Logical Agents

Chapter 7 (continued)

Outline: Inference

- Resolution in CNF
  - Sound and Complete
- Forward and Backward Chaining using Modus Ponens in Horn Form
  - Sound and Complete

Proof methods

- Proof methods divide into (roughly) two kinds:
  - Application of inference rules
    - Legitimate (sound) generation of new sentences from old
    - Proof = a sequence of inference rule applications
      Can use inference rules as operators in a standard search algorithm
    - Typically require transformation of sentences into a normal form
  - Model checking
    - truth table enumeration (always exponential in *n*)
    - improved backtracking, e.g., Davis-Putnam-Logemann-Loveland (DPLL)
    - heuristic search in model space (sound but incomplete)
      e.g., min-conflicts like hill-climbing algorithms

Inference by enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, α) returns true or false
    symbols ← a list of the proposition symbols in KB and α
    return TT-CHECK-ALL(KB, α, symbols, [])
```

```
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
    if EMPTY?(symbols) then
        if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
        else return true
    else do
        P ← FIRST(symbols); rest ← REST(symbols)
        return TT-CHECK-ALL(KB, α, rest, EXTEND(P, true, model)) and
                   TT-CHECK-ALL(KB, α, rest, EXTEND(P, false, model))
```

- For *n* symbols, time complexity is $O(2^n)$, space complexity is $O(n)$

This is a Model Checking version of proof

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i, j].
Let $B_{i,j}$ be true if there is a breeze in [i, j].

$\neg P_{1,1}$

$\neg B_{1,1}$

$B_{2,1}$

- "Pits cause breezes in adjacent squares"

$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB	α_1
$false$	$false$	$false$	$false$	$false$	$false$	$false$	$false$	$true$
$false$	$false$	$false$	$false$	$false$	$false$	$true$	$false$	$true$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$false$	$true$	$false$	$false$	$false$	$false$	$false$	$false$	$true$
$false$	$true$	$false$	$false$	$false$	$false$	$true$	$\underline{true}$	$\underline{true}$
$false$	$true$	$false$	$false$	$false$	$true$	$false$	$\underline{true}$	$\underline{true}$
$false$	$true$	$false$	$false$	$false$	$true$	$true$	$\underline{true}$	$\underline{true}$
$false$	$true$	$false$	$false$	$true$	$false$	$false$	$false$	$true$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$true$	$true$	$true$	$true$	$true$	$true$	$true$	$false$	$false$

$R_1 = \neg P_{1,1}$

$R_4 = \neg B_{1,1}$

$R_5 = B_{2,1}$

$\alpha_1 = P_{1,2}$?

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      e.g., min-conflicts like hill-climbing algorithms

Reasoning Patterns in Prop Logic

$$\frac{\text{Given(s)}}{\text{Conclusion}}$$

Rules that allow us to introduce new propositions while preserving truth values: logically equivalent

$$\frac{A \Rightarrow B,\ A}{B}$$

$$\frac{B \wedge A}{A}$$

Two Examples:

- Modus Ponens
- And Elimination

Logical equivalence

- Two sentences are logically equivalent iff true in same models: α ≡ β iff $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Resolution

Conjunctive Normal Form (CNF)

conjunction of disjunctions of literals

clauses

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

- Resolution inference rule (for CNF):

$$\frac{l_i \vee \ldots \vee l_k, \qquad m_1 \vee \ldots \vee m_n}{l_i \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n}$$

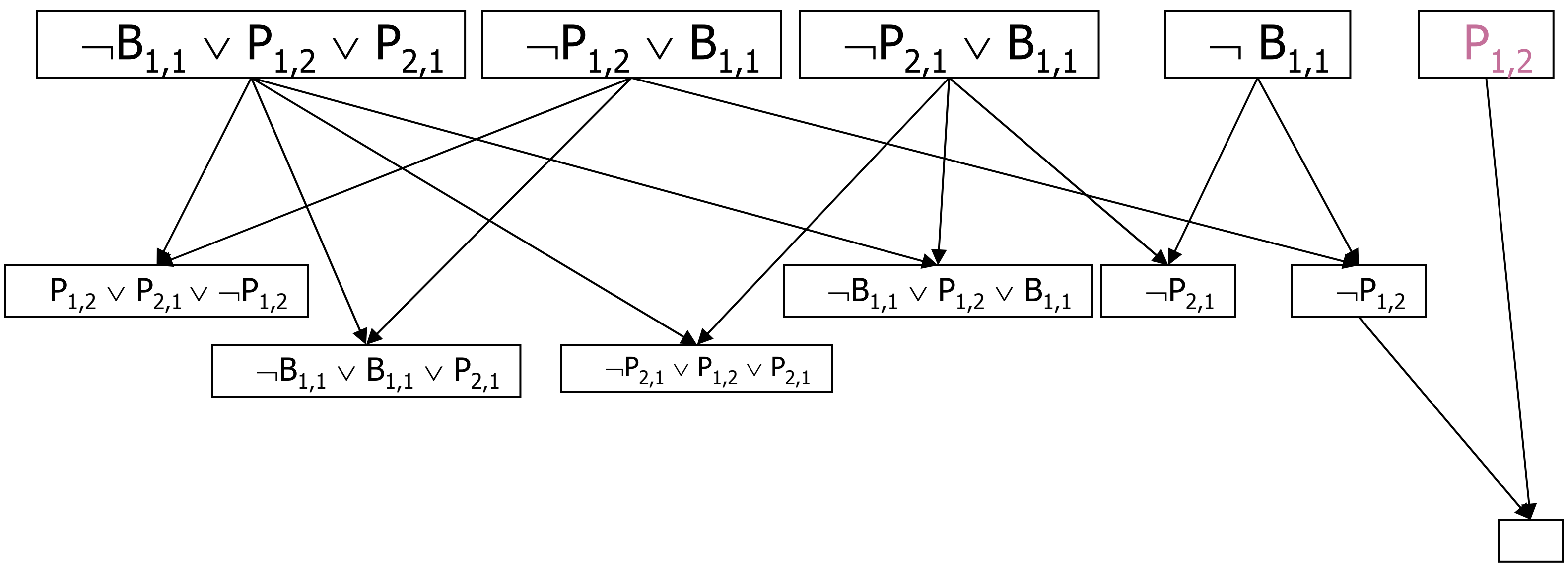
where l_i and m_j are complementary literals.

E.g., $\dfrac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$

- Resolution is sound and complete for propositional logic

Resolution example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$
- $\alpha = \neg P_{1,2}$ (negate the premise for proof by refutation)



The power of false

- Given: $(P) \wedge (\neg P)$
- Prove: Z

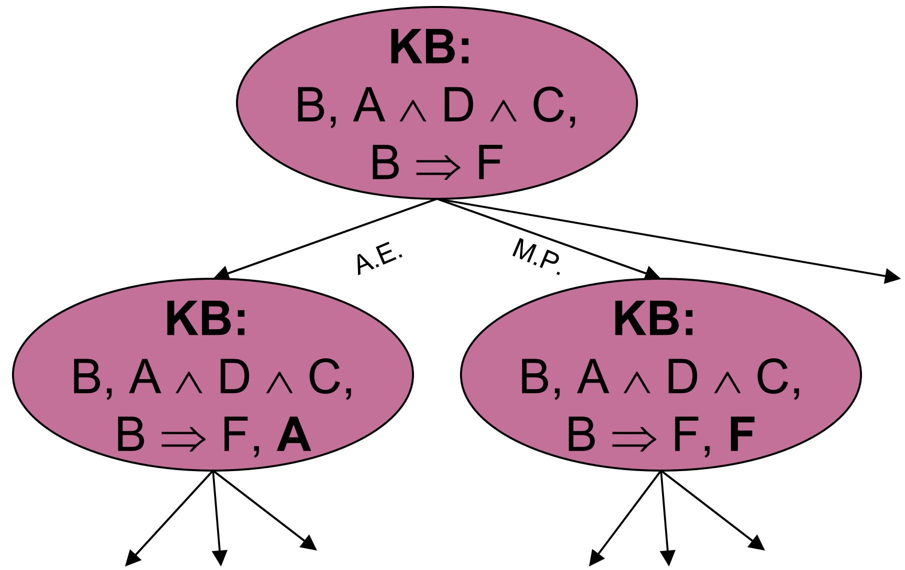
$\neg$ P	Given
P	Given
$\neg$ Z	Given
$\square$	Unsatisfiable

- Can we prove $\neg$Z using the givens above?

Applying inference rules

Equivalent to a search problem

- KB state = node
- Inference rule application = edge



Inference

Do the operators make conclusions that aren't always true?

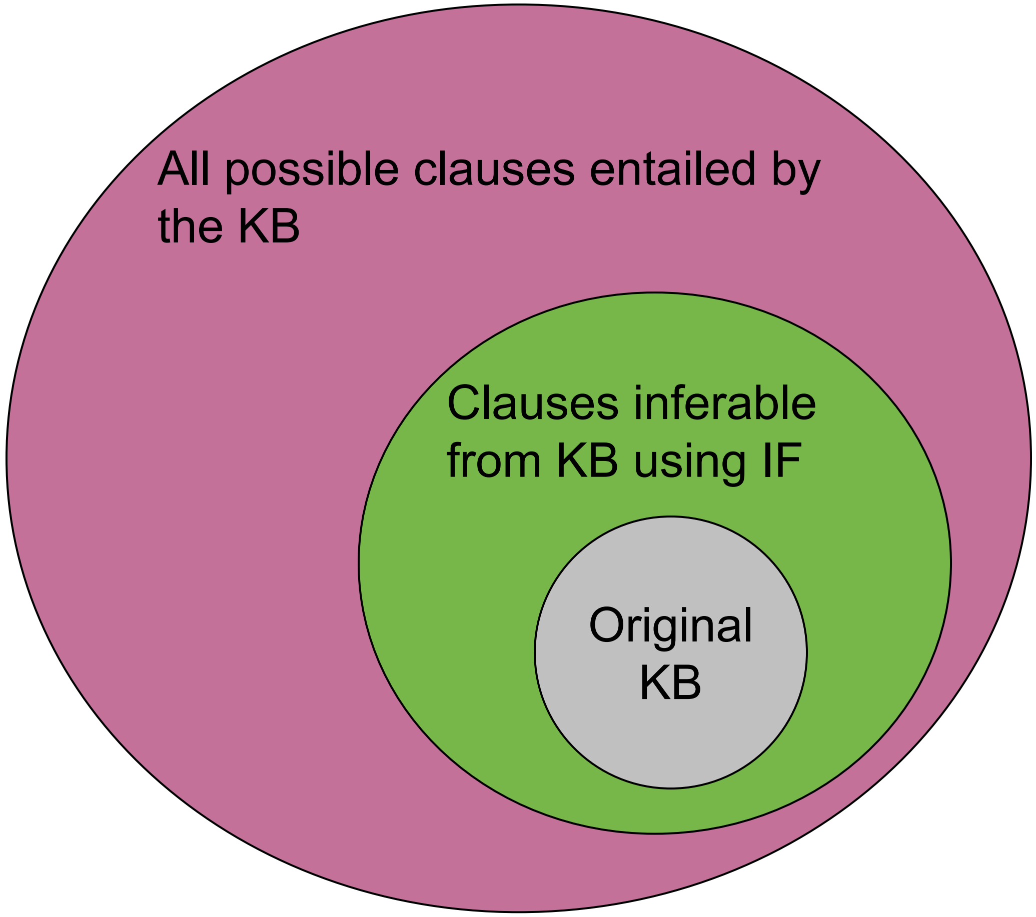
- Define: $KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i
- Soundness: i is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
- Completeness: i is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$
- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose ans

• Is a set of inference operators **complete** and **sound**?

Completeness

Completeness: i is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

- An incomplete inference algorithm cannot reach all possible conclusions
  - Equivalent to completeness in search (chapter 3)



Resolution

Conjunctive Normal Form (CNF)

conjunction of disjunctions of literals

clauses

E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$

- Resolution inference rule (for CNF):

$$\frac{\ell_1 \lor \dots \lor \ell_k, \qquad m_1 \lor \dots \lor m_n}{\ell_1 \lor \dots \lor \ell_{i-1} \lor \ell_{i+1} \lor \dots \lor \ell_k \lor m_1 \lor \dots \lor m_{j-1} \lor m_{j+1} \lor \dots \lor m_n}$$

where ℓ_i and m_j are complementary literals.

E.g., $\dfrac{P_{1,3} \lor P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$

- Resolution is **sound** and **complete** for propositional logic

Resolution

Soundness of resolution inference rule:

Same truth value

$$\frac{\neg(\ell_i \lor \ldots \lor \ell_{i-1} \lor \ell_{i+1} \lor \ldots \lor \ell_k) \Rightarrow \ell_i \qquad \neg m_j \Rightarrow (m_1 \lor \ldots \lor m_{j-1} \lor m_{j+1} \lor \ldots \lor m_n)}{\neg(\ell_i \lor \ldots \lor \ell_{i-1} \lor \ell_{i+1} \lor \ldots \lor \ell_k) \Rightarrow (m_1 \lor \ldots \lor m_{j-1} \lor m_{j+1} \lor \ldots \lor m_n)}$$

where ℓ_i and m_j are complementary literals.

- What if ℓ_i and $\neg m_j$ are false?
- What if ℓ_i and $\neg m_j$ are true?

Completeness of Resolution

- That is, that resolution can decide the truth value of S

- S = set of clauses
- RC(S) = Resolution closure of S = Set of all clauses that can be derived from S by the resolution inference rule.
- RC(S) has finite cardinality (finite number of symbols P_1, P_2, … P_k), thus resolution refutation must terminate.

Completeness of Resolution (cont)

- Ground resolution theorem = if S unsatisfiable, RC(S) contains empty clause.
- Prove by proving contrapositive:
  - i.e., if RC(S) doesn't contain empty clause, S is satisfiable
  - Do this by constructing a model:
    - For each P_i, if there is a clause in RC(S) containing $\neg P_i$ and all other literals in the clause are false, assign P_i = false
    - Otherwise P_i = true
  - This assignment of P_i is a model for S.

Forward and backward chaining

- Horn Form (restricted)

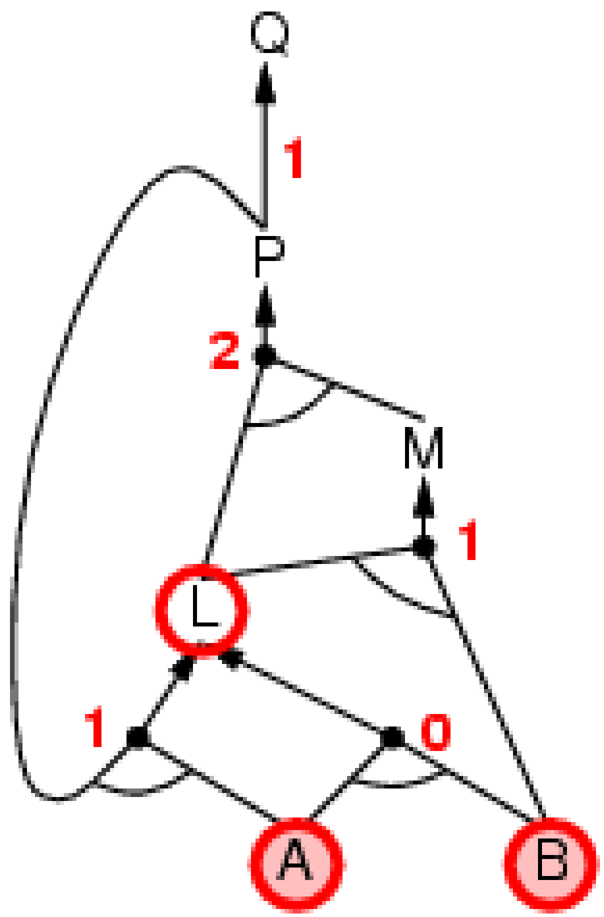
  KB = conjunction of Horn clauses

  - Horn clause =
    - proposition symbol; or
    - (conjunction of symbols) $\Rightarrow$ symbol
  - E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$
- Modus Ponens (for Horn Form): complete for Horn KBs

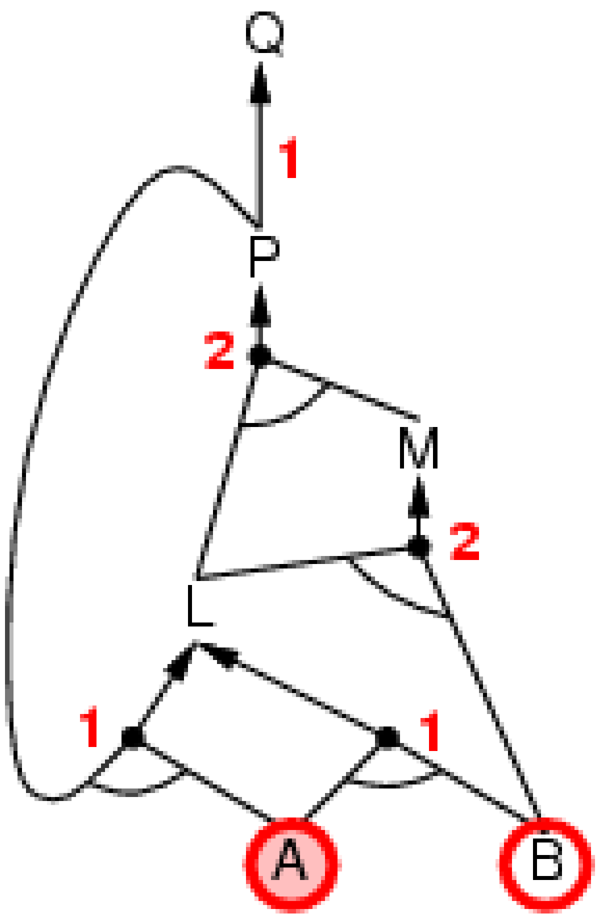
$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \ldots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

- Can be used with forward chaining or backward chaining.
- These algorithms are very natural and run in linear time

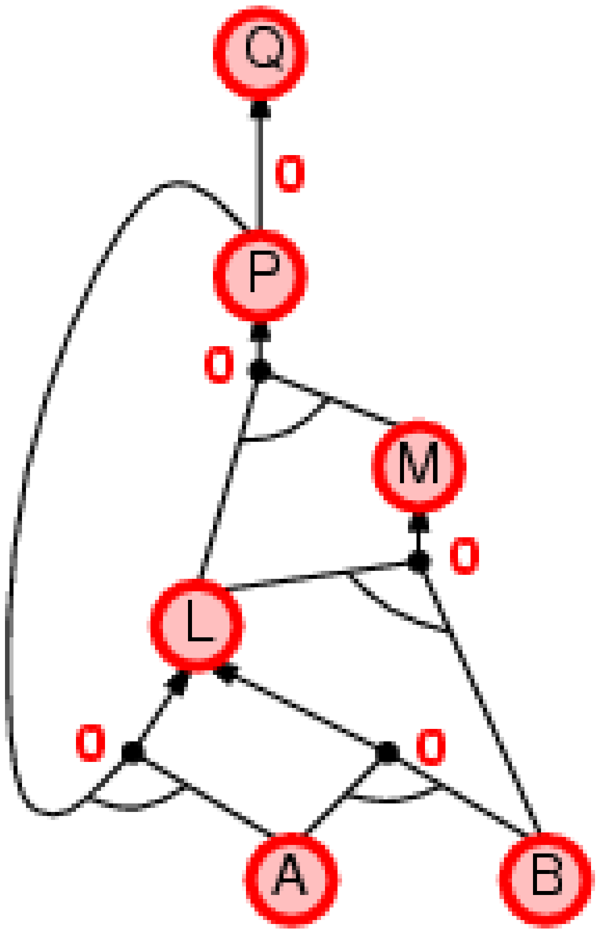
Forward chaining example



Forward chaining example



Forward chaining example

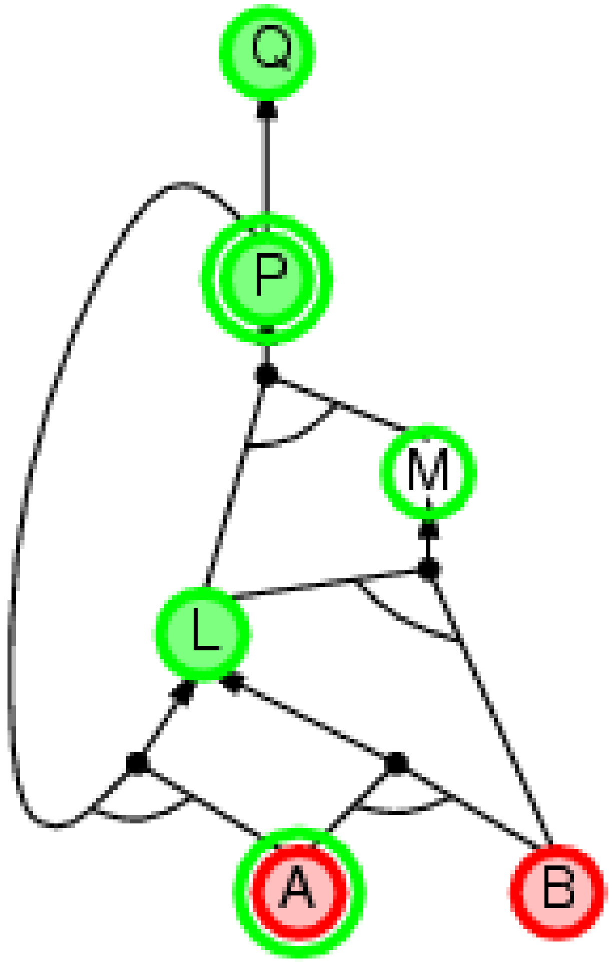


Proof of completeness

- FC derives every atomic sentence that is entailed by *KB* (only for clauses in Horn form)
  1. FC reaches a fixed point **(the deductive closure)** where no new atomic sentences are derived
  2. Consider the final state as a model *m*, assigning true/false to symbols
  3. Every clause in the original *KB* is true in *m*

     $a_1 \wedge \ldots \wedge a_k \Rightarrow b$
  4. Hence *m* is a model of *KB*
  5. If $KB \models q$, q is true in every model of *KB*, including *m*

Backward chaining example



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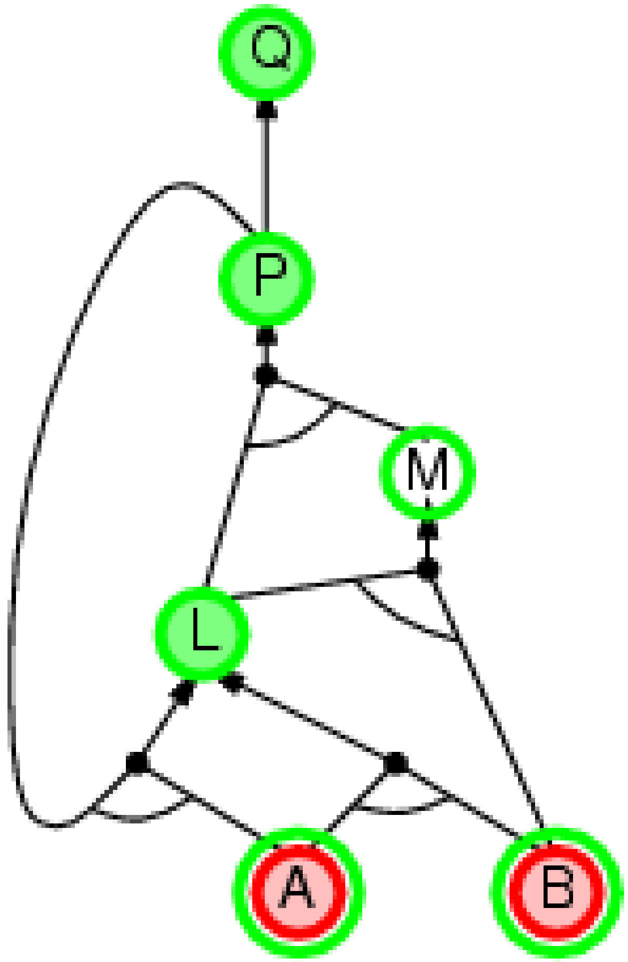
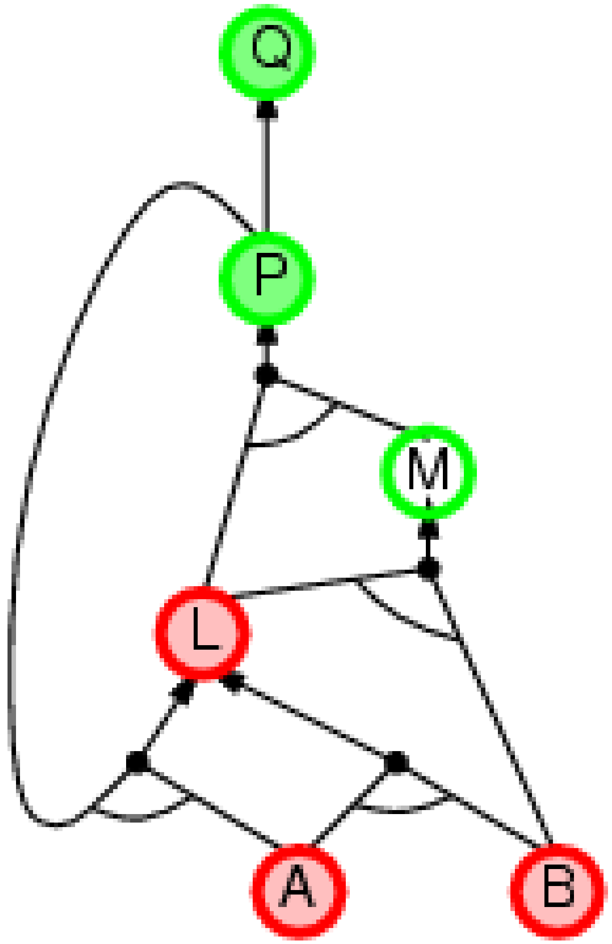


Backward chaining example



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Efficient propositional inference

Two families of efficient algorithms for propositional inference:

Complete backtracking search algorithms

- DPLL algorithm (Davis, Putnam, Logemann, Loveland)
- Incomplete local search algorithms
  - `WalkSAT` algorithm

The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable.

Improvements over truth table enumeration:

1. Early termination
   A clause is true if any literal is true.
   A sentence is false if any clause is false.

2. Pure symbol heuristic
   Pure symbol: always appears with the same "sign" in all clauses.
   e.g., In the three clauses $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, A and B are pure, C is impure.
   Make a pure symbol literal true.

3. Unit clause heuristic
   Unit clause: only one literal in the clause
   The only literal in a unit clause must be true.

Least constraining value

Most constrained value

What are correspondences between DPLL and in general CSPs?

The DPLL algorithm

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(*clauses*, *symbols*, [ ])

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

  if every clause in *clauses* is true in *model* **then return** *true*
  if some clause in *clauses* is false in *model* **then return** *false*
  P, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols*–P, [P = *value*|*model*])
  P, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols*–P, [P = *value*|*model*])
  P ← FIRST(*symbols*); *rest* ← REST(*symbols*)
  return DPLL(*clauses*, *rest*, [P = *true*|*model*]) **or**
      DPLL(*clauses*, *rest*, [P = *false*|*model*])

The `WalkSAT` algorithm

- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness

The `WalkSAT` algorithm

```
function WALKSAT(clauses, p, max-flips) returns a satisfying model or failure
   inputs: clauses, a set of clauses in propositional logic
           p, the probability of choosing to do a "random walk" move
           max-flips, number of flips allowed before giving up

   model ← a random assignment of true/false to the symbols in clauses
   for i = 1 to max-flips do
       if model satisfies clauses then return model
       clause ← a randomly selected clause from clauses that is false in model
       with probability p flip the value in model of a randomly selected symbol
             from clause
      else flip whichever symbol in clause maximizes the number of satisfied clauses
   return failure
```

Let's ask ourselves: Why is it **incomplete**?

Hard satisfiability problems

- Consider random 3-CNF sentences. e.g.,

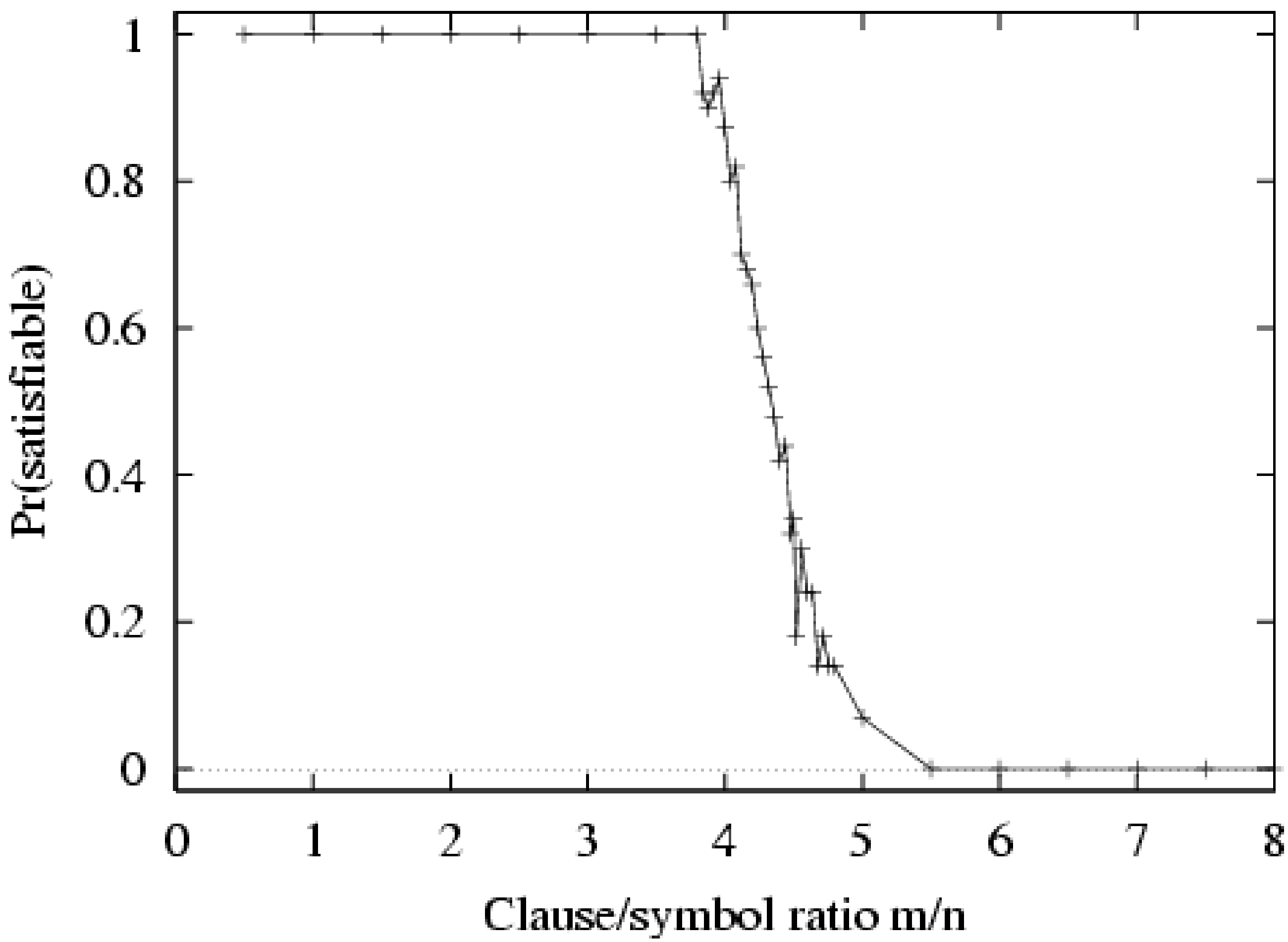
  $(\neg D \lor \neg B \lor C) \land (B \lor \neg A \lor \neg C) \land (\neg C \lor \neg B \lor E) \land (E \lor \neg D \lor B) \land (B \lor E \lor \neg C)$

  m = number of clauses

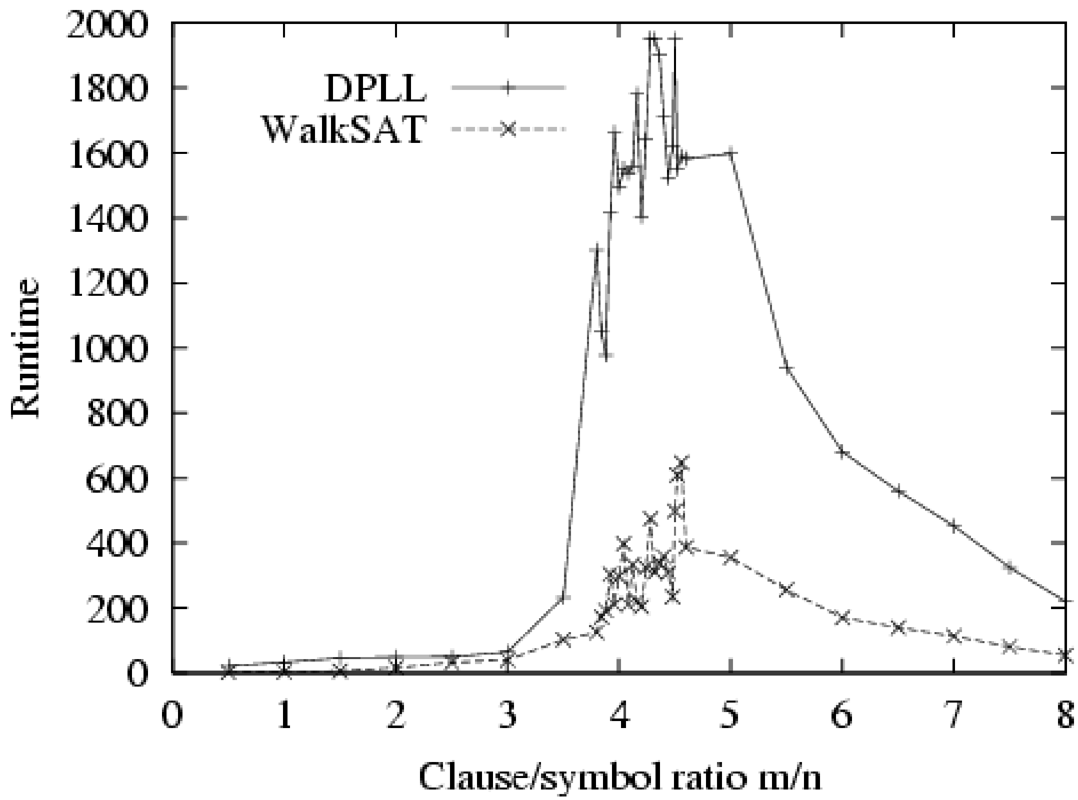
  n = number of symbols

  - Hard problems seem to cluster near $m/n = 4.3$ (critical point)

Hard satisfiability problems



Hard satisfiability problems



- Median runtime for 100 satisfiable random 3-CNF sentences, $n = 50$

Inference-based agents in the wumpus world

A wumpus-world agent using propositional logic:

$\neg P_{1,1}$
$\neg W_{1,1}$
$B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y})$
$S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y})$
$W_{1,1} \vee W_{1,2} \vee \ldots \vee W_{4,4}$
$\neg W_{1,1} \vee \neg W_{1,2}$
$\neg W_{1,1} \vee \neg W_{1,3}$
…

$\Rightarrow$ 64 distinct proposition symbols, 155 sentences

```
function PL-WUMPUS-AGENT(percept) returns an action
    inputs: percept, a list, [stench, breeze, glitter]
    static: KB, initially containing the "physics" of the wumpus world
            x, y, orientation, the agent's position (init. [1,1]) and orient. (init. right)
            visited, an array indicating which squares have been visited, initially false
            action, the agent's most recent action, initially null
            plan, an action sequence, initially empty

    update x,y,orientation, visited based on action
    if stench then TELL(KB, S_x,y) else TELL(KB, ¬ S_x,y)
    if breeze then TELL(KB, B_x,y) else TELL(KB, ¬ B_x,y)
    if glitter then action ← grab
    else if plan is nonempty then action ← POP(plan)
    else if for some fringe square [i,j], ASK(KB, (¬ P_i,j ∧ ¬ W_i,j)) is true or
            for some fringe square [i,j], ASK(KB, (P_i,j ∨ W_i,j)) is false then do
        plan ← A*-GRAPH-SEARCH(ROUTE-PB([x,y], orientation, [i,j], visited))
        action ← POP(plan)
    else action ← a randomly chosen move
    return action
```

Expressiveness limitation of propositional logic

- We didn't keep track of location and time in the KB. To do this we need more variables:
  - $L_{1,1}$ to show that agent in $L_{1,1}$. Does this work?

- KB contains "physics" sentences for every single square

- For every time *t* and every location [*x*,*y*],

$$L^{t}_{x,y} \wedge FacingRight^{t} \wedge Forward^{t} \Rightarrow L^{t}_{x+1,y}$$

- Rapid proliferation of clauses

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences wrt models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is complete for propositional logic
- Forward, backward chaining are linear-time, complete for Horn clauses
- Propositional logic lacks expressive power