Plane-Fitting Robust Registration for Complex 3D Models

Yuan Cheng, Shudong Xie, Wee Kheng Leow, and Kun Zhang

Dept. of Computer Science, National University of Singapore
{cyuan,xshudong,leowwk,zhangkun}@comp.nus.edu.sg

Abstract. In surgery planning, forensic and archeology, there is a need to perform analysis and synthesis of complex 3D models. One common first step of 3D model analysis and synthesis is to register a reference model to a target model using similarity transformation. In practice, the models usually contain noise and outliers, and are sometimes incomplete. These facts make the 3D similarity registration challenging. Existing similarity registration methods such as Iterative Closest Point algorithm (ICP) [1] and Fractional Iterative Closest Point algorithm (FICP) [2] are misled by the outliers and are not able to register these models properly. This paper presents a plane-fitting registration algorithm that is more robust than existing registration algorithms. It achieves its robustness by ensuring that the symmetric plane of the reference model is registered to the planar landmarks of the target model. Experiments on patients' skull models show that the proposed algorithm is robust, accurate and efficient in registering complex models.

Keywords: robust registration, symmetric plane, defected skull

1 Introduction

In surgery planning, forensic and archeology, there is a need to perform 3D model analysis and synthesis such as skull and face reconstruction, pose estimation by registration, etc. [3–5]. 3D model analysis and synthesis are challenging tasks because the models' shapes can be very complex and can vary from case to case. For example, human skull contain 22 complex 3D bones that are fused together (Fig. 1). It is much more complex than other 3D models such as face model which contains only a single surface.

One common first step of 3D model analysis and synthesis is to register a reference model to a target model using similarity transformation. In practice, due to noise and outliers caused by deformities and incompleteness of models, it is difficult to perform 3D similarity registration. For example, in craniomaxillofacial surgery planning, patients' skull models are usually incomplete, because only the region to be operated on are scanned so as to reduce radiology exposure. In addition, patients' skull models are either deformed congenitally or fractured due to injuries. The skull shown in Fig. 1(c) is fractured into several pieces, which are separated from each other and displaced away from their normal positions. Consequently, existing similarity registration methods such as Iterative Closest Point (ICP) [1], and Fractional Iterative Closest Point (FICP) [2] are misled and not able to register these models properly.

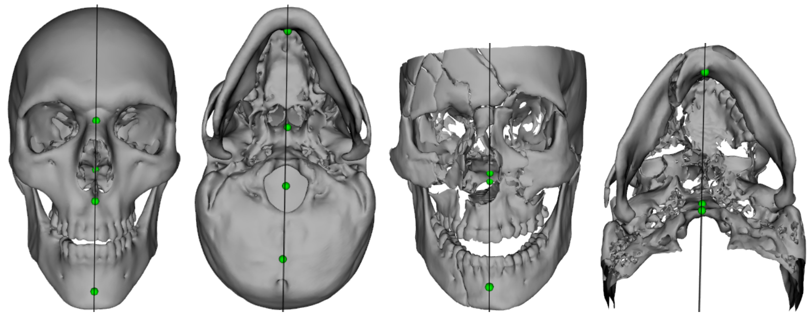

Fig. 1. Skull Models. (a, b) normal skull model. (c, d) fractured skull model. Green landmarks are planar landmarks that should lie on the symmetric plane (grey lines).

Fortunately, in complex 3D models like skulls, there is an approximate lateral symmetry with respect to a symmetric plane (Fig. 1). This symmetric plane is identified by a set of landmarks on the models [6], and these planar landmarks (Fig. 1) should approximately lie on the symmetric plane.

This paper presents a plane-fitting registration algorithm that is more robust than existing registration algorithms such as ICP and FICP. It achieves robustness by ensuring that the symmetric plane of the reference model is registered to the planar landmarks of the target model. Experiments on patients' skull models show that the proposed algorithm is robust, accurate and efficient in registering complex models.

2 Related Work

ICP algorithm [1] is popular in solving similarity registration problems. It formulates the problem as minimizing the mean-square distance from the points of the reference model to their closest points on the target model. ICP then solves the problem by iteratively finding optimal closest points and computing the optimal transformation. It converges to a local minimum and provides a standard solution to similarity registration problem.

Variances of ICP algorithm have been proposed to improve robustness [2, 7–10] and efficiency [11–13]. Robustness improvement is necessary for real applications, because the original ICP algorithm is not robust against noise and outliers [9].

One kind of robust ICP methods uses statistical methods. Gruen and Akca [10] proposed a method based on a generalized Gauss-Markov model to model noise statistically and reduce its effect on the registration result. This kind of methods is able to overcome noise, but it is still sensitive to outliers.

Another kind of robust ICP methods explicitly identifies outliers and excludes them from transformation computation. Zhang's method [7] rejects pairs of points that are too far from each other. Pajdla and Gool's method [8] rejects pairs of points by reciprocal correspondence distance. The idea is that inlier pairs computed from reference to target or from target to reference should be relatively the same. Therefore, pairs that do not satisfy this property can be identified and excluded. Chetverikov et al.'s method [9] trims the correspondence set to a fixed fraction of it. This method requires prior knowledge about the fraction of inliers, which is not available in most applications. Phillips et al. proposed an algorithm called Fractional Iterative Closest Point (FICP) [2]. FICP extends the objective function in ICP algorithm to fractional mean-square distance. By minimizing the extended objective function, FICP algorithm tends to find a large set of inlier pairs separated by small distances to compute transformation.

This paper presents a robust registration algorithm that ensures the matching of reference symmetric plane to target model's planar landmarks that are not affected by outliers. Ensuring planar constraints on symmetric plane and planar landmarks helps the algorithm to explicitly differentiate outliers and inliers and thus improves robustness.

3 Fractional Iterative Closet Point Method

Fractional Iterative Closest Point (FICP) algorithm is a variant of ICP that is more robust than ICP. Like ICP, FICP iteratively computes the best similarity transformation that registers the reference model to the target model. The difference is that in each iteration, FICP computes the transformation using only a subset of mesh points on the reference model whose distances to the target model are the smallest. This set of mesh points is called the inlier set.

Given two point sets , the reference model F and the target model D with unknown correspondence, FICP minimizes the fractional mean-square distance

$$E_1 = \left(\frac{|F|}{|G|}\right)^{\lambda} \frac{1}{|G|} \sum_{p \in G} \|T(p) - c(p)\|^2 \tag{1}$$

where G is a subset of F containing only the inliers, $c()$ is the correspondence mapping function of p that finds p's closest point on the target model, λ is a constant positive parameter, and T is the similarity transformation to be optimized. By minimizing E_1, FICP finds a large inlier set G with small errors and outputs the transformation computed only on G.

4 Plane-Fitting Registration

Reference model F and target model D can differ in size and shape details due to deformation caused by injury, normal variation among individuals, and possible incomplete scanning of the target skull. The registration algorithm should be robust to these variations, and find the common parts between the two models to align them. In addition, the plane P_F of the reference model should match the planar landmarks $v \in L$ of the target model.

The proposed plane-fitting registration algorithm enforces the matching of the reference plane to the planar landmarks of target model. In addition to the fractional mean-square distance (Eq. 1), a plane-fitting error E_2 is added to the objective function:

$$E_2 = \frac{1}{|L|} \sum_{v \in L} d_{\pi}^2(v) \tag{2}$$

where L is the set of planar landmarks on the target model, and $d_\pi(v)$ is the distance from target landmark v to the plane of the transformed reference model F. The overall objective function to be minimized becomes

$$E_r = E_1 + E_2. \tag{3}$$

The transformation T that minimizes E_r is the optimal transformation that registers F to D. Optimizing E_r is a difficult problem to solve. The proposed algorithm extends FICP algorithm to iteratively minimize E_r (Eq. 3) instead of E_1.

FICP finds the similarity transformation T that minimizes E_1 in Eq. 1 by iteratively performing four steps until convergence:

1. Finds correspondence between F and D.
2. Finds inlier subset G of F.
3. Computes similarity transformation T using correspondence of points in G.
4. Applies T on all points of F.

Phillips et al. [2] proved that the objective function E_1 decreases in each step of the iteration and the algorithm converges to a local minimum.

Out proposed algorithm has the same structure as FICP, but differs in algorithm details. In the first step, FICP finds the closest point as corresponding points only for the set F of mesh points. In the proposed algorithm, for a mesh vertex $p \in F$, its corresponding point $q \in D$ is the closest point of p. On the other hand, for a landmark $q \in L$, its corresponding point p of the reference model is the orthogonal projection of q on the plane P_F of the reference model. Let us denote the set of points p as P, and the set of corresponding points q as Q.

In the third step, FICP applies Horn et al.'s algorithm [14] to solve for the optimal similarity transformation T between P and Q that minimizes:

$$E = \sum_{p \in P} \|sRp + t - q\|^2 \tag{4}$$

where s, R and t are the scale factor, rotation matrix and translation vector of the similarity transformation T.

To minimize E_1 and E_2 in Eq. 3 at the same time, we introduce weights to Eq. 4 and reformulate Eq. 3 as:

$$E = \sum_{p \in P} w_p^2 \|sRp + t - q\|^2 \tag{5}$$

where

$$w_p^2 = \begin{cases} \left(\dfrac{|F|}{|G|}\right)^\lambda \dfrac{1}{|G|}, & \text{for } p \in G, \\ \\ \dfrac{1}{|L|}, & \text{for other } p. \end{cases}$$

This minimization can be achieved by adding the weights w_p into Horn et al.'s algorithm [14] where the rotation R and translation t are computed [15, 16]. After computing s, R and t, the points p are transformed by the similarity transformation:

$$q = sRp + t. \tag{6}$$

The proposed algorithm converges to a local minimum because registration error E_r decreases in each step of the iteration. In the first step, finding closest points for mesh vertices would reduce E_1 because the new closest points for vertices in F are closer than the closest points in the previous iteration. Finding new corresponding points for planar landmarks also reduces registration error because the orthogonal projection p on the symmetric plane of reference model is closer to q than the previous corresponding point on the same plane. In the second step, E_r decreases because E_1 decreases as proved in [2] and E_2 is unchanged in this step. Finally, in the third and forth steps, the optimal transformation that minimizes E_r (in the form of Eq. 5) is applied to the reference model. E_r is also reduced in these two steps. Therefore, same as ICP and FICP, the proposed plane-fitting registration algorithm also converges to a local minimum.

5 Experiments and Discussion

Experiments on complex skull models were conducted to evaluate the proposed plane-fitting registration algorithm by comparing it against existing algorithms. In this section, we first evaluate the registration accuracy of the proposed algorithm. Then, we study the robustness of the algorithm, and finally show its application in skull reconstruction.

5.1 Registration Quality

The first experiment evaluated the accuracy and efficiency of the proposed registration method. In this experiment, 124 full normal skull models were used. One of them was used as the reference (Fig. 1(a)). In practice, target skull models are usually patients skulls with deformities. For this reason, 5 skulls were manually cut and displaced in a manner similar to real fractures in patients to synthesize 5 fractured skull models. One of the manually cut skulls is presented in Fig. 2(2). Moreover, 5 skull models of real patients from a local hospital were also used for testing. Fig. 2(3-5) shows three of them. In addition to deformities caused by fracture, real patients' skulls were also incomplete because on the parts of the skulls under treatment were scanned.

The resolution of the CT images used to generate mesh models ranged from 0.31 to 3 mm/pixel. The CT images were segmented and 3D mesh models were reconstructed from them.

The proposed plane-fitting registration algorithm was applied to register the reference model to all the test target models. For comparison, two popular similarity registration algorithms, ICP [1] and FICP [2], were also tested.

To quantitatively assess the registration results produced by the algorithms, three errors were measured. First, surface error E_S measured the root-mean-square distance

Table 1. Comparison of registration methods. E_S denotes surface error (mm), and E_P denotes plane-fitting error (mm). The proposed plane-fitting registration algorithm attained the overall best performance with the lowest plane-fitting error and satisfactory surface error E_S.

Skull Model	ICP		FICP		Proposed	
	E_S	E_P	E_S	E_P	E_S	E_P
Normal	**3.69**	3.54	4.16	3.49	6.55	**1.42**
Synthetic fractured	**3.83**	3.82	4.10	3.27	3.92	**1.10**
Patient	**6.25**	9.05	15.38	2.07	14.21	**0.86**

from the reference surface to the target surface. Second, plane-fitting error E_P measured the root-mean-square distance from target models' planar landmarks to the reference model's symmetric plane. Finally, to examine the convergence of the algorithms, registration errors of one severely fractured skull (Fig. 2(3)) were measured for the intermediate results after each iteration according to the algorithms' objective functions. For ICP, FICP and the proposed plane-fitting registration algorithm, the registration errors are mean-square error, fractional mean-square error, and the error E_r shown in Eq. 3, respectively. Execution time was measured on a PC with a 3.4GHz CPU.

Fig. 2 shows the skull registration results. ICP algorithm was able to find reasonable results for normal skulls (Fig. 2(1)). However, ICP registration results were greatly affected by outliers caused by incompleteness and fractures of fractured models. Synthetic fractured skulls and real fractured skulls were not properly aligned by ICP algorithm (Fig. 2(2-5)). For the case in Fig. 2(3), ICP shrunk the reference model to a small region, and failed to find reasonable alignment between the reference and the patient's model. Due to inaccurate alignment, the symmetric plane of the reference model were not accurately aligned with the planar landmarks of the patients' models. In all the three categories of skull models, ICP had the lowest surface error E_S because it minimized points distance without the symmetric plane, resulting in large plane-fitting error E_P (Table 1).

FICP was robust to outliers and aligned the reference model relatively well to synthetic and patients' models (Fig. 2). FICP had larger surface error E_S (Table 1) than ICP algorithm, because it identified a portion of outliers, which were discarded. Therefore, errors of the outliers may be arbitrarily large and result in large E_S, even though the overall shape and the inliers are properly registered. This contradiction shows that surface error E_S is not a reliable assessment of registration quality. Fig. 2(c-d) shows that the alignment between the reference model's symmetric plane and the target models' planar landmarks were not accurate. Some landmarks were obviously off from the planes. Table 1 also shows that FICP had large plane-fitting error E_P because FICP did not consider plane-fitting in the registration process.

The proposed plane-fitting registration algorithm inherited its robustness from FICP. In addition to robustly registering the reference models to the target model, it also matched the symmetric plane of the reference model to the patients' planar landmarks accurately (Fig. 2). It was more robust to outliers than FICP due to the fitting of symmetric plane. The outliers that violated the fitting of plane were also identified and excluded from the computation of similarity transformation. Therefore, the symmetric plane of

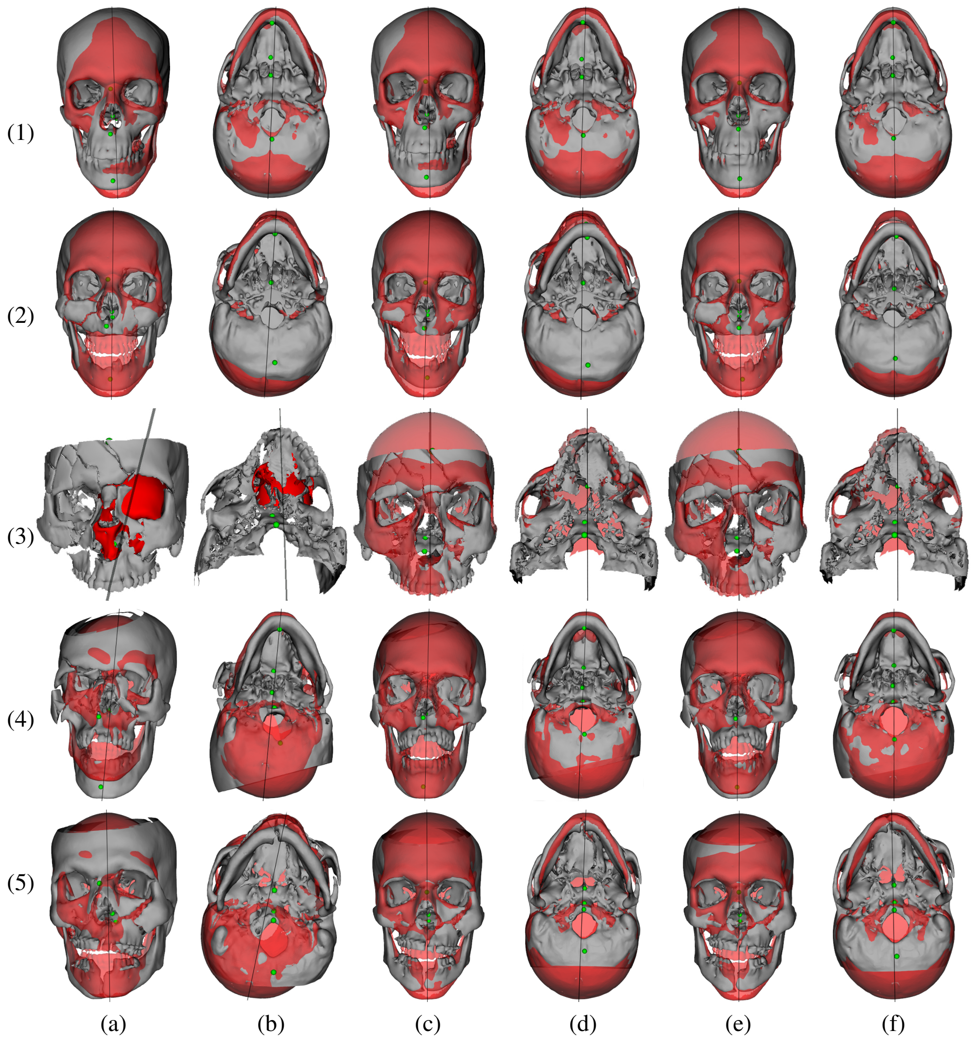

Fig. 2. Skull registration results. (1) normal skull. (2) synthetic fractured skull. (3-5) patients' fractured skulls. (a, b) ICP. (c, d) FICP. (e, f) proposed algorithm. The proposed plane-fitting registration algorithm attained the best overall performance compared to ICP and FICP. It aligned the reference skull models' symmetric planes (grey lines) accurately to the target skull models' planar landmarks (green balls). Surfaces of the reference models are colored red and the target models are colored grey.

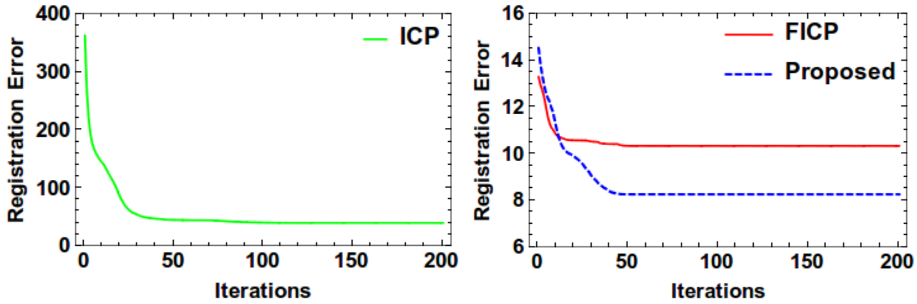

Fig. 3. Convergence curves. Same as ICP and FICP, the proposed algorithm converges to a stable value quickly.

the reference model was aligned accurately to the patients' planar landmarks, and the inlier surface points were robustly registered, resulting in the lowest plane-fitting error and similar surface error compared to FICP (Table 1).

Fig. 3 shows the convergence curves of the three algorithms on a patient's skull (Fig. 2(3)). The convergence curves validate that ICP (green) and FICP (red) converge quickly to local minimum. The convergence curve of the proposed algorithm (blue) shows that the proposed algorithm also converged quickly to a local minimum after a few iterations. As discussed in the previous paragraphs, ICP had the smallest E_S but the registration result may not be reliable, and the proposed algorithm was the most reliable one among the three methods.

In all the experiments, the convergence condition is to terminate when the reduction of registration error in an iteration was smaller than $1‰$. For the model in Fig. 2(3), ICP converged in 52 iterations and 0.48 second, FICP converged in 27 iterations and 0.60 second, and the proposed plane-fitting registration algorithm converged in 45 iterations and 1.01 second. The proposed algorithm is about 0.5 second slower than the other two algorithms, which is worthy considering the significant improvement in robustness and registration quality. In intensive applications where there are a large number of models to be registered, computational efficiency would become the main limitation of the proposed algorithm. Future research should be performed to improve the efficiency of the algorithm.

5.2 Robustness

To study the robustness of the proposed method, 124 normals skulls were manually cut off by different proportions f to synthesize 620 incomplete skull models with different levels of incompleteness. Together with the 124 original normal skulls, the experiment dataset contains skull models with 0% to 50% missing data. Due to the missing data in the target models, the reference model would have similar proportions of points that do not have corresponding parts in the target model.

We registered the reference model to the target models using three registration algorithms, ICP, FICP and plane-fitting registration. Plane-fitting error E_P was measured on

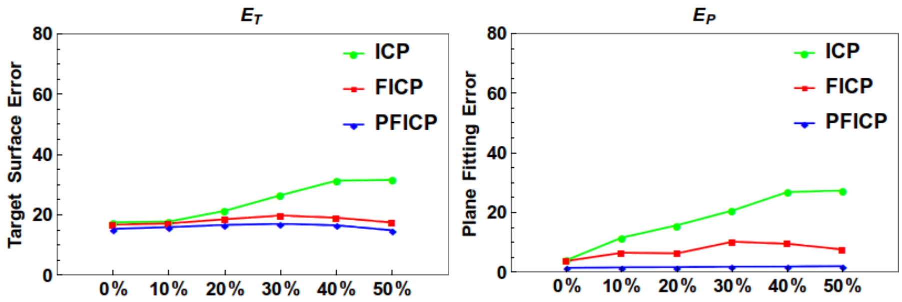

Fig. 4. Robustness. The proposed plane-fitting registration algorithm is more robust to incompleteness that ICP and FICP.

the registration results. However, surface error E_S was not computed because of the incomplete target models. Instead, target surface error E_T measured the root-mean-square distance from the target surface to the reference surface. It reflected the registration quality because it only used the surfaces of the target models that had correspondence in the reference model. They should have small errors in ideal registration.

Fig. 4 shows the errors measured for different proportions f of incompleteness. ICP is not robust to incompleteness because E_T and E_P increased as the proportion f of missing parts increased. FICP is more robust than ICP. Its E_T was quite stable when f increased, while there is a noticeable increase of E_P when f increased from 20% to 30%. The proposed method was the most robust. Its errors E_T and E_P were the smallest among all methods in all levels of incompleteness, and they did not increase when proportion f of incompleteness increased.

5.3 Application to Skull Reconstruction

This experiment compared the proposed method against FICP method in skull reconstruction that reconstructs normal complete skull models from fractured skull models. The experiment used the 5 synthetic fractured skull models described in Section 5.1. It also used the original models without fractures as ground truth to access the reconstruction accuracy.

We followed the statistical skull reconstruction framework described in [17] to reconstruct the fractured models. First, we registered the reference skull model to the fractured skull model. After registration, we built dense correspondences between the reference model and the target model using the Thin-Plate Spline (TPS) method developed in [18]. Then, we applied statistical shape model fitting method [17] to estimate complete normal model by fitting a statistical model to healthy parts of the fractured model.

Reconstruction error E_R measured the root-mean-square surface distance between ground truth and the reconstructed mesh. It was computed to quantitatively evaluate the reconstructed skull models generated.

Table 2. Comparison of registration methods in application of skull reconstruction. E_S denotes surface error (mm), E_P denotes plane-fitting error (mm), and E_R denotes restoration error (mm).

Registration method	E_S	E_P	E_R
FICP	33.57	2.36	7.47
Proposed	36.38	1.64	6.26

Table 2 shows the results. Although the proposed plane-fitting registration method had larger surface error, it produced smaller plane-fitting error E_P and smaller reconstruction error E_R than did FICP. The robust plane-fitting registration algorithm helped to improve the skull reconstruction accuracy.

6 Conclusion

Similarity registration is a common first step in complex 3D model analysis and synthesis. In many applications, 3D models to be registered contain large amount of noise and outliers, and are sometimes incomplete. Existing similarity registration methods are not able to register these models properly. This paper presents a plane-fitting registration algorithm that is more robust than existing registration algorithms. It achieves its robustness by ensuring that the symmetric plane of the reference model is registered to the planar landmarks of the target model. Quantitative and qualitative experimental results on real patients' skull models showed that the proposed algorithm is efficient and can robustly align the overall structures of the models while matching the symmetric plane of the reference model to the planar landmarks of target model accurately. Experimental results also showed that the proposed robust registration algorithm can benefit applications such as skull reconstruction.

References

1. Besl, P.J., McKay, N.D.: A method for registration of 3d shapes. IEEE transactions on Pattern Analysis and Machine Intellinegce **14**(2) (1992) 239–256
2. Phillips, J.M., Liu, R., Tomasi, C.: Outlier robust icp for minimizing fractional rmsd. In: Proc. of International Conference on 3D Digital Imaging and Modeling. (2007)
3. Jayaratne, Y.S., Zwahlen, R.A., Lo, J., Tam, S.C., Cheung, L.K.: Compute-aided maxillofacial surgery: An update. Surgical Innovation (2010)
4. Claes, P., Vandermeulen, D., De Greef, S., Willems, G., Clement, J.G., Suetens, P.: Computerized craniofacial reconstruction: conceptual framework and review. Forensic Science International **201**(1) (2010) 138–145
5. Benazzi, S., Stansfield, E., Milani, C., Gruppioni, G.: Geometric morphometric methods for three-dimensional virtual reconstruction of a fragmented cranium: the case of angelo poliziano. International Journal of Legal Medicine **123**(4) (2009) 333–344
6. Taylor, K.T.: Forensic Art and Illustration. CRC Press (2010)
7. Zhang, Z.: Iterative point matching for registration of free-form curves and surfaces. International Journal of Computer Vision **13**(2) (1994) 119–152
8. Pajdla, T., Gool, L.V.: Matching of 3-d curves using semi-differential invariants. In: Proc. of International Conference on Computer Vision. (1995)

9. Chetverikov, D., Stepanov, D., Krsek, P.: Robust euclidean alignment of 3d point sets: the trimmed iterative closest point algorithm. Image and Vision Computing **23**(3) (2005) 299–309
10. Gruen, A., Akca, D.: Least squares 3d surface and curve matching. ISPRS Journal of Photogrammetry and Remote Sensing **59**(3) (2005) 151–174
11. Chen, Y., Medioni, G.: Object modelling by registration of multiple range images. Image and vision computing **10**(3) (1992) 145–155
12. Fitzgibbon, A.W.: Robust registration of 2d and 3d point sets. Image and Vision Computing **21**(13) (2003) 1145–1153
13. Pottmann, H., Huang, Q., Yang, Y., Hu, S.: Geometry and convergence analysis of algorithms for registration of 3d shapes. International Journal of Computer Vision **67**(3) (2006) 277–296
14. Horn, B.: Closed-form solution of absolute orientation using unit quaternions. JOSA A **4**(4) (1987) 629–642
15. Kabsch, W.: A solution for the best rotation to relate two sets of vectors. Acta Crystallographica Section A: Crystal Physics, Diffraction, Theoretical and General Crystallography **32**(5) (1976) 922–923
16. Damm, K.L., Carlson, H.A.: Gaussian-weighted rmsd superposition of proteins: a structural comparison for flexible proteins and predicted protein structures. Biophysical journal **90**(12) (2006) 4558–4573
17. Zhang, K., Leow, W.K., Cheng, Y.: Performance analysis of active shape reconstruction of fractured, incomplete skulls. In: Submitted to International Conference on Computer Analysis of Images and Patterns. (2015)
18. Zhang, K., Cheng, Y., Leow, W.K.: Dense correspondence of skull models by automatic detection of anatomical landmarks. In: Proc. of Computer Analysis of Images and Patterns. (2013)