CS224d
Deep Learning
for Natural Language Processing

Lecture 3:
More Word Vectors

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Refresher: The simple word2vec model

- Main cost function J:

$$J(\theta) = \frac{1}{T}\sum_{t=1}^{T}\sum_{-m\leq j\leq m, j\neq 0} \log p(w_{t+j}|w_t)$$

- With probabilities defined as: $p(o|c) = \frac{\exp\left(u_o^T v_c\right)}{\sum_{w=1}^{W}\exp\left(u_w^T v_c\right)}$

- We derived the gradient for the internal vectors v_c

Calculating all gradients!

- We went through gradients for each center vector v in a window
- We also need gradients for outside vectors u
- Derive at home!

- Generally in each window we will compute updates for all parameters that are being used in that window.
- For example window size c = 1, sentence:
  "I like learning ."
- First window computes gradients for:
  - internal vector v_{like} and external vectors u_I and $u_{learning}$
- Next window in that sentence?

Compute all vector gradients!

- We often define the set of ALL parameters in a model in terms of one long vector θ
- In our case with d-dimensional vectors and V many words:

$$\theta = \begin{bmatrix} v_{aardvark} \\ v_a \\ \vdots \\ v_{zebra} \\ u_{aardvark} \\ u_a \\ \vdots \\ u_{zebra} \end{bmatrix} \in \mathbb{R}^{2dV}$$

Gradient Descent

- To minimize $J(\theta)$ over the full batch (the entire training data) would require us to compute gradients for all windows

- Updates would be for each element of μ :

$$\theta_j^{new} = \theta_j^{old} - \alpha \frac{\partial}{\partial \theta_j^{old}} J(\theta)$$

- With step size ®
- In matrix notation for all parameters:

$$\theta^{new} = \theta^{old} - \alpha \frac{\partial}{\partial \theta^{old}} J(\theta)$$

$$\theta^{new} = \theta^{old} - \alpha \nabla_{\theta} J(\theta)$$

Vanilla Gradient Descent Code

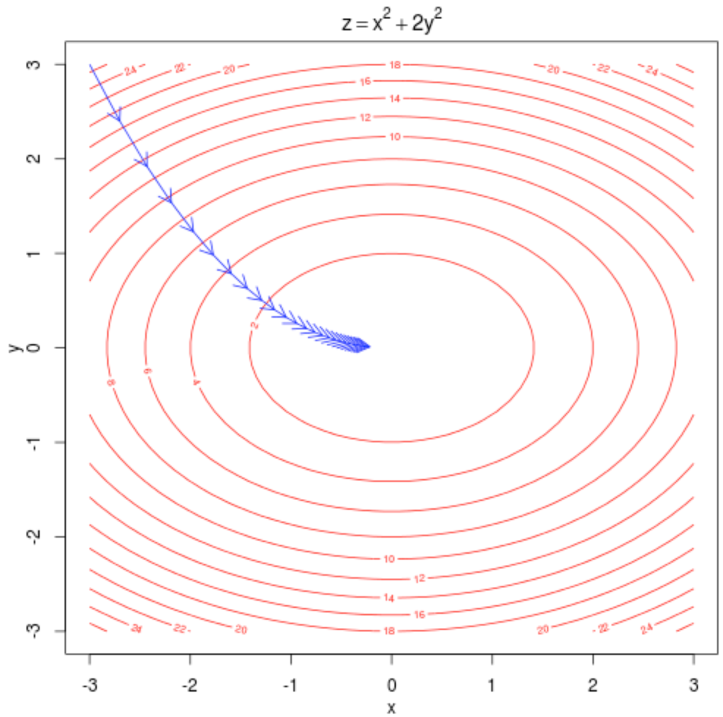
$$\theta^{new} = \theta^{old} - \alpha \nabla_{\theta} J(\theta)$$

```
while True:
    theta_grad = evaluate_gradient(J,corpus,theta)
    theta = theta - alpha * theta_grad
```

Intuition

- For a simple convex function over two parameters.

- Contour lines show levels of objective function
-



Stochastic Gradient Descent

- But Corpus may have 40B tokens and windows
- You would wait a very long time before making a single update!

- Very bad idea for pretty much all neural nets!
- Instead: We will update parameters after each window t
  → Stochastic gradient descent (SGD)

$$\theta^{new} = \theta^{old} - \alpha \nabla_\theta J_t(\theta)$$

```
while True:
    window = sample_window(corpus)
    theta_grad = evaluate_gradient(J,window,theta)
    theta = theta - alpha * theta_grad
```

Stochastic gradients with word vectors!

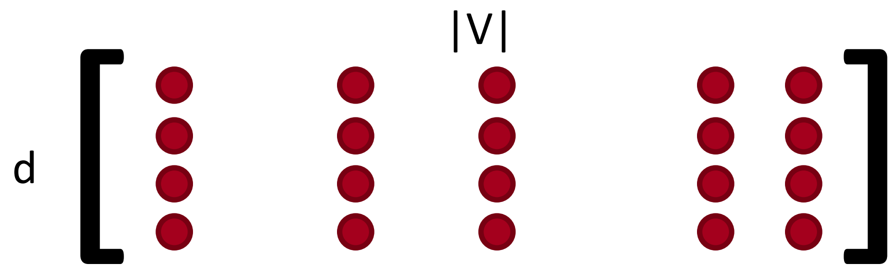
- But in each window, we only have at most 2c -1 words, so $\nabla_\theta J_t(\theta)$ is very sparse!

$$\nabla_\theta J_t(\theta) = \begin{bmatrix} 0 \\ \vdots \\ \nabla_{v_{like}} \\ \vdots \\ 0 \\ \nabla_{u_I} \\ \vdots \\ \nabla_{u_{learning}} \\ \vdots \end{bmatrix} \in \mathbb{R}^{2dV}$$

Stochastic gradients with word vectors!

- We may as well only update the word vectors that actually appear!

- Solution: either keep around hash for word vectors or only update certain columns of full embedding matrix U and V



- Important if you have millions of word vectors and do distributed computing to not have to send gigantic updates around.

Approximations: PSet 1

- The normalization factor is too computationally expensive

$$p(o|c) = \frac{\exp\left(u_o^T v_c\right)}{\sum_{w=1}^{W} \exp\left(u_w^T v_c\right)}$$

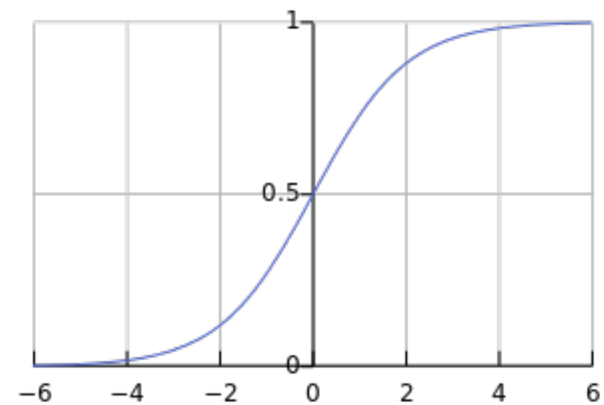
- Hence, in PSet1 you will implement the skip-gram model
- Main idea: train binary logistic regressions for a true pair (center word and word in its context window) and a couple of random pairs (the center word with a random word)

PSet 1: The skip-gram model and negative sampling

- From paper: "Distributed Representations of Words and Phrases and their Compositionality" (Mikolov et al. 2013)
- Overall objective function: $J(\theta) = \frac{1}{T}\sum_{t=1}^{T} J_t(\theta)$

$$J_t(\theta) = \log \sigma\left(u_o^T v_c\right) + \sum_{i=1}^{k} \mathbb{E}_{j \sim P(w)} \left[\log \sigma\left(-u_j^T v_c\right)\right]$$

- Where k is the number of negative samples and we use,
- The sigmoid function! $\sigma(x) = \frac{1}{1+e^{-x}}$ (we'll become good friends soon)
- So we maximize the probability of two words co-occurring in first log →



PSet 1: The skip-gram model and negative sampling

- Slightly clearer notation:

$$J_t(\theta) = \log \sigma\left(u_o^T v_c\right) + \sum_{j \sim P(w)} \left[\log \sigma\left(-u_j^T v_c\right)\right]$$

- Max. probability that real outside word appears, minimize prob. that random words appear around center word

- $P(w)=U(w)^{3/4}/Z$,
  the unigram distribution U(w) raised to the 3/4rd power
  (We provide this function in the starter code).
- The power makes less frequent words be sampled more often

PSet 1: The continuous bag of words model

- Main idea for continuous bag of words (CBOW): Predict center word from sum of surrounding word vectors instead of predicting surrounding single words from center word as in skip-gram model

- To make PSet slightly easier:

  The implementation for the CBOW model is not required and for bonus points!

Count based vs direct prediction

LSA, HAL (Lund & Burgess),
COALS (Rohde et al),
Hellinger-PCA (Lebret & Collobert)

- Fast training
- Efficient usage of statistics
- Primarily used to capture word similarity
- Disproportionate importance given to large counts

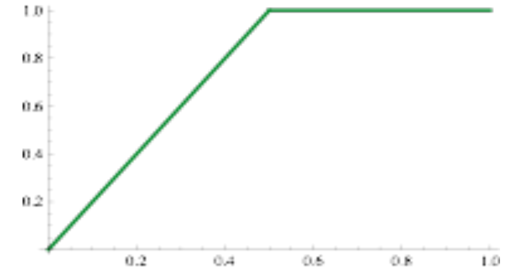
- NNLM, HLBL, RNN, Skip-gram/CBOW, (Bengio et al; Collobert & Weston; Huang et al; Mnih & Hinton; Mikolov et al; Mnih & Kavukcuoglu)

- Scales with corpus size
- Inefficient usage of statistics
- Generate improved performance on other tasks
- Can capture complex patterns beyond word similarity

Combining the best of both worlds: GloVe

$$J(\theta) = \frac{1}{2} \sum_{i,j=1}^{W} f(P_{ij})(u_i^T v_j - \log P_{ij})^2$$

$f \sim$



- Fast training
- Scalable to huge corpora
- Good performance even with small corpus, and small vectors

Glove results

Nearest words to
frog:

1. frogs
2. toad
3. litoria
4. leptodactylidae
5. rana
6. lizard
7. eleutherodactylus

litoria

leptodactylidae

rana

eleutherodactylus

What to do with the two sets of vectors?

- We end up with U and V from all the vectors u and v (in columns)

- Both capture similar co-occurrence information. It turns out, the best solution is to simply sum them up:

$$X_{final} = U + V$$

- One of many hyperparameters explored in *GloVe: Global Vectors for Word Representation* (Pennington et al. (2014)

How to evaluate word vectors?

- Related to general evaluation in NLP: Intrinsic vs extrinsic
- Intrinsic:
  - Evaluation on a specific/intermediate subtask
  - Fast to compute
  - Helps to understand that system
  - Not clear if really helpful unless correlation to real task is established
- Extrinsic:
  - Evaluation on a real task
  - Can take a long time to compute accuracy
  - Unclear if the subsystem is the problem or its interaction or other subsystems
  - If replacing one subsystem with another improves accuracy → Winning!

Intrinsic word vector evaluation

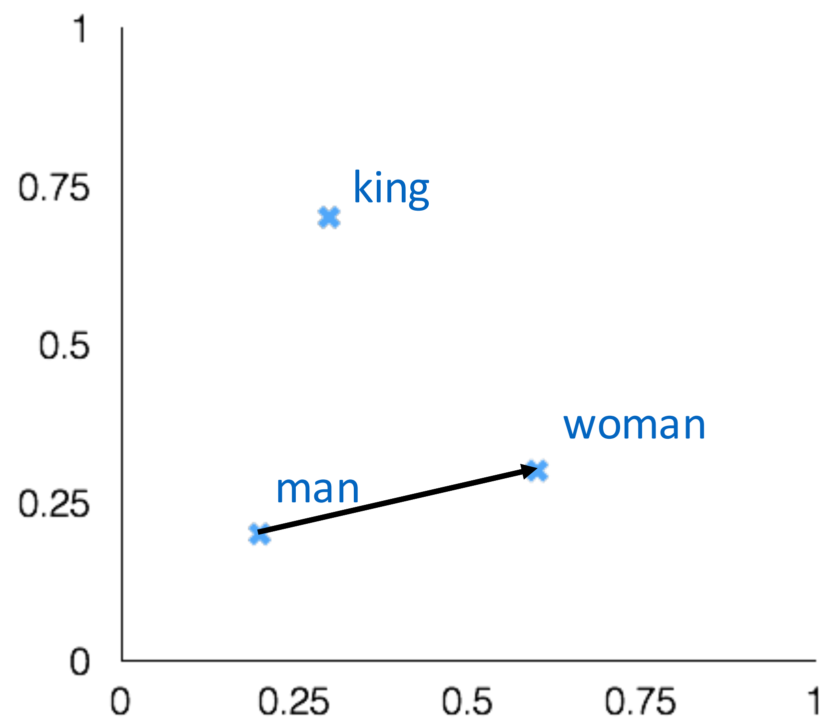
- Word Vector Analogies

a:b :: c:?

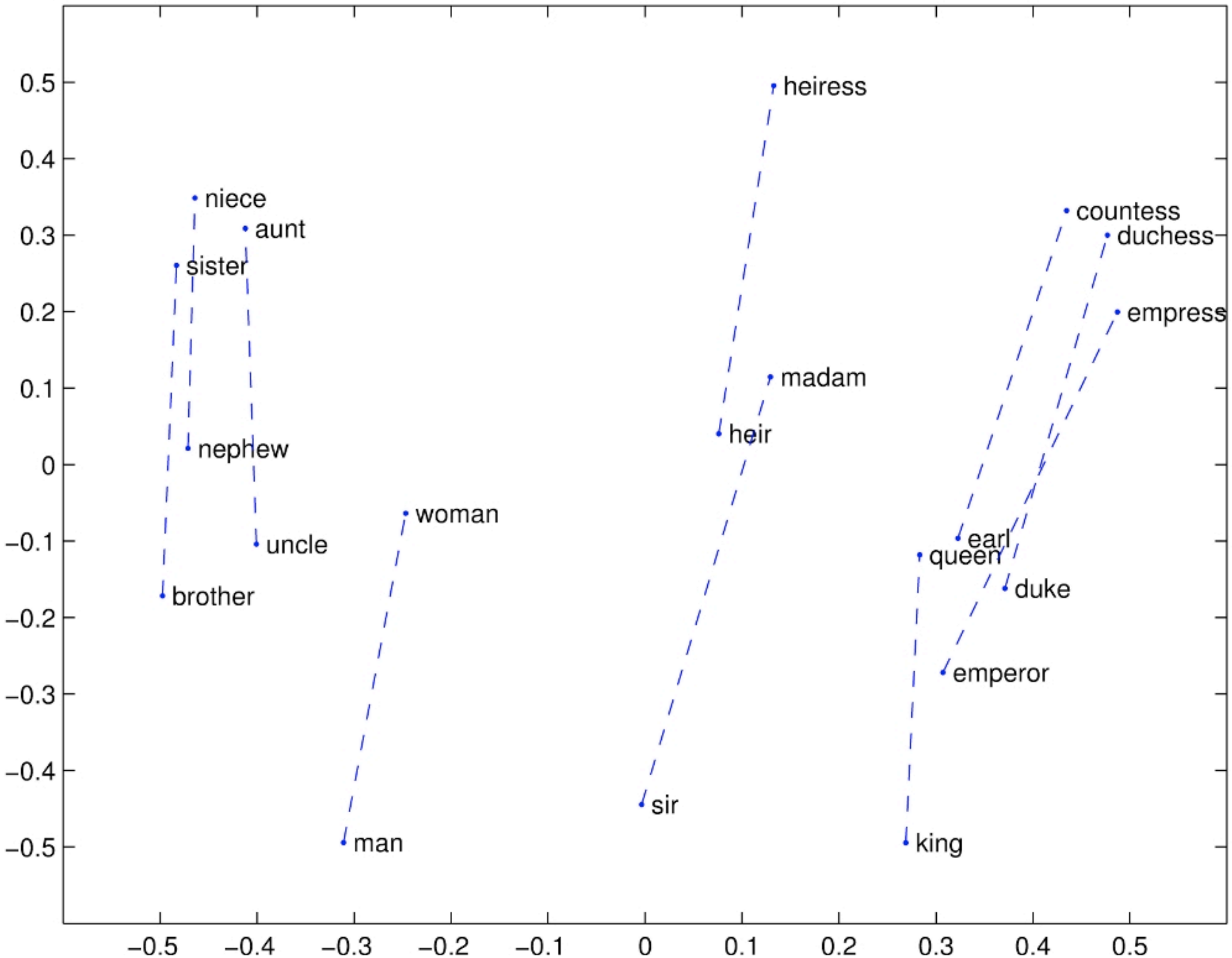
man:woman :: king:?

$$d = \arg\max_i \frac{(x_b - x_a + x_c)^T x_i}{||x_b - x_a + x_c||}$$

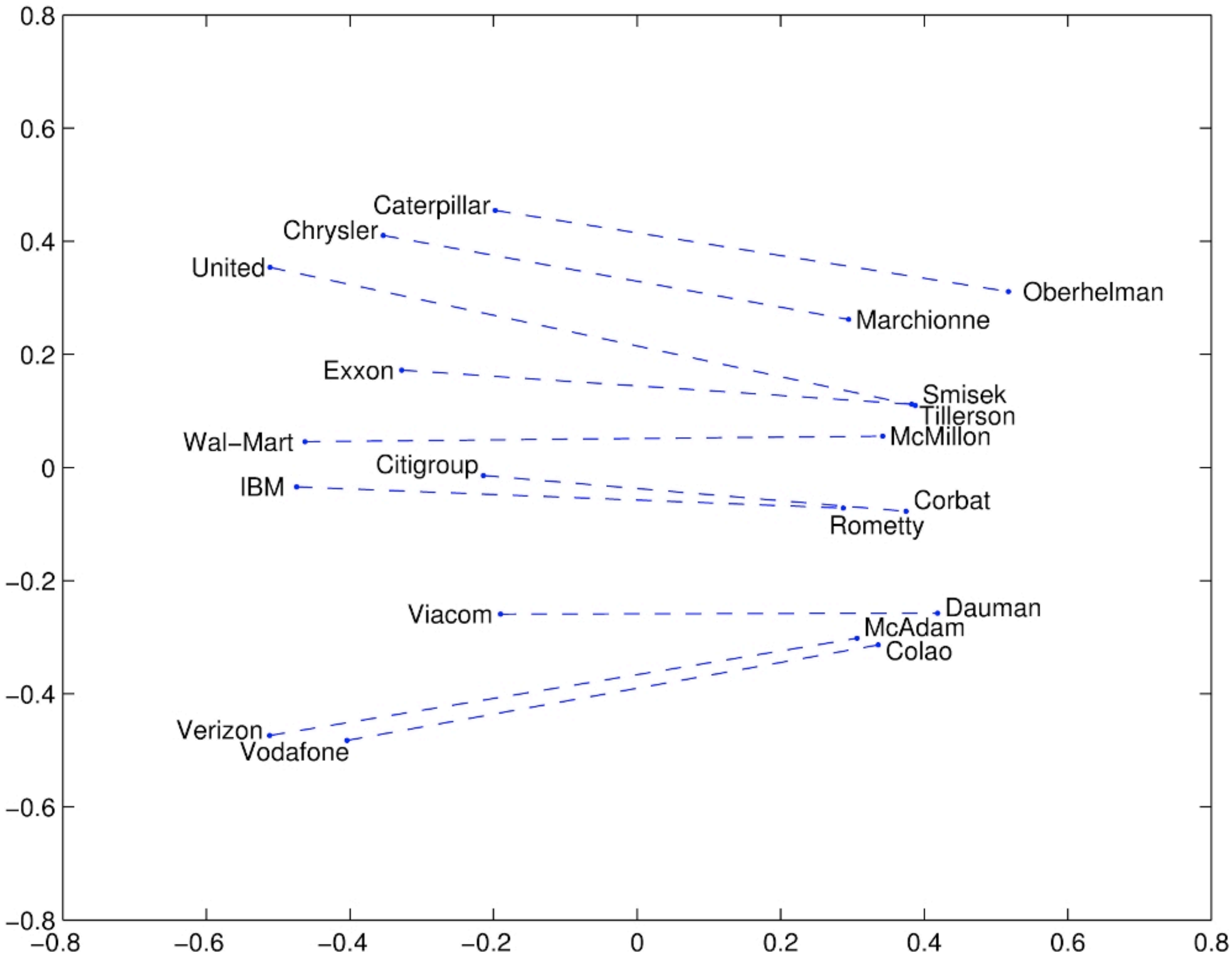
- Evaluate word vectors by how well their cosine distance after addition captures intuitive semantic and syntactic analogy questions
- Discarding the input words from the search!
- Problem: What if the information is there but not linear?



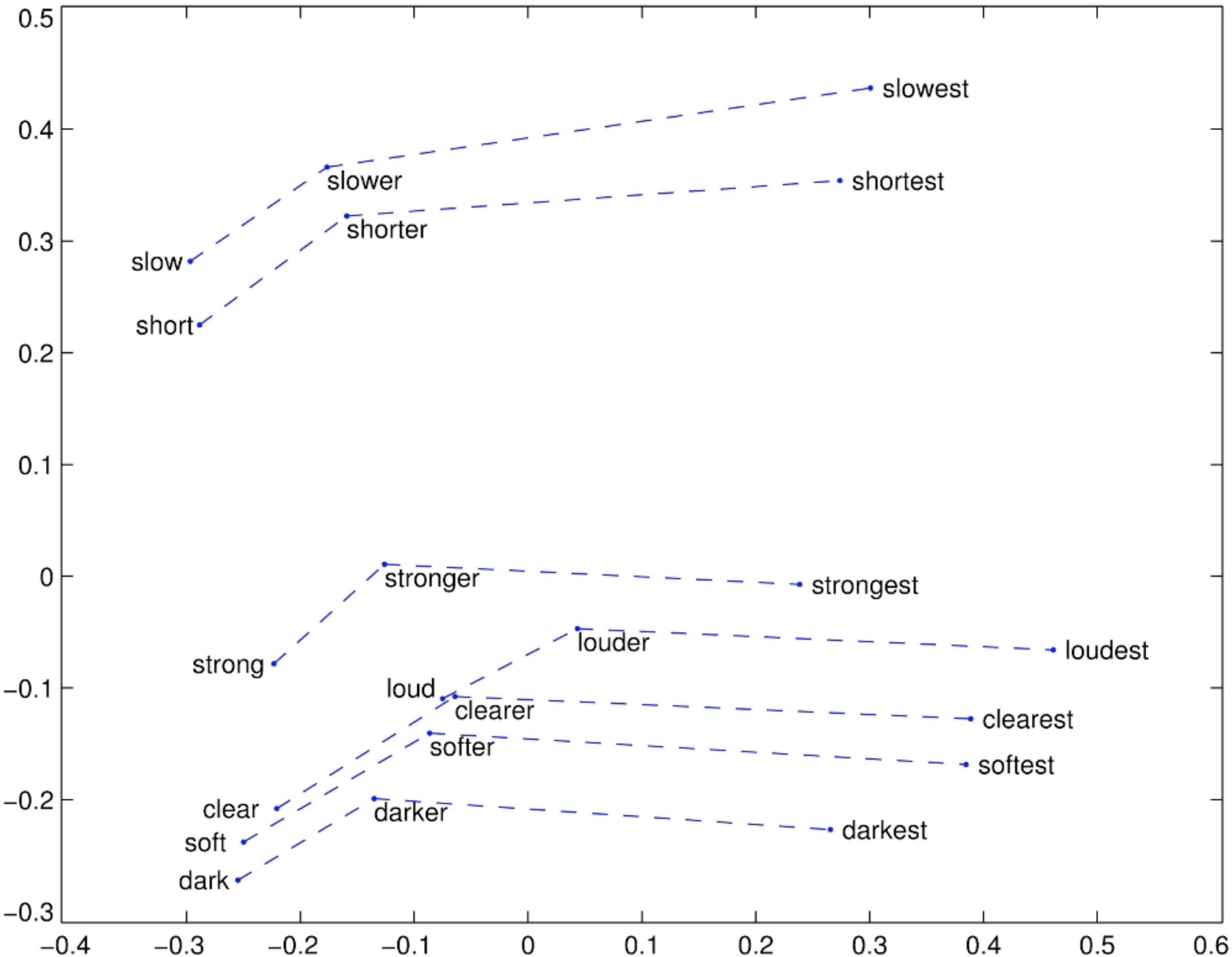
Glove Visualizations



Glove Visualizations: Company - CEO



Glove Visualizations: Superlatives



Details of intrinsic word vector evaluation

- Word Vector Analogies: Syntactic and **Semantic** examples from http://code.google.com/p/word2vec/source/browse/trunk/questions-words.txt

: city-in-state
Chicago Illinois Houston Texas
Chicago Illinois Philadelphia Pennsylvania
Chicago Illinois Phoenix Arizona
Chicago Illinois Dallas Texas
Chicago Illinois Jacksonville Florida
Chicago Illinois Indianapolis Indiana
Chicago Illinois Austin Texas
Chicago Illinois Detroit Michigan
Chicago Illinois Memphis Tennessee
Chicago Illinois Boston Massachusetts

problem: different cities may have same name

Details of intrinsic word vector evaluation

- Word Vector Analogies: Syntactic and **Semantic** examples from

: capital-world
Abuja Nigeria Accra Ghana
Abuja Nigeria Algiers Algeria
Abuja Nigeria Amman Jordan
Abuja Nigeria Ankara Turkey
Abuja Nigeria Antananarivo Madagascar
Abuja Nigeria Apia Samoa
Abuja Nigeria Ashgabat Turkmenistan
Abuja Nigeria Asmara Eritrea
Abuja Nigeria Astana Kazakhstan

problem: can change

Details of intrinsic word vector evaluation

- Word Vector Analogies: **Syntactic** and Semantic examples from

: gram4-superlative
bad worst big biggest
bad worst bright brightest
bad worst cold coldest
bad worst cool coolest
bad worst dark darkest
bad worst easy easiest
bad worst fast fastest
bad worst good best
bad worst great greatest

Details of intrinsic word vector evaluation

- Word Vector Analogies: **Syntactic** and Semantic examples from

: gram7-past-tense
dancing danced decreasing decreased
dancing danced describing described
dancing danced enhancing enhanced
dancing danced falling fell
dancing danced feeding fed
dancing danced flying flew
dancing danced generating generated
dancing danced going went
dancing danced hiding hid
dancing danced hitting hit

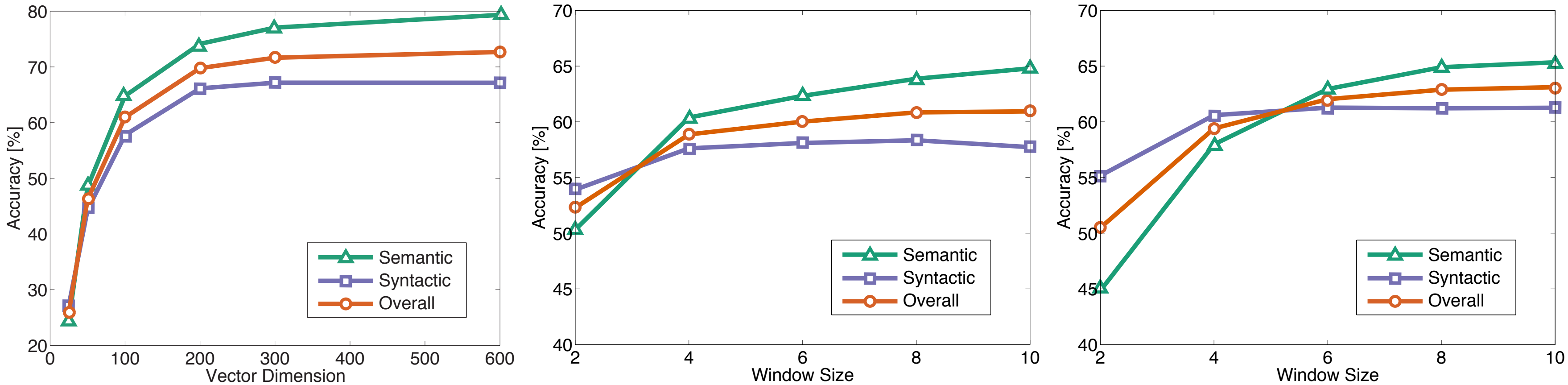
Analogy evaluation and hyperparameters

- Very careful analysis: Glove word vectors

Model	Dim.	Size	Sem.	Syn.	Tot.
ivLBL	100	1.5B	55.9	50.1	53.2
HPCA	100	1.6B	4.2	16.4	10.8
GloVe	100	1.6B	<u>67.5</u>	<u>54.3</u>	<u>60.3</u>
SG	300	1B	61	61	61
CBOW	300	1.6B	16.1	52.6	36.1
vLBL	300	1.5B	54.2	<u>64.8</u>	60.0
ivLBL	300	1.5B	65.2	63.0	64.0
GloVe	300	1.6B	<u>80.8</u>	61.5	<u>70.3</u>
SVD	300	6B	6.3	8.1	7.3
SVD-S	300	6B	36.7	46.6	42.1
SVD-L	300	6B	56.6	63.0	60.1
CBOW†	300	6B	63.6	<u>67.4</u>	65.7
SG†	300	6B	73.0	66.0	69.1
GloVe	300	6B	<u>77.4</u>	67.0	<u>71.7</u>
CBOW	1000	6B	57.3	68.9	63.7
SG	1000	6B	66.1	65.1	65.6
SVD-L	300	42B	38.4	58.2	49.2
GloVe	300	42B	<u>**81.9**</u>	<u>**69.3**</u>	<u>**75.0**</u>

Analogy evaluation and hyperparameters

- Asymmetric context (only words to the left) are not as good



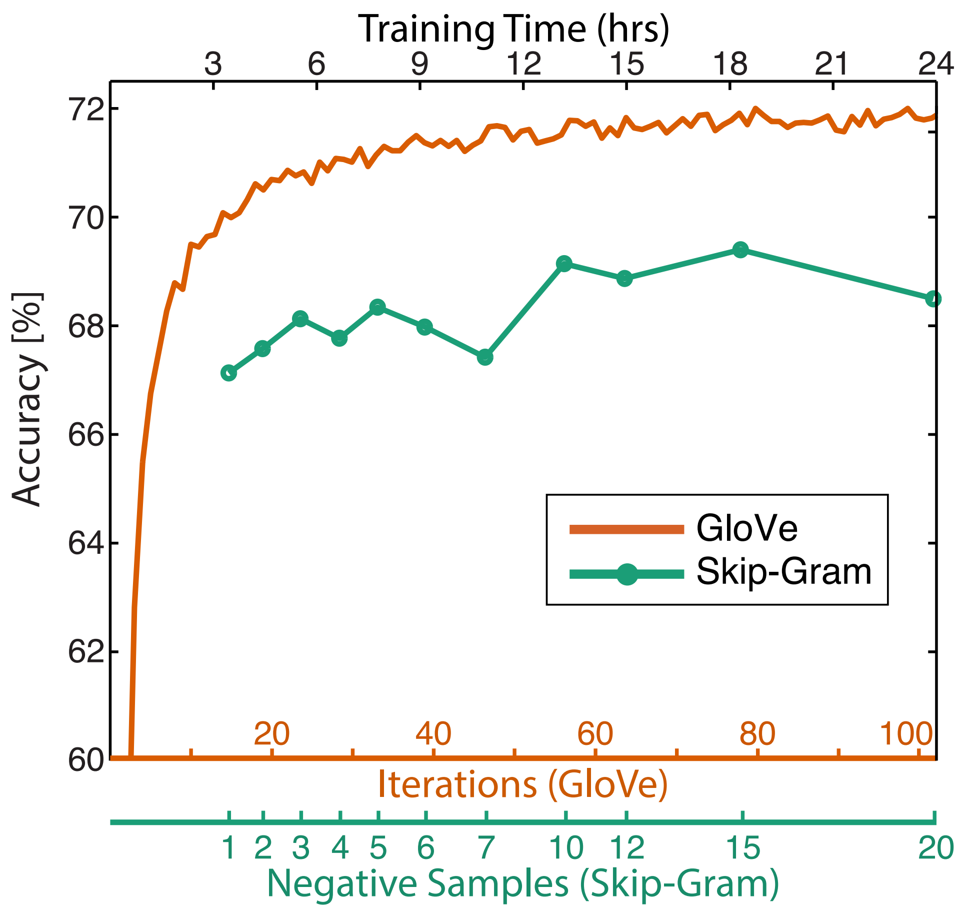
(a) Symmetric context (b) Symmetric context (c) Asymmetric context

- Best dimensions ~300, slight drop-off afterwards
- But this might be different for downstream tasks!

- Window size of 8 around each center word is good for Glove vectors

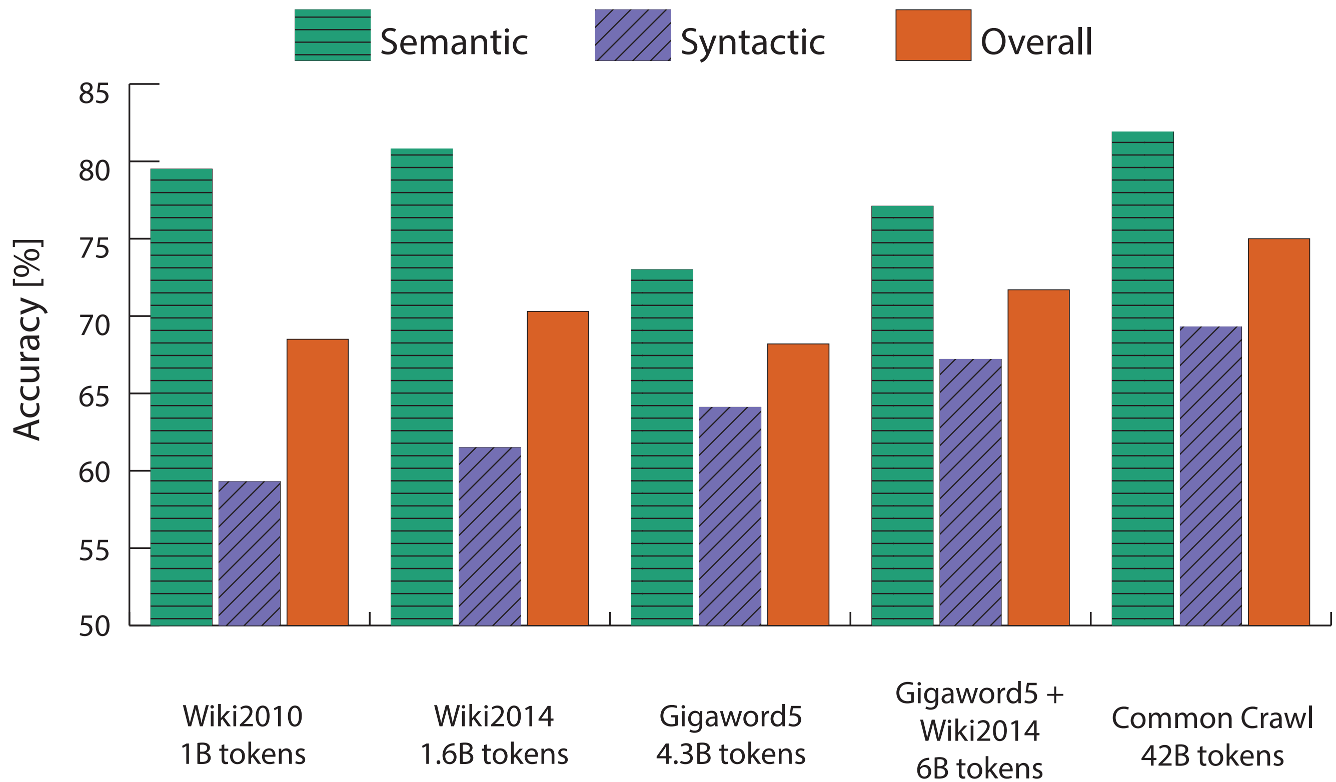
Analogy evaluation and hyperparameters

- More training time helps



Analogy evaluation and hyperparameters

- More data helps, Wikipedia is better than news text!



Intrinsic word vector evaluation

- Word vector distances and their correlation with human judgments
- Example dataset: WordSim353
  http://www.cs.technion.ac.il/~gabr/resources/data/wordsim353/

Word 1	Word 2	Human (mean)
tiger	cat	7.35
tiger	tiger	10.00
book	paper	7.46
computer		internet 7.58
plane	car	5.77
professor		doctor 6.62
stock	phone	1.62
stock	CD	1.31
stock	jaguar	0.92

Correlation evaluation

- Word vector distances and their correlation with human judgments

Model	Size	WS353	MC	RG	SCWS	RW
SVD	6B	35.3	35.1	42.5	38.3	25.6
SVD-S	6B	56.5	71.5	71.0	53.6	34.7
SVD-L	6B	65.7	<u>72.7</u>	75.1	56.5	37.0
CBOW†	6B	57.2	65.6	68.2	57.0	32.5
SG†	6B	62.8	65.2	69.7	<u>58.1</u>	37.2
GloVe	6B	<u>65.8</u>	<u>72.7</u>	<u>77.8</u>	53.9	<u>38.1</u>
SVD-L	42B	74.0	76.4	74.1	58.3	39.9
GloVe	42B	<u>**75.9**</u>	<u>**83.6**</u>	<u>**82.9**</u>	<u>**59.6**</u>	<u>**47.8**</u>
CBOW*	100B	68.4	79.6	75.4	59.4	45.5

- Some ideas from Glove paper have been shown to improve skip-gram (SG) model also (e.g. sum both vectors)

But what about ambiguity?

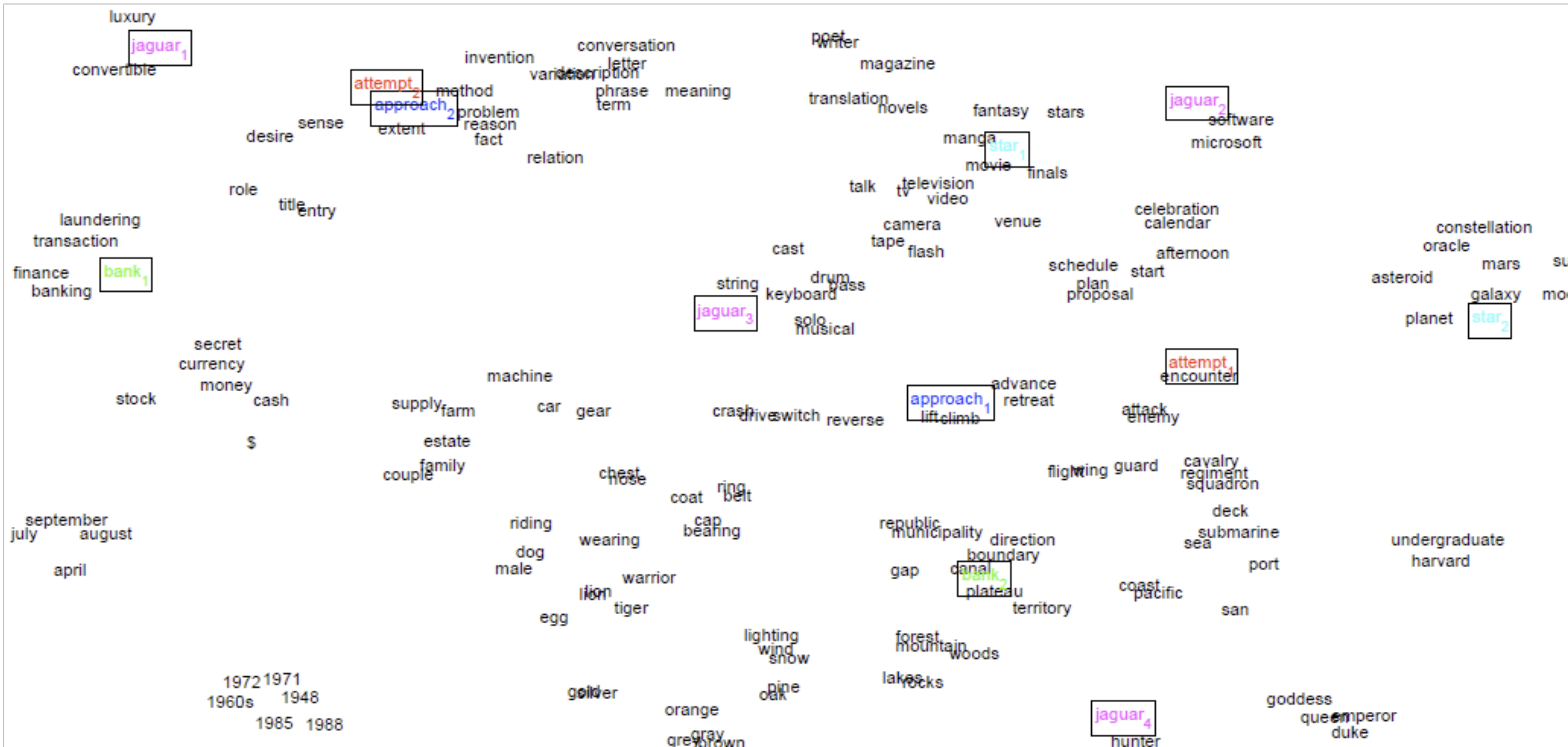
- You may hope that one vector captures both kinds of information (run = verb and noun) but then vector is pulled in different directions

- Alternative described in: *Improving Word Representations Via Global Context And Multiple Word Prototypes* (Huang et al. 2012)

- Idea: Cluster word windows around words, retrain with each word assigned to multiple different clusters $bank_1$, $bank_2$, etc

But what about ambiguity?

- *Improving Word Representations Via Global Context And Multiple Word Prototypes* (Huang et al. 2012)



Extrinsic word vector evaluation

- Extrinsic evaluation of word vectors: All subsequent tasks in this class
- One example where good word vectors should help directly: named entity recognition: finding a person, organization or location

Model	Dev	Test	ACE	MUC7
Discrete	91.0	85.4	77.4	73.4
SVD	90.8	85.7	77.3	73.7
SVD-S	91.0	85.5	77.6	74.3
SVD-L	90.5	84.8	73.6	71.5
HPCA	92.6	**88.7**	81.7	80.7
HSMN	90.5	85.7	78.7	74.7
CW	92.2	87.4	81.7	80.2
CBOW	93.1	88.2	82.2	81.1
GloVe	**93.2**	88.3	**82.9**	**82.2**

- Next: How to use word vectors in neural net models!

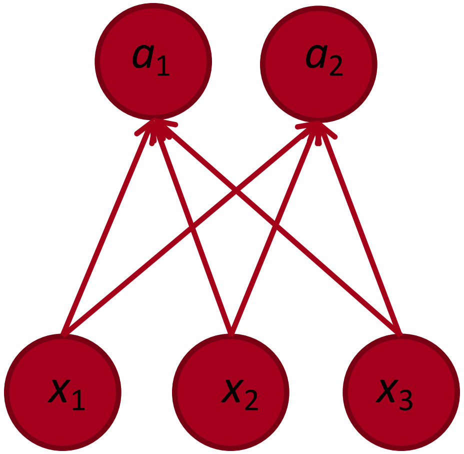
Simple single word classification

- What is the major benefit of deep learned word vectors?
  - Ability to also classify words accurately
    - Countries cluster together → classifying location words should be possible with word vectors
  - Incorporate **any** information into them other tasks
    - Project sentiment into words to find most positive/negative words in corpus

The softmax

Logistic regression = Softmax classification on word vector x to obtain probability for class y:

$$p(y|x) = \frac{\exp(W_y.x)}{\sum_{c=1}^{C} \exp(W_c.x)}$$



where: $W \in \mathbb{R}^{C \times d}$

Generalizes >2 classes
(for just binary sigmoid unit would suffice as in skip-gram)

The softmax - details

- Terminology: Loss function = cost function = objective function
- Loss for softmax: Cross entropy

- To compute p(y|x): first take the y'th row of W and multiply that with row with x:

$$W_{y \cdot} x = \sum_{i=1}^{d} W_{yi} x_i = f_y$$

- Compute all f_c for c=1,…,C
- Normalize to obtain probability with softmax function:

$$p(y|x) = \frac{\exp(f_y)}{\sum_{c=1}^{C} \exp(f_c)}$$

The softmax and cross-entropy error

- The loss wants to maximize the probability of the correct class y

- Hence, we minimize the negative log probability of that class:

$$-\log p(y|x) = -\log\left(\frac{\exp(f_y)}{\sum_{c=1}^{C}\exp(f_c)}\right)$$

- As before: we sum up multiple cross entropy errors if we have multiple classifications in our total error function over the corpus (**more next lecture**)

Background: The Cross entropy error

- Assuming a ground truth (or gold or target) probability distribution that is 1 at the right class and 0 everywhere else: p = [0,…,0,1,0,…0] and our computed probability is q, then the cross entropy is:

$$H(p,q) = -\sum_{c=1}^{C} p(c) \log q(c)$$

- Because of one-hot p, the only term left is the negative probability of the true class
- Cross-entropy can be re-written in terms of the entropy and Kullback-Leibler divergence between the two distributions:

$$H(p,q) = H(p) + D_{KL}(p||q)$$

The KL divergence

- Cross entropy: $H(p,q) = H(p) + D_{KL}(p||q)$
- Because p is zero in our case (and even if it wasn't it would be fixed and have no contribution to gradient), to minimize this is equal to minimizing the KL divergence

- The KL divergence is **not a distance** but a non-symmetric measure of the difference between two probability distributions *p* and *q*

$$D_{KL}(p||q) = \sum_{c=1}^{C} p(c) \log \frac{p(c)}{q(c)}$$

PSet 1

- Derive the gradient of the cross entropy error with respect to the input word vector x and the matrix W

Simple single word classification

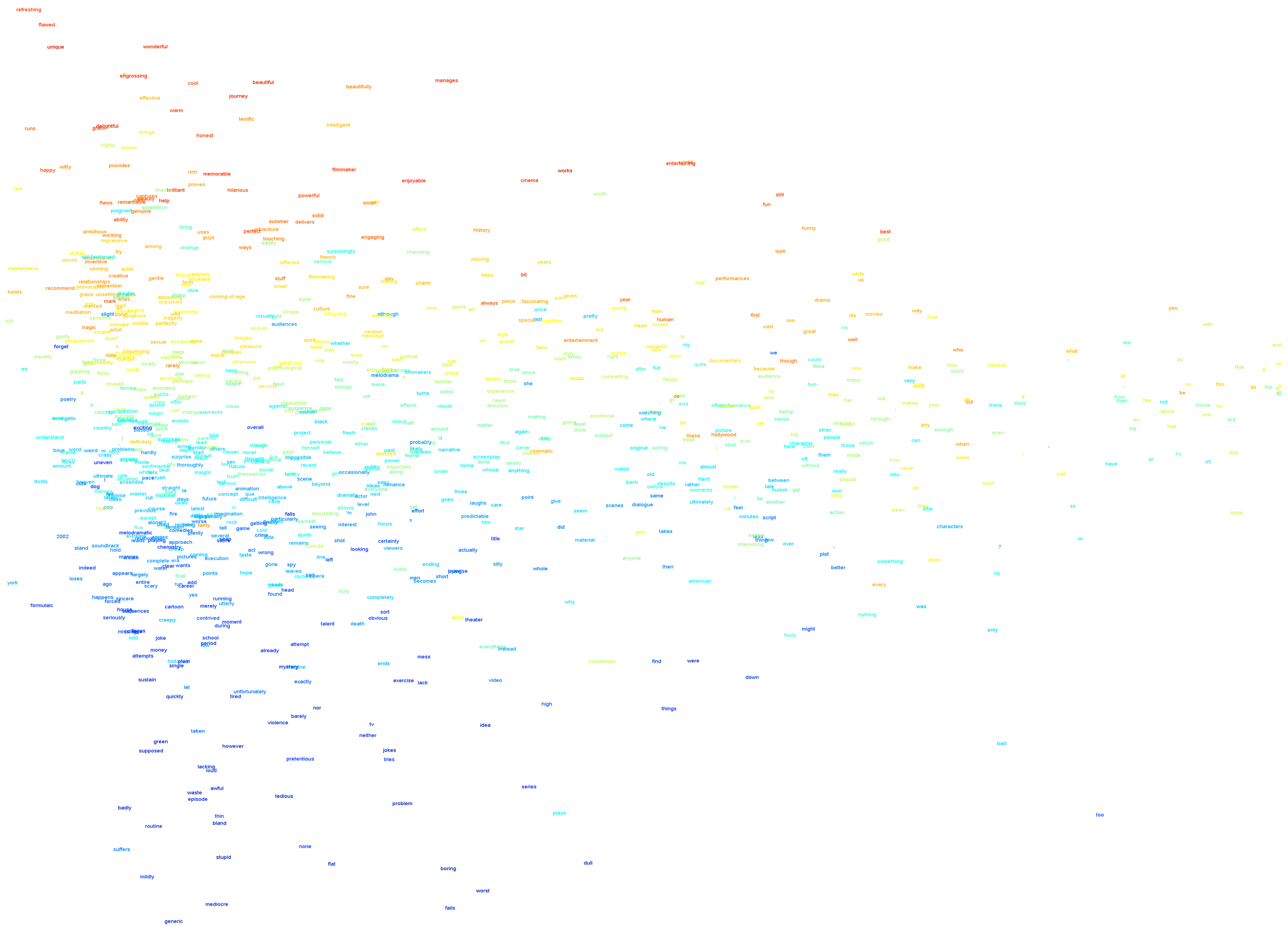
- Example: Sentiment

- Two options: train only *softmax* weights W and fix word vectors or also train word vectors

- Question: What are the advantages and disadvantages of training the word vectors?

- Pro: better fit on training data
- Con: Worse generalization because the words move in the vector space

Visualization of sentiment trained word vectors



Next level up: Window classification

- Single word classification has no context!
- Let's add context by taking in windows and classifying the center word of that window!
- Possible: Softmax and cross entropy error or **max-margin loss**
- Next class!

References