} \frac{1}{\deg(w_h)}$$

- Steps end up on same side of the bipartite graph

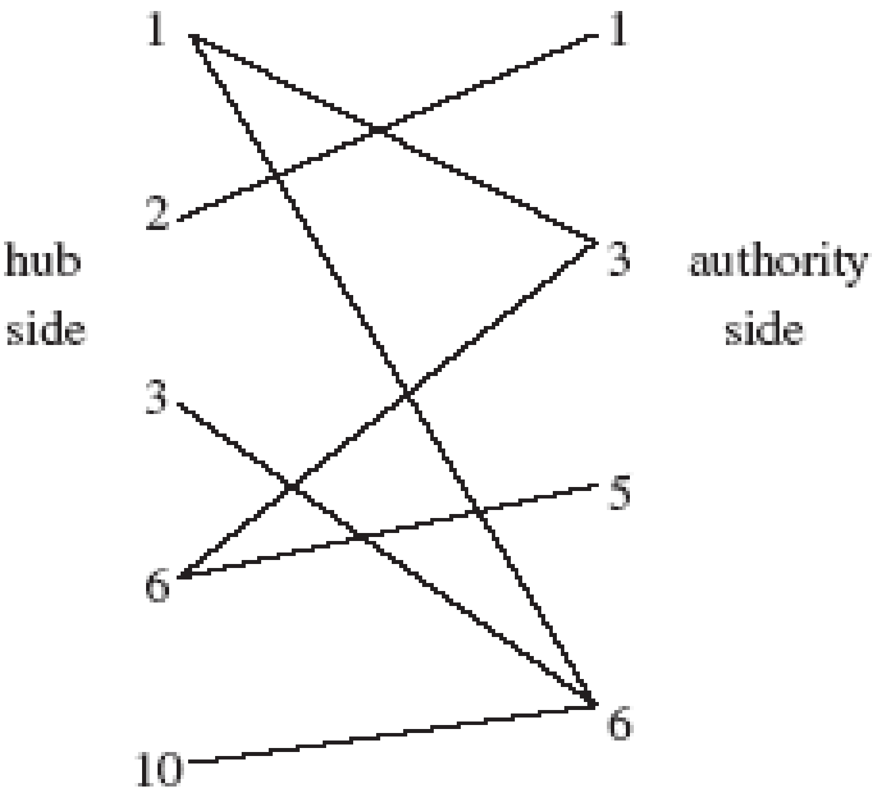
Completing SALSA

- Use same power method as in previous methods to compute principal eigenvector
  - Caveat: have to deal with disconnected components!

{1},{2}

{1,3,6,10},{3,5,6}

  - Link them together in some way



Where does SALSA fit in?

- Matrices H’ and A’ can be derived from the adjacency matrix used in both methods
- HITS used unweighted matrix
- PageRank uses a row weighted version of matrix A
- SALSA uses both row and column weighting

Why do we say this?

Strengths and Weaknesses

- Not affected as much by topic drift like HITS
- Handles Tightly knit communities better (spammers)
- It gives authority and hub scores.
- Query dependence

Summary

- Ranking needs to account for the graph structure
- Directed structure of the web leads to dichotomy in treatment (giving/receiving ends)
- Global models (propagation) and local models (at run time)
- Linear Algebra strikes again: SVD and Eigenvectors

Still more work to do here:

- Not yet convincingly coupled with standard retrieval models; “content” not really factored in

References

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