CS3243 Foundations of Artificial Intelligence (2005/2006 Semester 2)
Tutorial 6

1. Determine using a truth table whether the following sentence is valid, satisfiable, or unsatisfiable:
(a) $(P \wedge Q) \vee \neg Q$
(b) $((P \wedge Q) \Rightarrow R) \Leftrightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$

2. Assume that a knowledge base *KB* contains the following rules:
$poor \Rightarrow \neg worried$
$rich \Rightarrow scared$
$\neg rich \Rightarrow poor$
Show that *KB* |= (*worried* ⇒ *scared*), using the model checking approach.

3. Repeat question 2 above, but use resolution to prove *worried* ⇒ *scared*.

4. Someone says: "On either Saturday or Sunday, if I am free, I will go to the concert". Using prepositional logic, the statement is represented as:
$(saturday \vee sunday) \Rightarrow (free \Rightarrow concert)$
Convert the above sentence into conjunctive normal form, and then into Horn form, by using the logical equivalences in Figure 7.11. Figure 7.11 is reproduced below.

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	de Morgan
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	de Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

5. (Question 7.9 from the textbook) (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

6. (Question 7.6 from the textbook) We have defined four different binary logical connectives.
(a) Are there any others that might be useful?
(b) How many binary connectives can there be?
(c) Why are some of them not very useful?