Fun and Creative Problem Solving in Mathematics and Computer Science

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Experience the fun of problem solving

Fun and CPS in Math-CS

Outline

- Polya's "Four Stages in Problem Solving"
- Creative Problem Solving Examples
  - *Inverting a Triangle of Coins*
  - *Good Coins, Bad Coins*

Experience the fun of problem solving

First, a Quote.

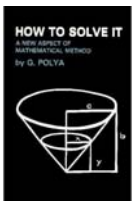
A great discovery solves a great problem,
but there is a grain of discovery
in the solution of any problem.

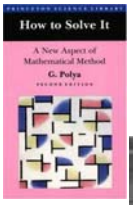
Your problem may be modest;
but if it challenges your curiosity and
bring into play your inventive faculties,
and if you solve it by your own means,
you may experience the tension and
enjoy the triumph of discovery.

The Aha! moment

G. Polya, 1945

"How to Solve It", by George Polya






The most widely cited reference
for problem solving in all disciplines

Polya's Four Stages of PS

I. UNDERSTANDING THE PROBLEM
You have to understand the problem

II. DEVISING A PLAN
Find the connection between the data and the unknown. …You should obtain eventually a *plan* for the solution

III. CARRYING OUT THE PLAN
Carry out your plan

III. LOOKING BACK
Examine the solution you obtained

I. UNDERSTANDING THE PROBLEM

- *What is the unknown?* • *What are the data?*
- *What is the condition?*
- *Is it possible to satisfy the condition?*
- *Is it sufficient? or redundant? or contradictory?*
- *Draw a figure?* • *Introduce suitable notations?*
- *Separate the various parts of the condition?*
- *Can you write them down?*

When you are stuck,
you should ask questions
to help generate ideas

II. DEVISING A PLAN

- *Have you seen it before?* • *or seen it in a slightly different form?*
- *Do you know a related problem?* • *or a useful theorem?*
- *Look at the unknown!* • *Problem with a similar unknown?*
- *Related problem and solved before? Can you use it?*
  or its results? or its method?
  or add auxiliary element to make it possible?
- *Can you re-state the problem? Re-state it still differently?*
  go back to definitions
- *Try to solve a related problem first? a more accessible related problem?*
  A more general problem? More special problem? An analogous problem?
- *Solve part of the problem first?*
  Keep only a part of the condition, drop the other part;
  How far is the unknown then determined, how can it vary?

II. DEVISING A PLAN

- *Can you derive something useful from the data?*
  What other data appropriate to determine the unknown
- *Can you change the unknown or the data or both to get*
  the new unknown and the new data nearer to each other?
- *Did you use all the data? the whole condition?*
- *Have you taken into account all essential notions involved?*

Step 2 is the "hardest",
so it has the most questions
to help you get unstuck.

III. CARRYING OUT THE PLAN

- ***Carry out your plan of solution.***
- ***Check every step.***
- ***Can you see clearly that each step is correct?***
- ***Can you prove that it is correct?***

The plan is rough, so
here you carry out the plan
carefully, step-by-step

III. LOOKING BACK

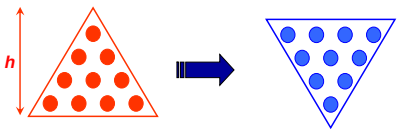
- ***Can you check the result?***
- ***Can you check the argument?***
- ***Can you derive the result differently?***
- ***Can you see it in a glance?***
- ***Can you use the result, or the method,***
  for other problem?

Looking back helps you
to gain insights into
your solution/approach

Creative Problem Solving Examples

- ❑ **Inverting a Triangle of Coins**
- ❑ **Good Coins, Bad Coin**
- ❑ **Man, Wolf, Goat, Cabbage**
- ❑ **Two containers**
- ❑ **…**

Problem: Inverting a Triangle of Coins



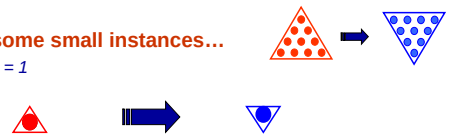
- "Invert" a triangle of coins by moving as few coins as possible
- Let *h* := "height of triangle"
  m := "# of coins to move"

Question: What is the value of *m*, when *h* = 263 ?

*NYGH cutoff for 2010: **263***

Inverting a Triangle of Coins (1)

- Try some small instances…
  - *h = 1*



- TRIVIAL:
  - *h = 1* *m = 0*

Inverting a Triangle of Coins (2)

- Now, try something bigger…
  - *h = 2*

- TRIVIAL:
  - *h = 1* *m = 0*
  - *h = 2* *m = 1*

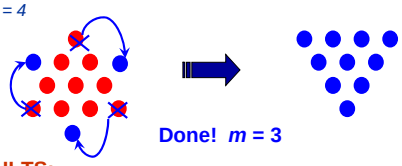
Inverting a Triangle of Coins (3)

- Now, try something bigger…
  - *h = 3*

- RESULTS:
  - *h = 1* *m = 0*
  - *h = 2* *m = 1*
  - *h = 3* *m = 2*

Inverting a Triangle of Coins (4)

- Now, try original problem…
  - *h = 4*



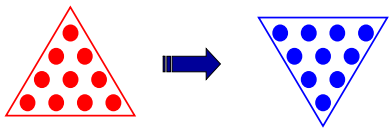
- RESULTS:
  - *h = 1* *m = 0*
  - *h = 2* *m = 1*
  - *h = 3* *m = 2*
  - *h = 4* *m = 3*

Is there a pattern?

What about *h = 5?* *h = 263*

We Need a Different Perspective!!

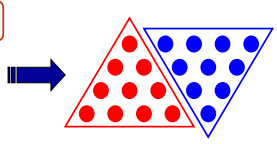
- Let's see….



- How about… seeing them *together?*
- Let's move *THEM…*

We Need a Different Perspective (2)

New Perspective

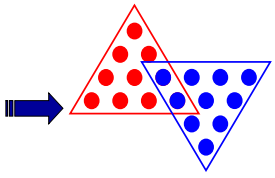


New Question:

How to turn the "orange coins" into the "blue coins"?

Need to move 10 coins!

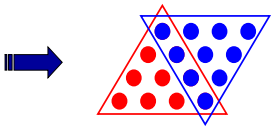
We Need a Different Perspective (3)



New Operation:
Move the orange triangle around the blue triangle.

Need to move 8 coins!

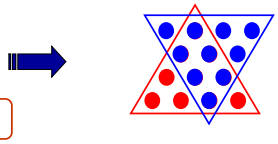
We Need a Different Perspective (4)



New Operation:
Move the orange triangle around the blue triangle.

Need to move 6 coins!

We Need a Different Perspective (5)

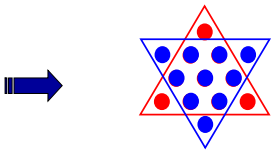


New Insight

Minimize "moves" = Maximize overlap

Need to move 4 coins!

We Need a Different Perspective (6)

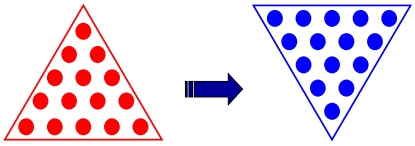


Is this the maximum overlap?

Need to move 3 coins!

Try new insight on larger problem

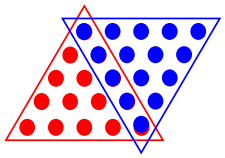
Now try *h* = 5



Move triangle & maximum overlap

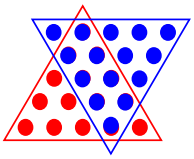
Maximizing Overlap for h=5 (1)

- **There are 3 sides in each triangle.**
  - *At each side, we have a "non-overlapping" triangle!*
  - *Denote their heights by (a,b,c)*



(0,0,4)

10 coins!

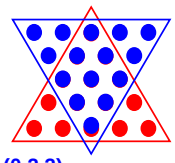


(0,1,3)

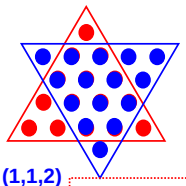
7 coins!

Maximizing Overlap for h=5 (2)

- More configurations…



(0,2,2) 6 coins!

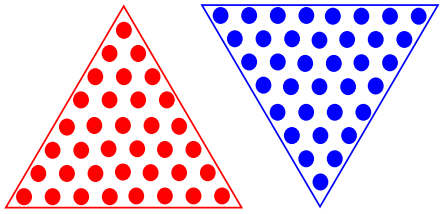


(1,1,2) 5 coins!

- Note that (a + b + c) = 4
- Which configuration gives minimum *m*?

Activity Period #1:

"Invert Triangle of Coins" for *h* = 8 (DIY)
(5 minutes)

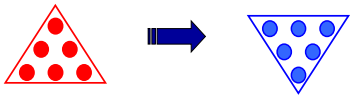


Review of Activity #1

- Did you try with real coins?
  - Yes: _____ No: _____
- What is the value of (a + b + c)?
- What configuration (a, b, c) is *minimum*?
  - (a, b, c) = (__ , __ ,__)
  - How many coins moved? _____ coins
- Was it easy?
- ***Was it fun?***

Maximizing Overlap is Easy…

- Try for previous cases h = 3



- "Non-overlapping triangles with hts (a,b,c)
  - *For h=2, we have (0,0,1) m=1*
  - *For h=3, we have (0,1,1) m=2*
  - *For h=4, we have (1,1,1) m=3*
  - *For h=5, we have (1,1,2) m=5*
  - *For h=6, we have (1,2,2) m=7*
  - *For h=7, we have (2,2,2) m=9*
  - *For h=8, we have (2,2,3) m=12*

Looking Back…(ala Polya)

- Can work out for h = 9, 10, 11, …, 263
- The ANSWER is NOT so important, the *METHOD IS more important !*
- Where is the *Key Step*? *The Aha! moment*
- Why was it *not apparent* to us *at the beginning* ?
- How *did* we get to this step?

Moral of the Story

- **Don't just solve *a* problem.**
- **Look for a *general* method.**
- **View problem from *different perspective*.**
- **Look for an *Aha moment*.**
- **Ask "How can I solve something similar if I see one in the future?"**

Good Coins, Bad Coin

You have 11 gold coins. All except one of them have the same weight; one is defective (meaning it is either slight heavier or lighter).
You have to find the defective gold coin among the eleven coins. You are given a balance (like the one shown in the picture) to help you.
Find the defective gold coin using the balance only three times.

Man, Wolf, Goat, Cabbage

- **Skipped (due to lack of time)**

Two mathematicians…

- **Two mathematicians met after 27 years…**

This was their conversation.

First: How are you doing?
Second: Fine I got married and have 3 daughters.
First: Fine. I also got married 2 years after the college. How old are your daughters?
Second: Well, Product of their ages is 72 and their sum is equal to that house number over there. (The actual number was not mentioned).
First: Wait, I am still counting.
Second: Sorry, youngest of them just started walking.
First: Is it! Youngest of mine is also of the same age.

How old are the three daughters?

References…. for More readings...

- G. Polya, "***How to Solve It***",
  - ***Princeton Univ Press (1945),***
  - ***re-published by Penguin Books (1990)***
- W. A. Wickelgren, ***"How to Solve Problems"***,
  - ***W. H. Freeman, 1974.***
- R. R. Kadesch, ***"Problem Solving Across the Disciplines"***,
  - ***Prentice Hall, 1997.***
- Edward De Bono,
  - ***Various Different Titles (from Lateral Thinking, to Teaching Thinking, to Hats, to Simplicity).***
- [EvAn] ***Everywhere and Anywhere -- Use your imagination!***

Thank you!

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