

Bill Gates and the Pancake Flipping Problem

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Experience the fun of problem solving

Pancake Flipping

Overview:

- ❑ When did Bill Gates flip pancakes**
- ❑ The Pancake Flipping Problem**
- ❑ What we know about Pancake Flipping**
- ❑ Why Study Pancake Flipping?**

Did you know that ...



I used to
flip pancakes
[翻煎饼].

Bill Gates [比尔 盖茨], *formerly* Microsoft CEO

What is *pancake flipping*?

Input:
“Messy” stack of
pancakes [煎饼]



Output: Neat,
sorted stack of
pancakes

Source: Neil Jones and Pavel Pevzner, 2004
“Introduction to BioInformatics Algorithms”.

pancake flipping



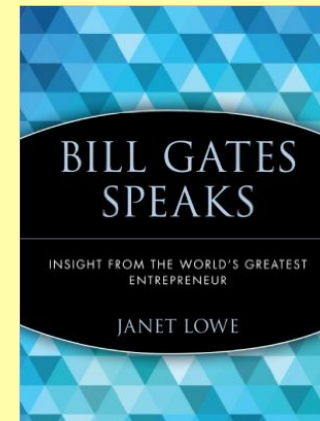
Did Bill Gates *really* flip pancakes?

THE PANCAKE PUZZLE

“The chef in our place is sloppy, and when he prepares a stack of pancakes they come out all different sizes. Therefore, when I deliver them to a customer, on the way to the table I rearrange them (so that the smallest winds up on top, and so on, down to the largest at the bottom) by grabbing several from the top and flipping them over, repeating this (varying the number I flip) as many times as necessary. If there are “ n ” pancakes, what is the maximum number of flips (as a function of “ n ”) that I will ever have to use to rearrange them?”



Source: Neil Jones and Pavel Pevzner, 2004 “Introduction to BioInformatics Algorithms”.



by Janet Lowe.
(see page 19)

<http://www.npr.org/templates/story/story.php?storyId=92236781>

Pancake Flipping

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Pancake Flipping Problem

Given an initial pancake configuration...

You want to get a “*sorted*” configuration ...

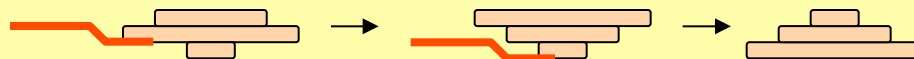
Constraints: can *only* flip ...



Source: Neil Jones and Pavel Pevzner,
 2004 “Introduction to BioInformatics
 Algorithms”.



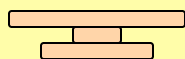
Example ...



2 flips

Bill Gates & Christos Papadimitriou:, “Bounds For Sorting By
 Prefix Reversal.” *Discrete Mathematics*, Vol 27, pp 47-57, 1979.

More pancake-flipping examples...



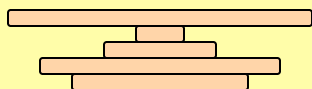
2 flips

**Abstraction skills,
Problem Solving skills**



3 flips

*Need a systematic
approach...
an algorithm!*

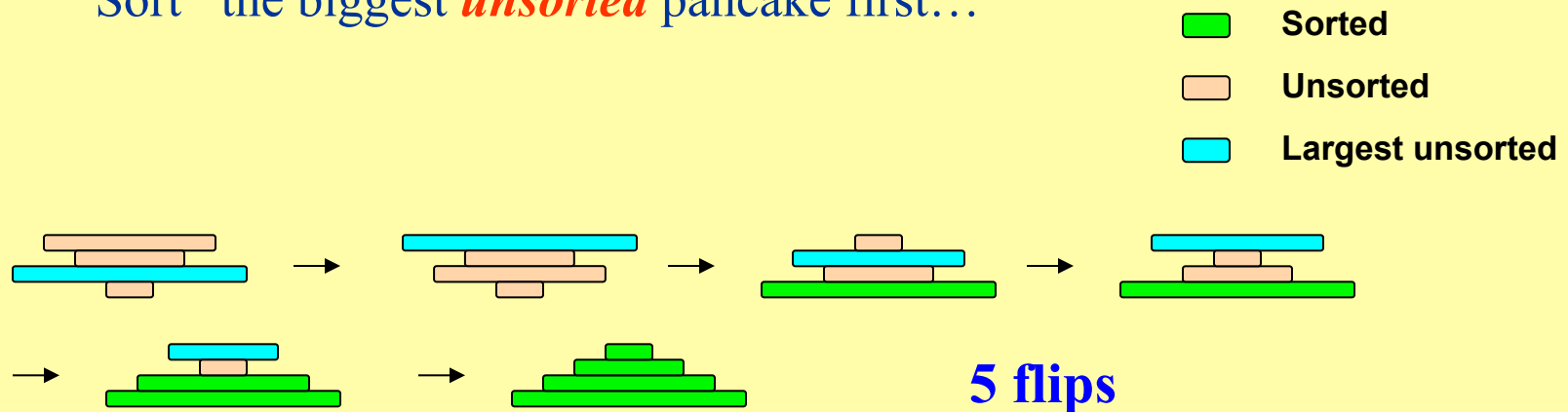


? flips

An Initial Algorithm (Greedy)

Simple Idea:

“Sort” the biggest *unsorted* pancake first...

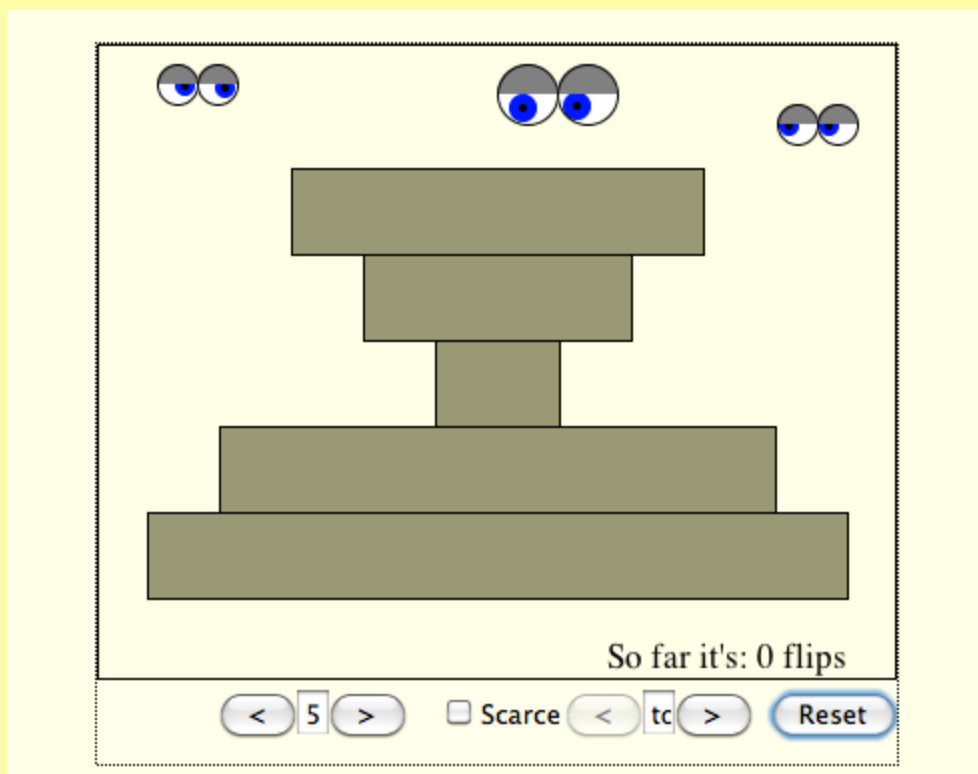


Greedy Algorithm:

Repeatedly “sort” the biggest pancake;

Try it Online:

<http://www.cut-the-knot.org/SimpleGames/Flipper.shtml>

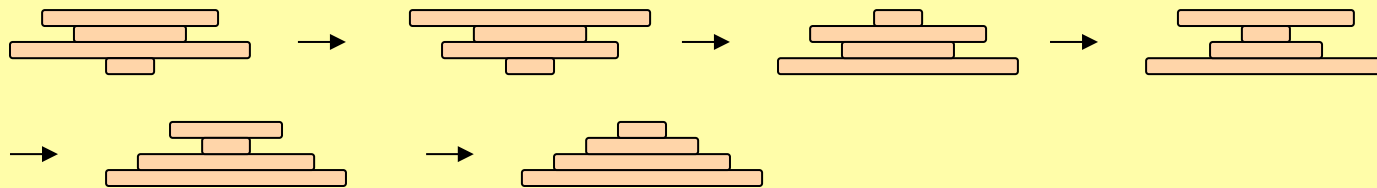


Is Greedy “the best” possible?

Answer: NO

A Counter Example:

Greedy method [5 flips]



Better way [3 flips]



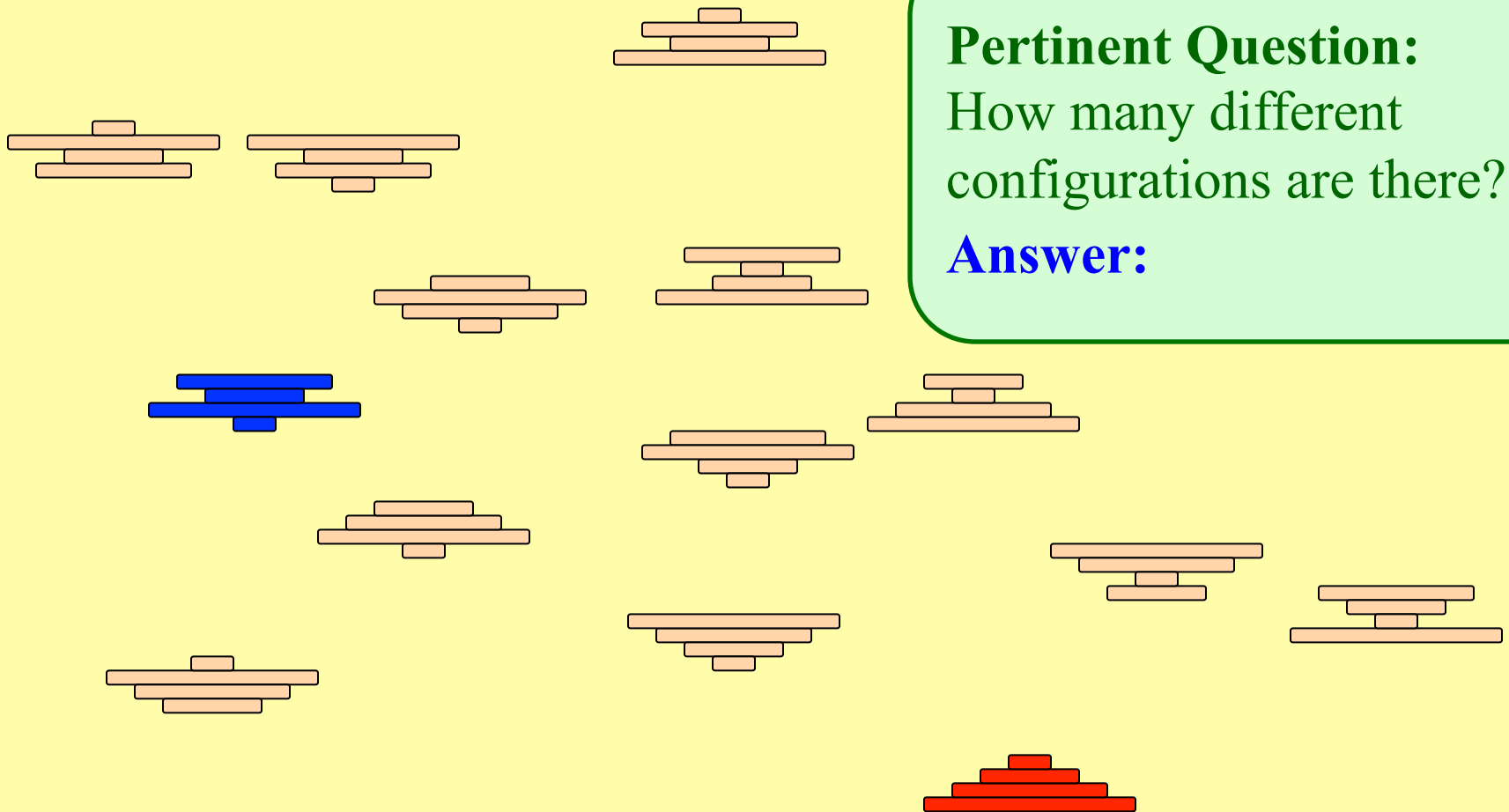
Question: Design an algorithm that solve the pancake flipping problems using the *minimum number of flips*.

Pancake Flipping Problem...

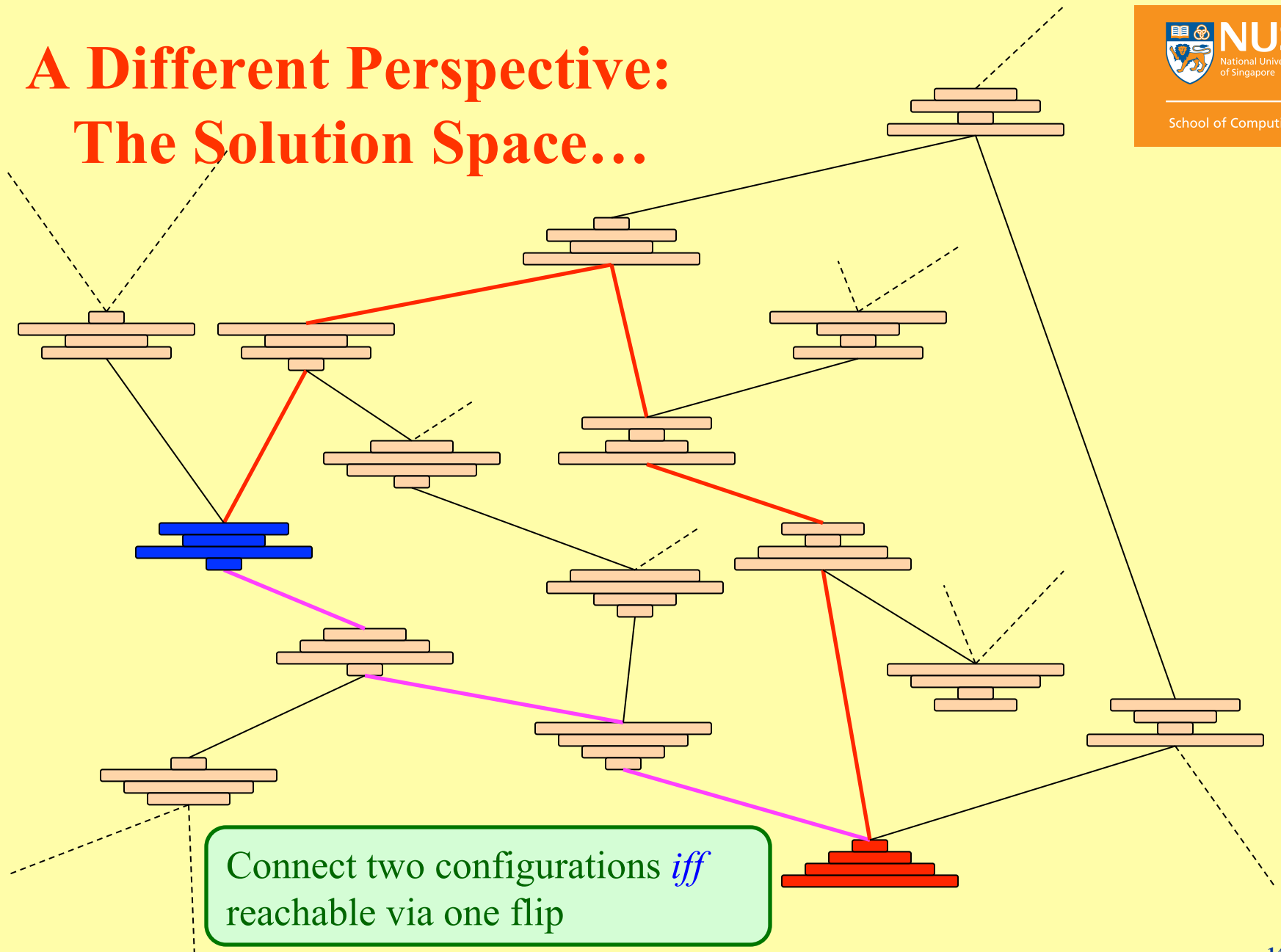


*Sometimes, it is
good to look from
another perspective!*

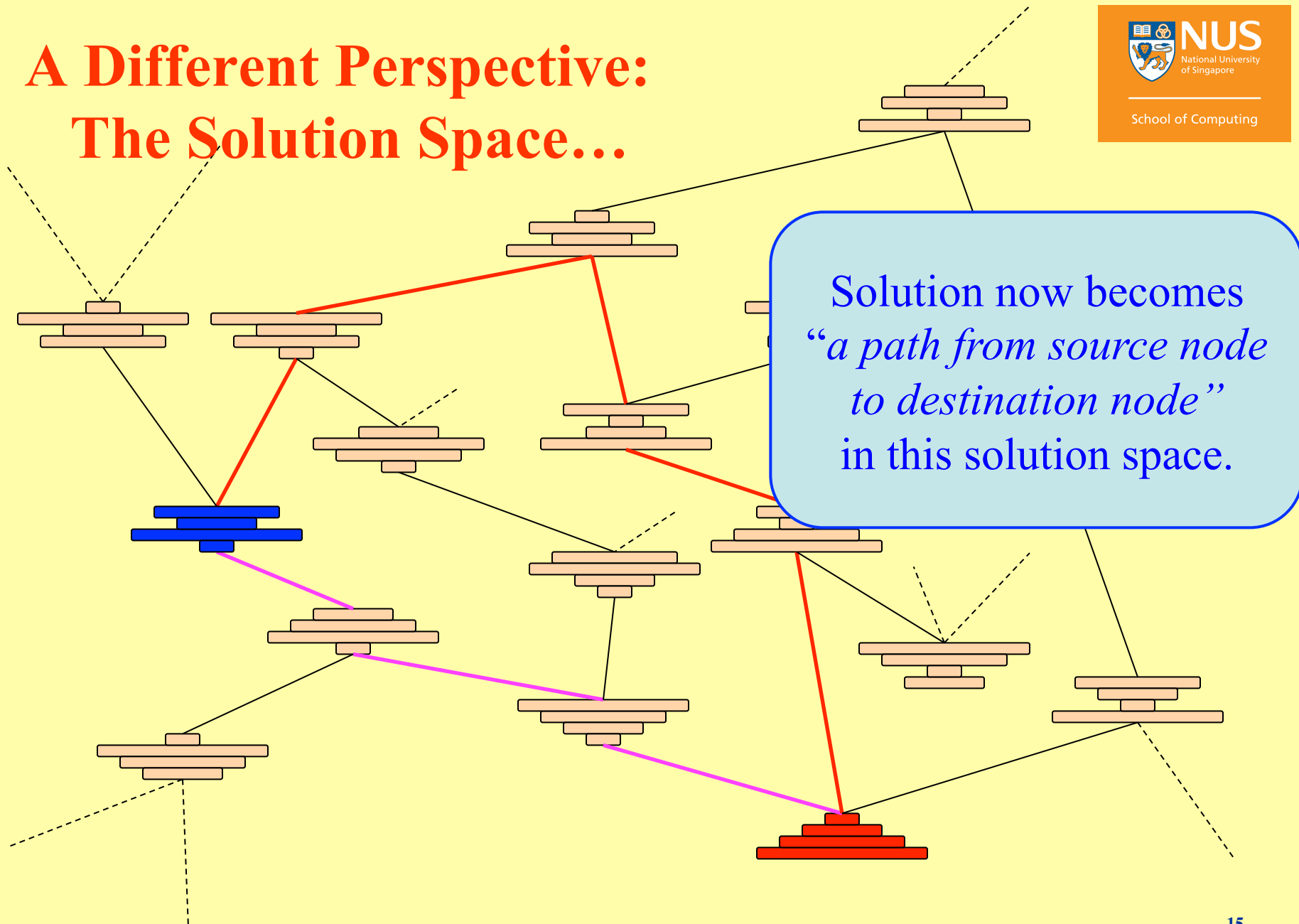
A Different Perspective: The Solution Space...



A Different Perspective: The Solution Space...

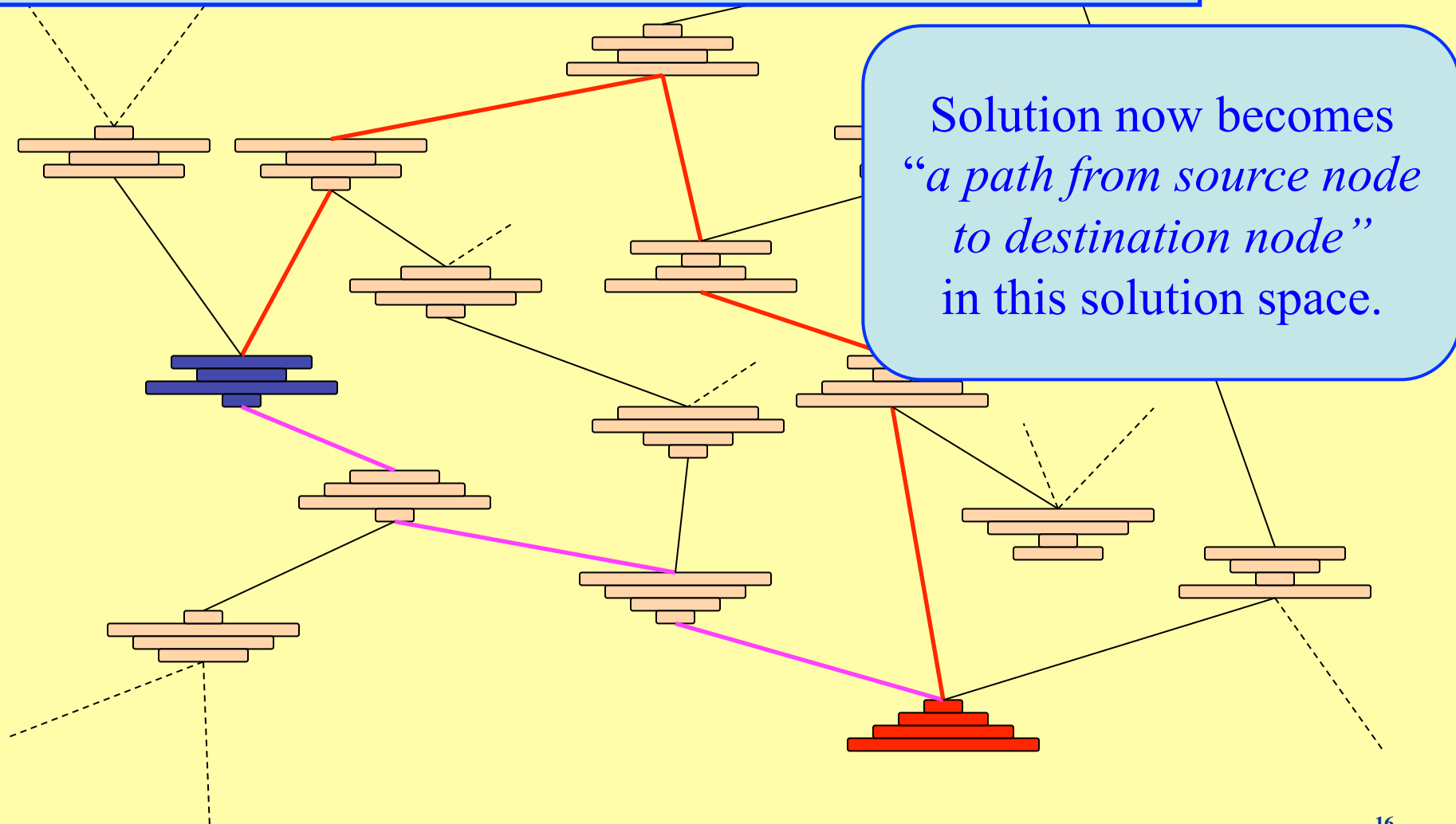


A Different Perspective: The Solution Space...



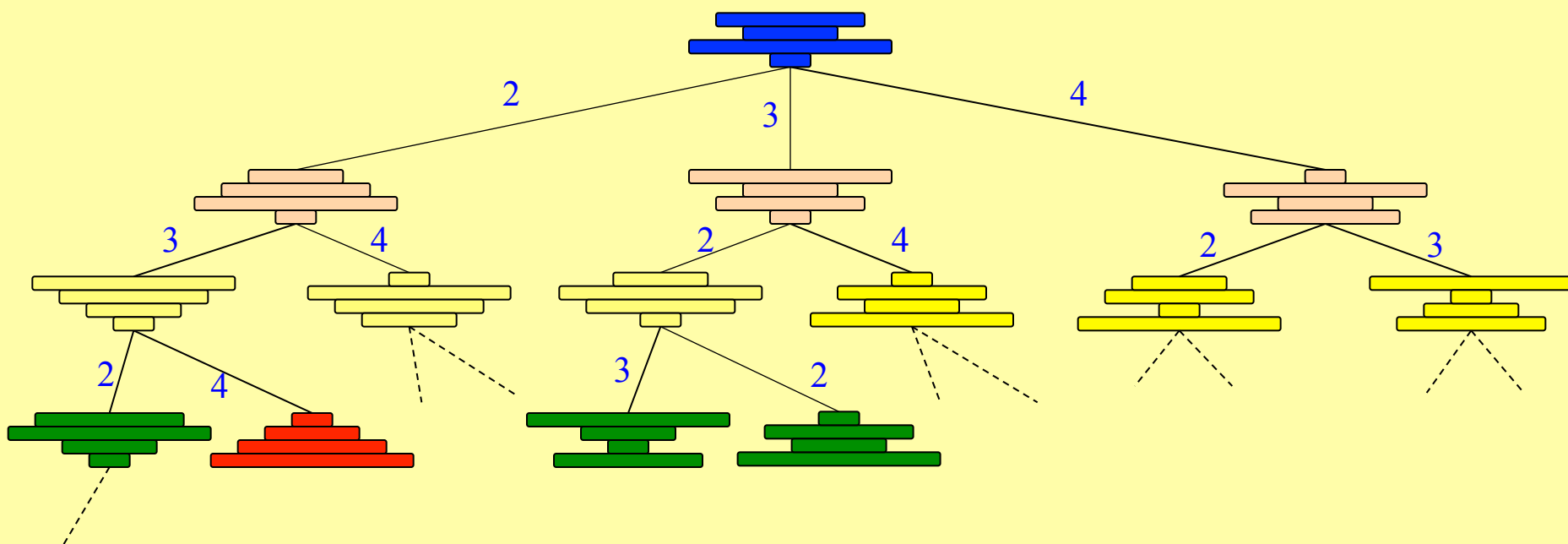
New Problem:

Want a smart method (algorithm) to search this space to find the optimal flipping solution.



A Search Tree Method:

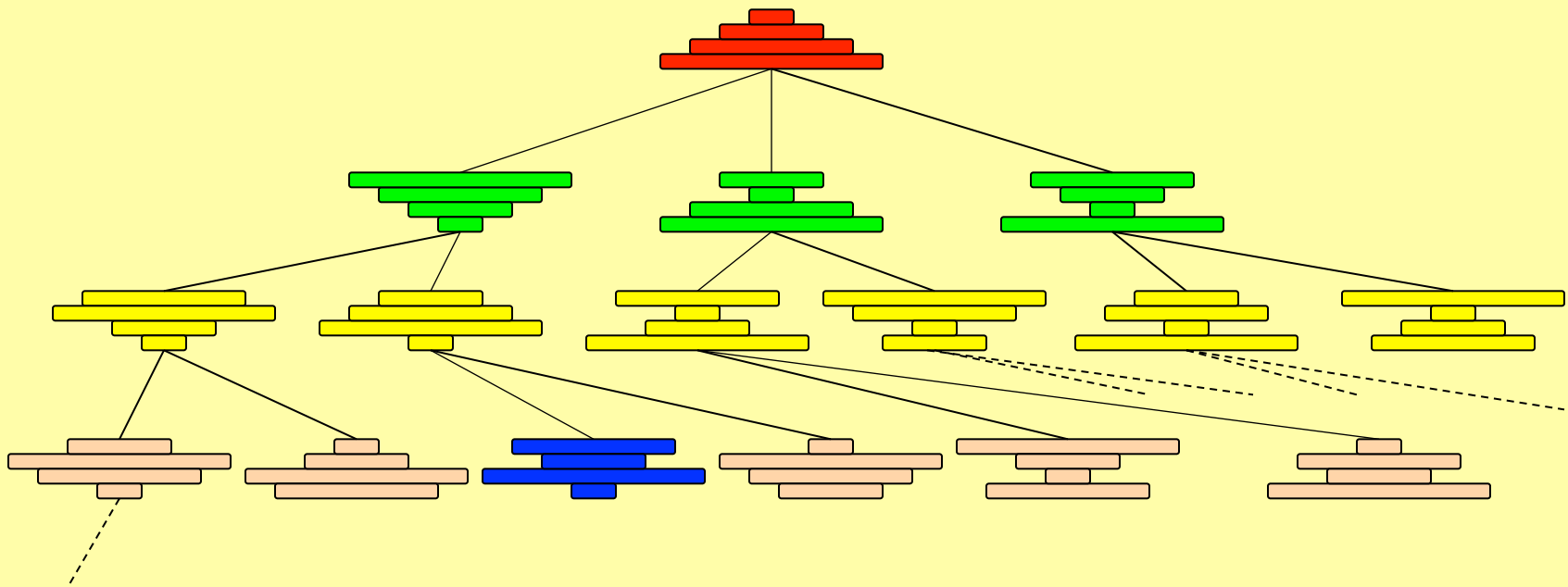
(systematically search the search space)



Problem: Want a smart method (algorithm) to search this space to find the optimal flipping solution.

A Search Tree Method:

(systematically search the search space)



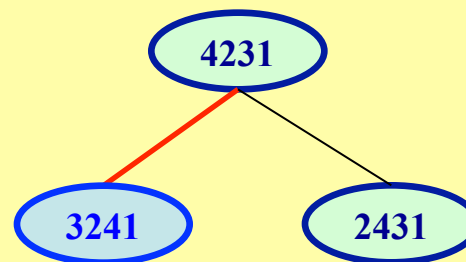
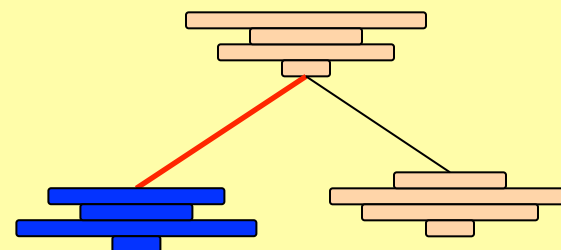
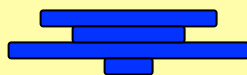
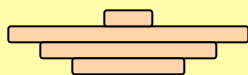
PQ: Can we work backward from the solution to the data?
(In AI, this is called “backward chaining”; the previous search [from start towards destination] is called “forward chaining”.)

A Side-Track...

Real-Life or Model World)
Which works better?

Real World (of Pancakes...) vs Model world (of Graph...)

Represent pancake sizes by numbers.



The Model World...

312

? flips

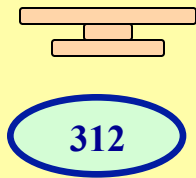
4213

? flips

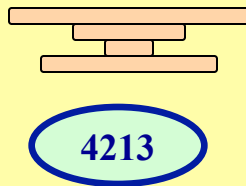
52143

? flips

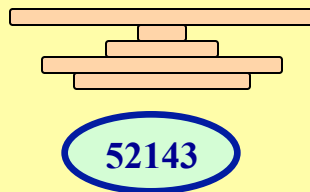
Let's see both together...



? flips



? flips



? flips

**Real-World
or
Model World?**

**Which works better
for YOU?**

Is Greedy “the best” possible?

Answer: NO

A Counter Example:

Greedy method [5 flips]

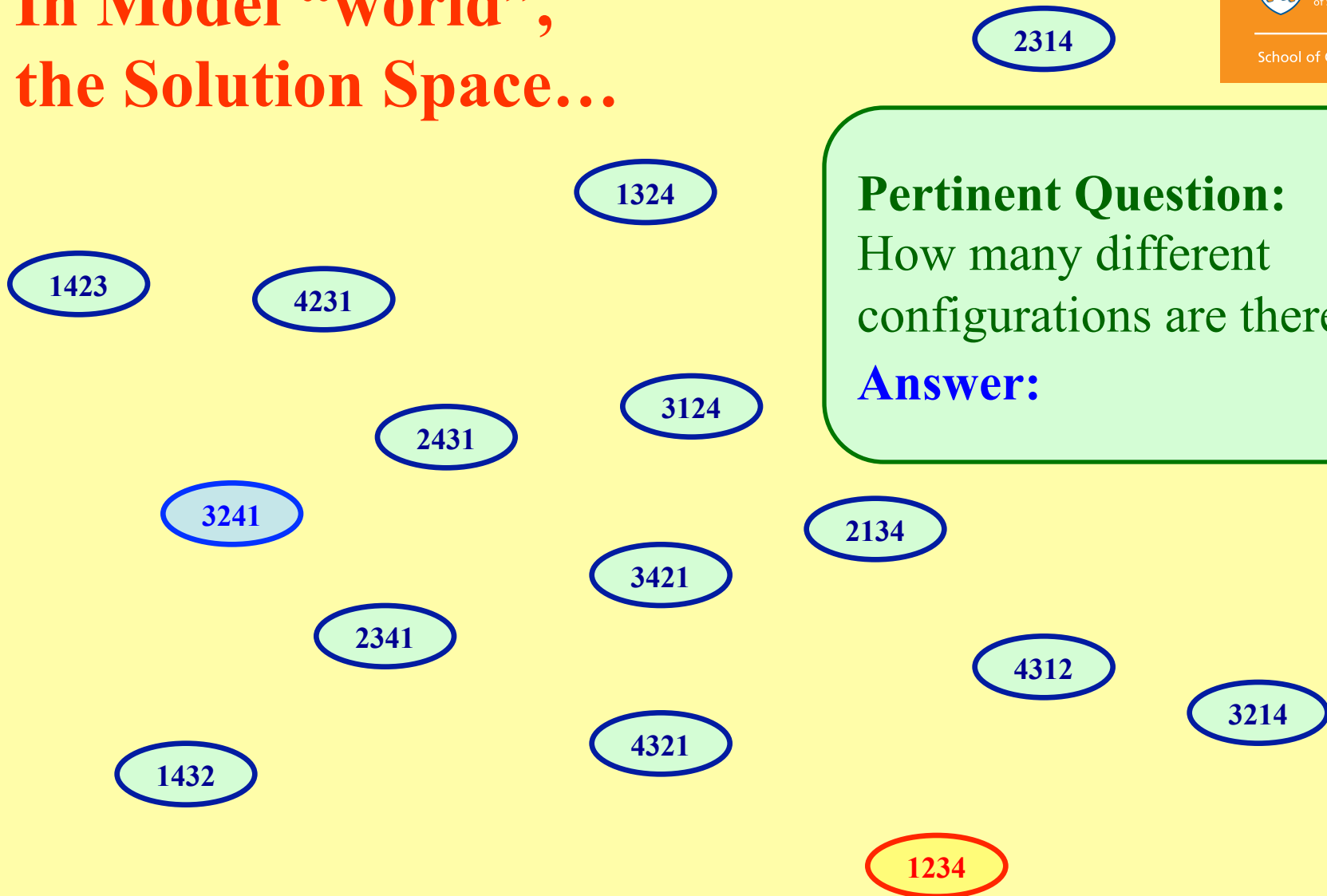


Better way [3 flips]



Question: Design an algorithm that solve the pancake flipping problems using the *minimum number of flips*.

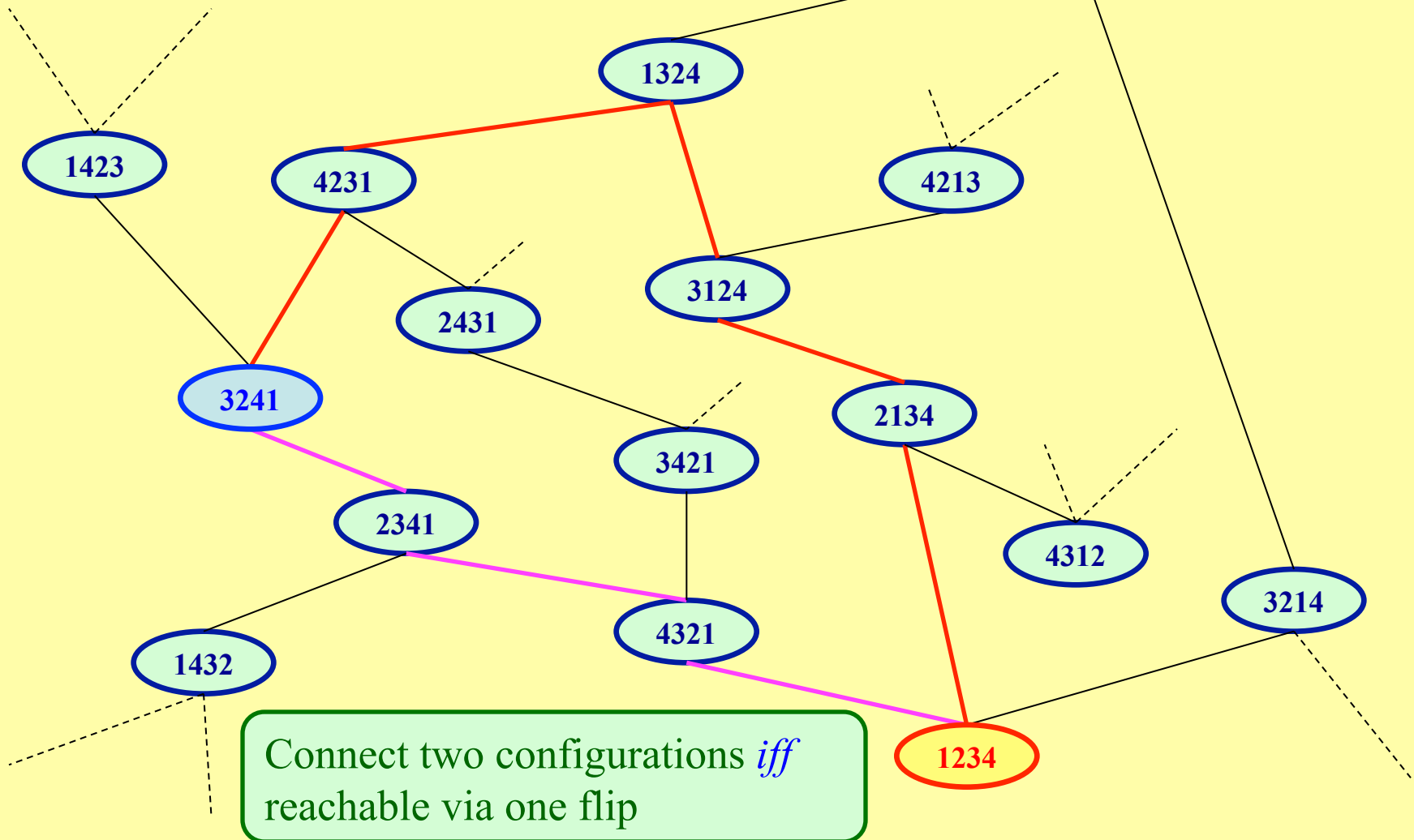
In Model “world”, the Solution Space...



Pertinent Question:
How many different
configurations are there?

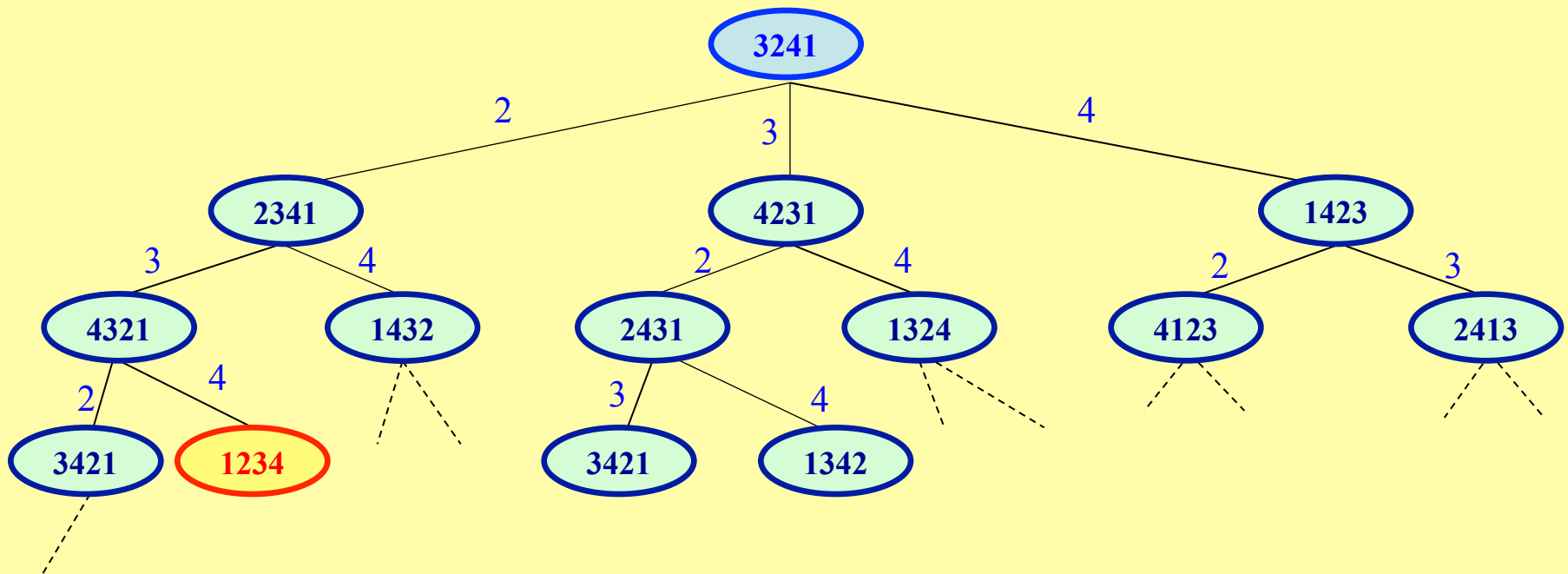
Answer:

In Model “world”, the Solution Space...



A Search Tree Method:

(systematically search the search space)



Problem: Want a smart method (algorithm) to search this space to find the optimal flipping solution.

Pancake Flipping

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- ❑ What we know about Pancake Flipping
- ❑ Why Study Pancake Flipping?

Pancake Flipping Problem: Known Results

- Greedy Algorithm uses at most $2n-3$ flips
- For n pancakes, at most $(5n+5)/3$ flips are needed
[Bill Gates and Papadimitriou, 1979] $\sim 1.666n$
- 2008 (about 30 years later), at most $18n/11$ needed
[a team from UT-Dallas, 2008] $\sim 1.6363n$ (*diff*: $< 2\%$)
- Pancake flipping problem is an open problem in math
and an NP-hard problem in computer science

Pancake Flipping

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Why study pancake flipping

- **Mathematics – Study its properties**
 - define $f(n)$ to be the minimum of number of flip for any configuration of n pancakes
- **Computing – Want an algorithm to solve it**
 - Given any n pancake configuration, sort it with the *minimum* number of flips

Math vs Computer Science

Problem:

We need to drive from NUS-SOC to Kuala Kubu Bahru in West Malaysia?

Mathematicians:

may give a Theorem

There exist a path with total distance no more than 450km.

But, a computer scientist wants the exact route (and more)

← from National University of Singapore School of Com...
to Kuala Kubu Bharu, Selangor, Malaysia

4 h 58 min (440 km)

via Lebuhraya Utara-Selatan/AH2/E2

4 h 20 min without traffic

⚠ This route has tolls.

⚠ This route crosses a country border.

National University of Singapore School of Computing

13 Computing Drive, Singapore 117417

- Get on AYE from Kent Ridge Dr and Clementi Rd

6 min (2.1 km)

- Take Lebuhraya Utara-Selatan/AH2/E2, ELITE/E6 and AH2/E1 to Route 3208 in Bukit Beruntung, Rawang, Malaysia. Take exit 118-Bkt. Beruntung from Lebuhraya Utara - Selatan/AH2/E1

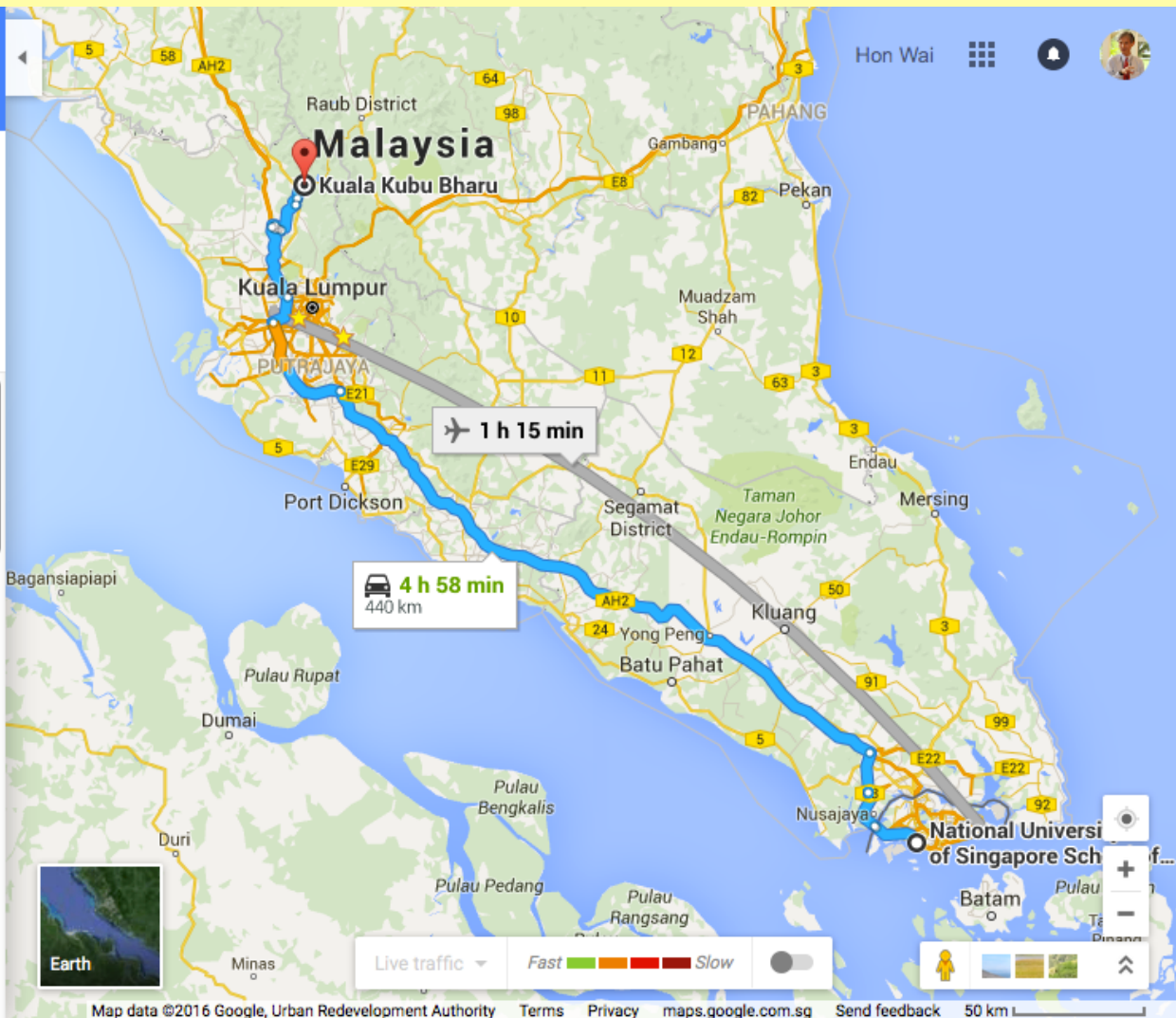
4 h 1 min (409 km)

- Take Route 1 to Jalan Bukit Kerajaan in Pekan Kuala Kubu Bharu, Kuala Kubu Baru

31 min (29.1 km)

Kuala Kubu Bharu

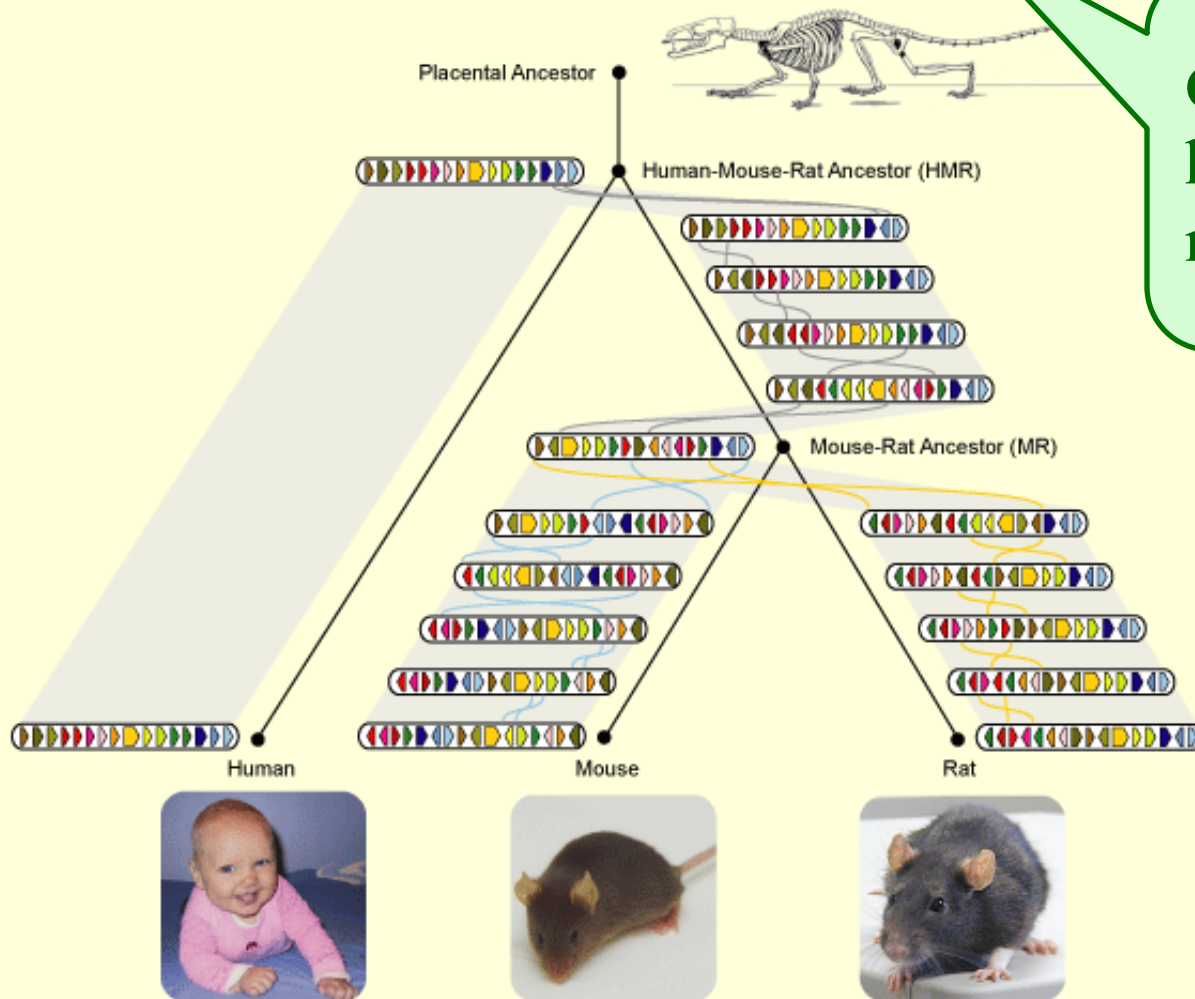
Selangor, Malaysia



Why study pancake flipping

- **Mathematics – Study its properties**
 - define $f(n)$ to be the minimum of number of flip for any configuration of n pancakes
- **Computing – Want an algorithm to solve it**
 - Given any n pancake configuration, sort it with the *minimum* number of flips
- **Applications**
 - sorting by prefix reversal
 - used to study evolution of species in biology

Sorting by Reversals used here...



Question: Is human closer to mouse or rat?

Application of Sorting by Reversals

SIAM Journal on Computing

SIAM J. Comput. / Volume 25 / Issue 2

▲ HITS

Genome Rearrangements and Sorting by Reversals

SIAM J. Comput. Volume 25, Issue 2, pp. 272-289 (1996)

Issue Date: 1996

ABSTRACT

REFERENCES (28)

CITING ARTICLES

Vineet Bafna and Pavel A. Pevzner

Sequence comparison in molecular biology is in the beginning of a major paradigm shift—a shift from gene comparison based on local mutations (i.e., insertions, deletions, and substitutions of nucleotides) to chromosome comparison based on global rearrangements (i.e., inversions and transpositions of fragments). The classical methods of sequence comparison do not work for global rearrangements, and little is known in computer science about the edit distance between sequences if global rearrangements are allowed. In the simplest form, the problem of gene rearrangements corresponds to sorting by reversals, i.e., sorting of an array using reversals of arbitrary fragments. Recently, Kececioğlu and Sankoff gave the first approximation algorithm for sorting by reversals with guaranteed error bound 2 and identified open problems related to chromosome rearrangements. One of these problems is Gollan's conjecture on the reversal diameter of the symmetric group. This paper proves the conjecture. Further, the problem of expected reversal distance between two random permutations is investigated. The reversal distance between two random permutations is shown to be very close to the reversal diameter,

Important Application in Computational Biology:
 Used to study the evolution from one species to another.

Curious?
 See more details in the extra slides at the end.

Relevant Skills and Courses

- **Pancake flipping is a *model* for**
 - sorting by prefix-reversals
- **Many CS problems are model in *similar* ways**
 - sending files over internet (routing problems)
 - time table scheduling (graph colouring, 图着色问题)
- **Courses to learn these things**
 - CS1231 (Discrete Mathematics, 离散数学) [Blogs: [1](#), [2](#),]
 - CS3230 (Analysis of Algorithms, 算法设计与分析)
 - new!

More on Pancake Flipping

Have some fun with pancake flipping:

<http://www.cut-the-knot.org/SimpleGames/Flipper.shtml>

Read about Pancake Flipping:

<http://www.amazon.com/Bill-Gates-Speaks-Greatest-Entrepreneur/dp/0471401692>

Listen to the story on NPR (July-2008):

<http://www.npr.org/templates/story/story.php?storyId=92236781>

Youtube Video:

<https://www.youtube.com/watch?v=oDzauRFiWFU>

How are the following people connected to Bill Gates?

Jacob Goodman, Harry Lewis, Harry Dweigher,
David Cohen, Manuel Blum

More on Genome Rearrangements

Genome Rearrangement:

<http://www.cut-the-knot.org/SimpleGames/Flipper.shtml>

Read about Pancake Flipping:

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Thank you!

**If you want to contact me,
email, FB: leonghw@comp.nus.edu.sg**

or

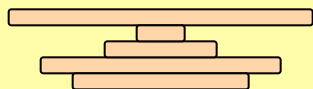
Google “Leong Hon Wai”

The Remaining slides are

FOR YOUR FUN READING ONLY

Activity: Fun with Pancake Flipping

*Try your hand on this example.
How many flips are needed?*



? flips

What are Reversals in the study of Genomics?

Example: (2 genomes G and H , from different species)

$$G = \{ 1, -5, 4, -3, 2 \} \quad H = \{ 1, 2, 3, 4, 5 \}$$

A **genome** consists of a sequence of **genes**;

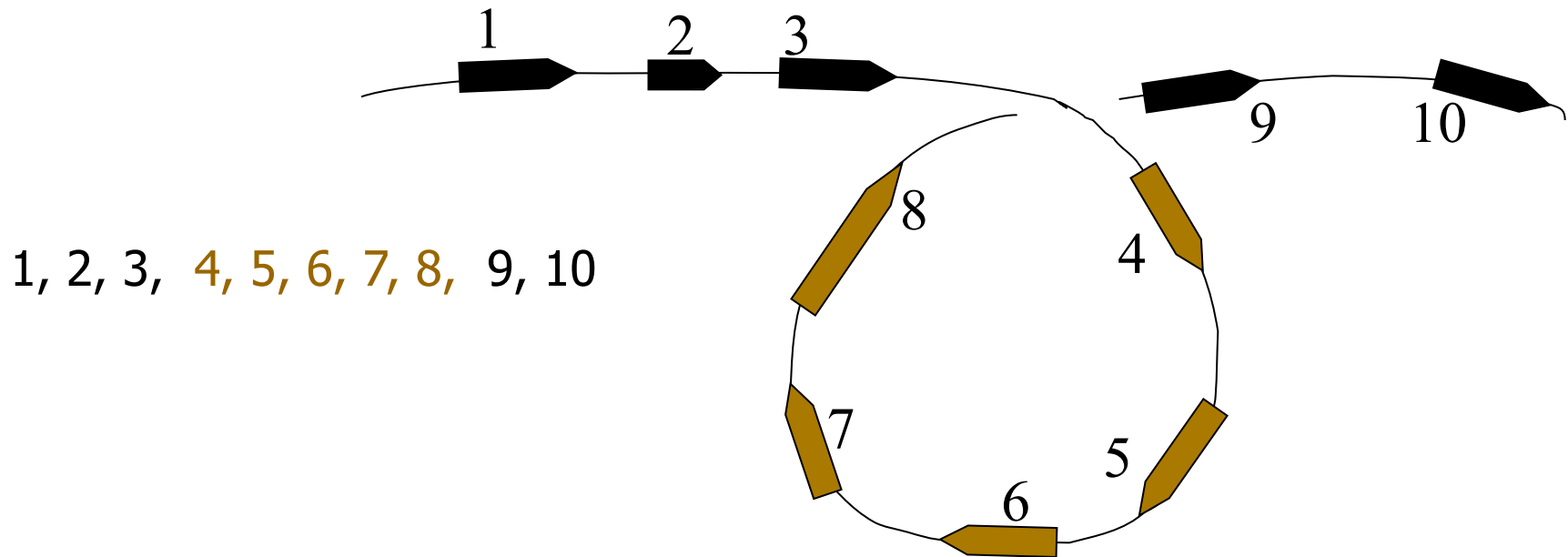
Represent each gene by a integer $\{1, 2, \dots, n\}$.

Each gene has a “*direction*” indicated by the sign (like -3).

Then a genome is a “like” a stack of pancakes of diff sizes.

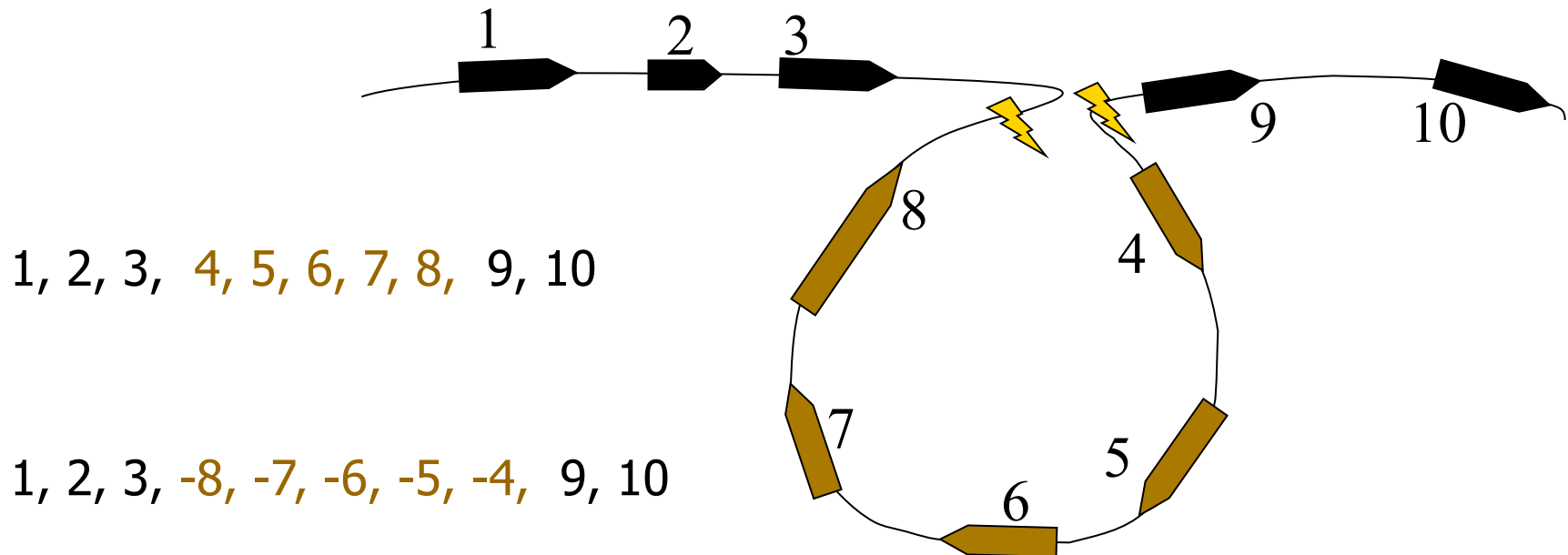
(each pancake is burnt on one side, indicate “direction”)

Reversals in the Gene Sequence



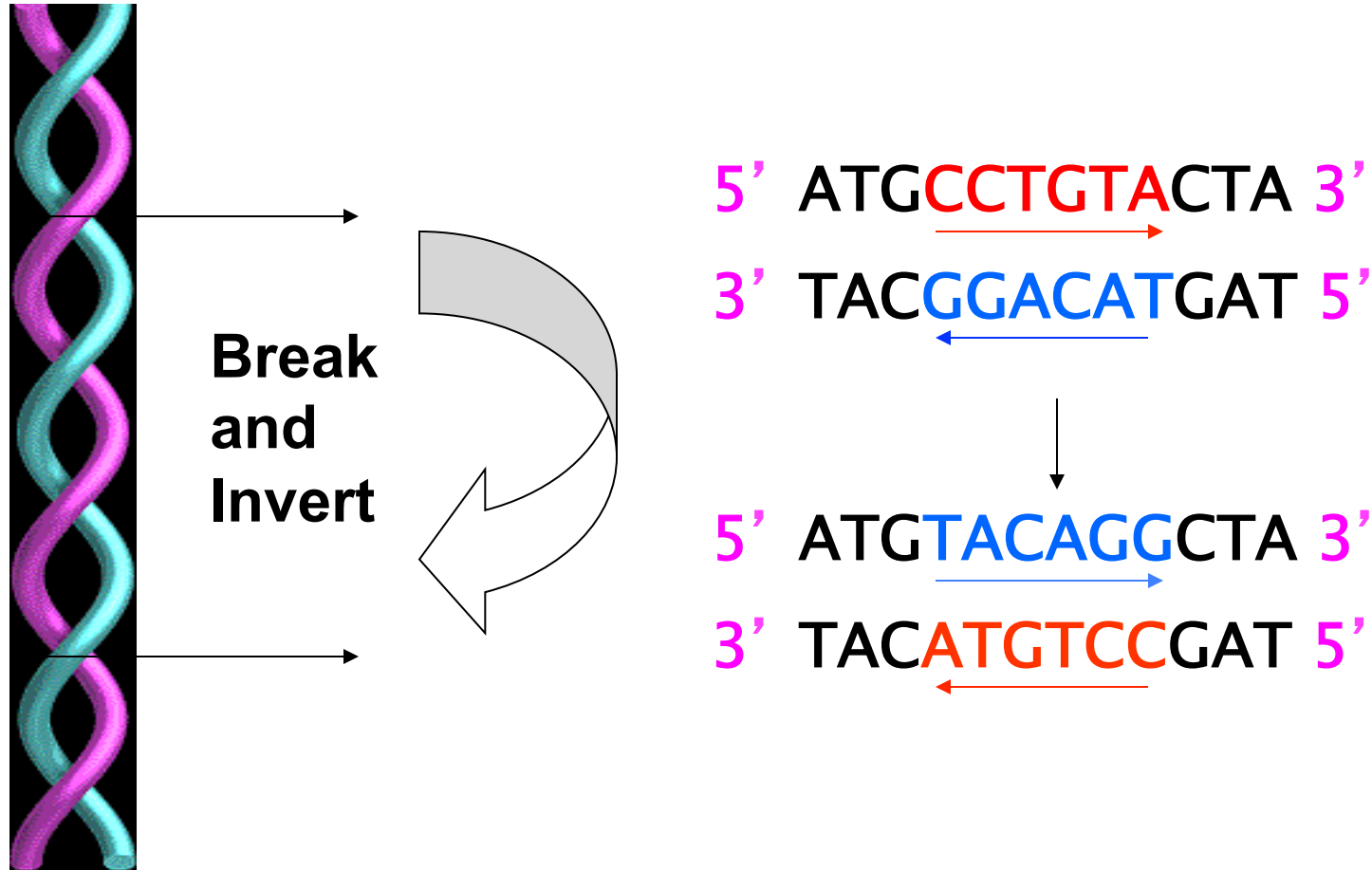
* Consider the block of genes (4, 5, 6, 7, 8,)

Reversals and Breakpoints



- * Consider the block of genes (4, 5, 6, 7, 8,)
- * The reversal introduced two *breakpoints* ⚡
- * Order of genes reversed, and direction inverted
(4, 5, 6, 7, 8,) → (-8, -7, -6, -5, -4,)

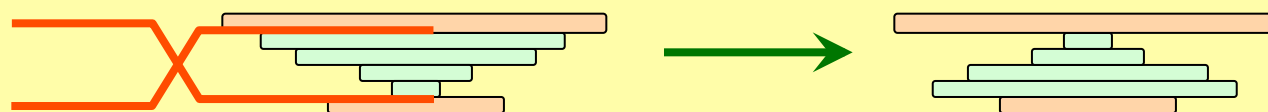
Reversals: Example



Reversals are Special Pancake Flips



*Need a special spatula
With 2-prongs*



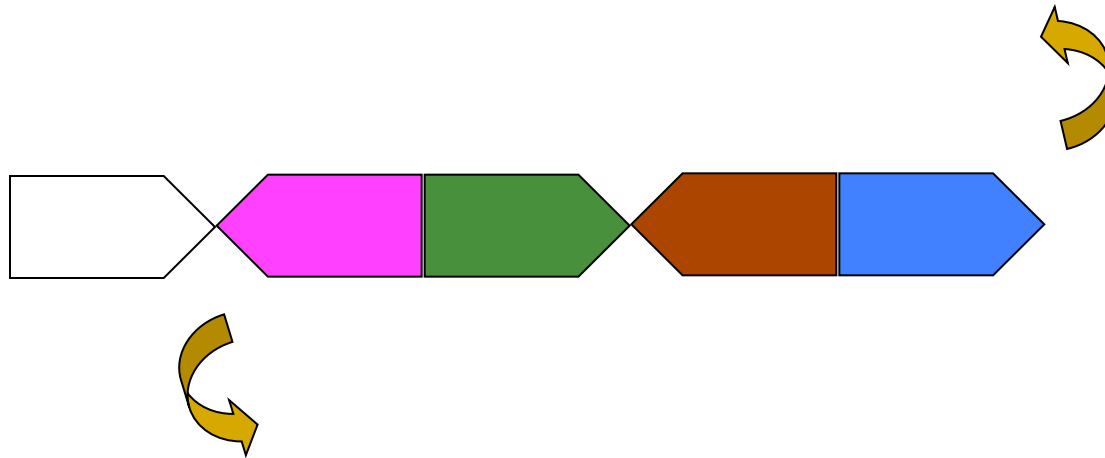
6 | 5421 | 3



6 | -1 -2 -4 -5 | 3

Evolution of the gene sequence via reversals

- Gene order comparison:



Evolution of the gene sequence via reversals

- Gene order comparison:



Evolution of the gene sequence via reversals

- Gene order comparison:



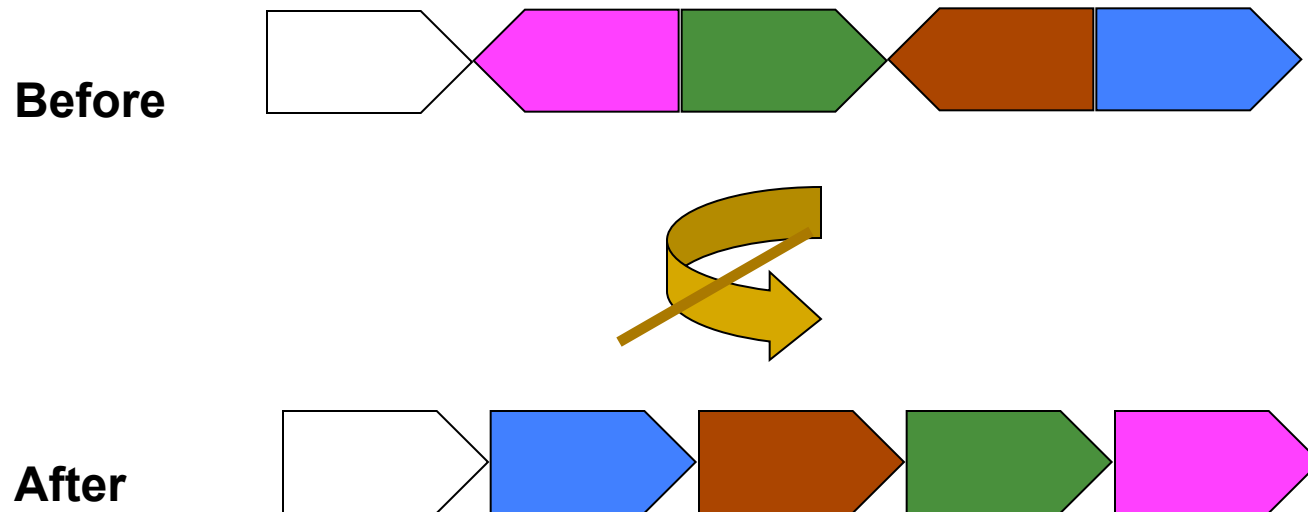
Evolution of the gene sequence via reversals

- Gene order comparison:



Evolution of the gene sequence via reversals

- Gene order comparison:

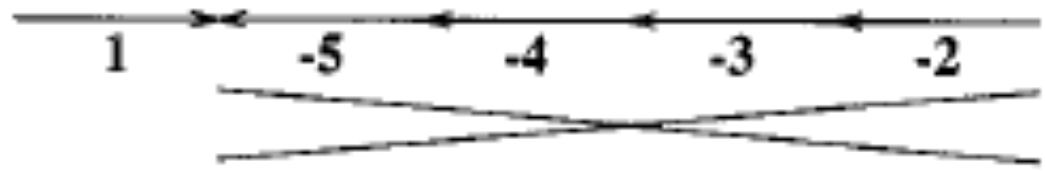
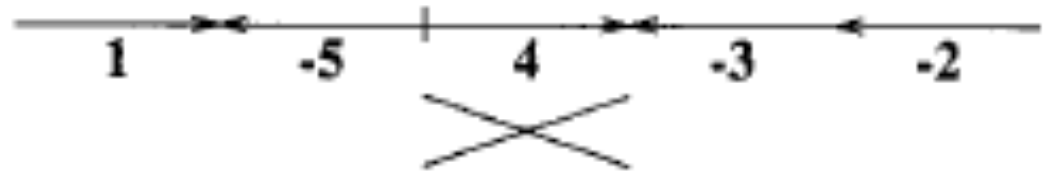
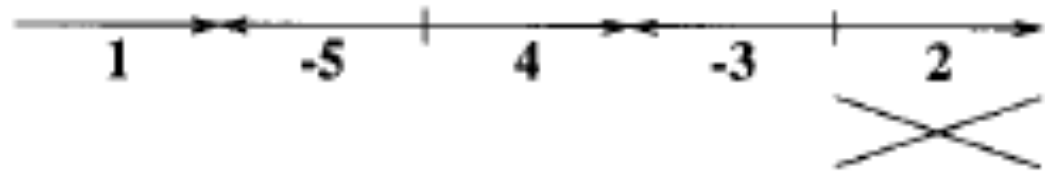


Evolution is manifested as the divergence in gene order

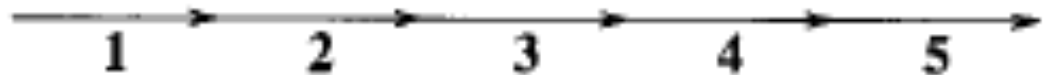
Transforming Cabbage into Turnip

(with 3 reversal operations)

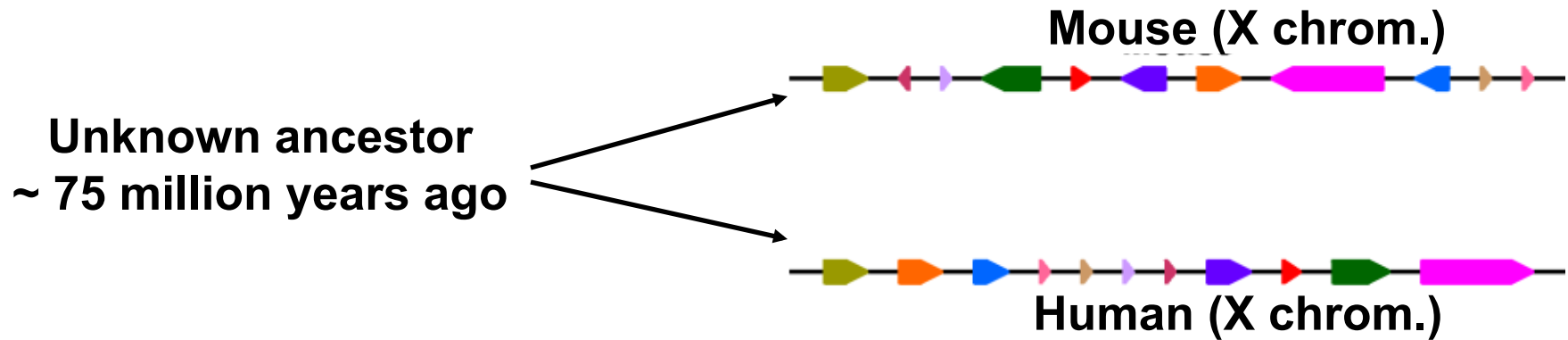
B. oleracea
(cabbage)



B. campestris
(turnip)



Genome rearrangements



- What are the similarity blocks and how to find them?
- What is the architecture of the ancestral genome?
- What is the evolutionary scenario for transforming one genome into the other?

History of Chromosome X

