Stabilization of Temporal Medical Image Sequence

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## Contents

1 Introduction ...................................................... 1  
  1.1 Motivation .................................................... 1  
  1.2 Related Problems ........................................... 2  
  1.3 Organization of the Paper ................................. 3  

2 Existing Work .................................................. 4  
  2.1 Image Registration Algorithms ............................ 4  
    2.1.1 Parametric Transformation ........................... 5  
    2.1.2 Iterative Closest Points and Its Variations ........ 6  
    2.1.3 Variational Approach .................................. 12  
    2.1.4 Differential Elastic Registration .................... 15  
    2.1.5 Demons Registration .................................. 19  
    2.1.6 Summary ................................................. 20  
  2.2 Image Sequence Stabilization Algorithms ................. 21  
    2.2.1 Stabilization by Fourier Transform ................. 21  
    2.2.2 Registration-Based Stabilization .................... 23  
    2.2.3 Motion Model-Based Stabilization .................... 27  
    2.2.4 Summary ................................................. 27  

3 Possible Research Topics ...................................... 29  
  3.1 Stabilization of Blood Vessels in a Temporal Sequence of  
      Cardiac Angiogram ......................................... 29  
  3.2 Stabilization of Sequential Kidney Slices in a 4-D MR Sequence .... 31  

4 Preliminary Work ............................................... 32  
  4.1 Characteristics of Input Image Sequence ................. 32  
  4.2 Problem Description ....................................... 32  
  4.3 Experiment and Discussion .................................. 33  

5 Conclusion ....................................................... 37
Chapter 1

Introduction

1.1 Motivation

The development of radiology has revolutionized the field of medical diagnosis and surgery in the last few decades [12]. Medical imaging techniques, such as computed tomography (CT) and magnetic resonance imaging (MRI), play an important role in various medical procedures. They provide us a powerful way to look inside the human body, acquire 2-D slices and 3-D volume images of regions of interest (ROI), and perform both qualitative and quantitative analysis on the images. With the help of medical imaging techniques, doctors can not only make a more exact diagnosis of the patients, but also make a more precise surgical plan.

Temporal image sequence is a series of images that are taken on the same object or the same scene over time (Figure 1.1). Temporal image sequence can provide a dynamic view of the object of interest, such as the object movement, the intensity change and the shape change. By comparing the differences between image frames in the sequence, doctors can analysis the object’s motion, shape change and intensity change over time.

There are many factors that can cause differences between frames in a medical image sequence, such as altered positions of the patient, organ intensity change due to drug injection and organ shape change. Organ shape change can be classified into three types. The first type is the change within an individual’s anatomical structures due to growth, surgery, or disease. The second type is the warping due to the distortion during the imaging process, such as in echo-planar MR imaging [18]. The third type is the difference due to physiological processes such as breathing and pumping of heart.
In most of the medical image sequence, these factors occur all together. However, in a certain medical application, the doctors may be interested in differences caused only by some of these factors, while differences caused by the other factors are considered as the unwanted difference [51]. These unwanted differences should be eliminated before the medical image sequence is analyzed.

Medical image stabilization is the procedure to eliminate the difference caused by the factors that the doctors are not interested in. For example, if the doctors want to analyze the organ intensity change over time, the organ shape change is considered as unwanted and should be eliminated by stabilization procedure. On the other hand, if the doctors want to analyze the organ’s anatomical structure over time, the stabilization procedure should be applied to stabilize the organ’s intensity change due to physiological processes.

1.2 Related Problems

There are two other problems that are related to but different from stabilization. For clarity, these problems are highlighted in this section.
Correspondence
Given two sets of points (or entities), and a measure of the similarity between the points in the two sets, find a mapping function from one point set to the other that maximizes the average similarity between the points.

Registration
Given two images of an object that are acquired at different time, from different view points, or by different sensors, find a transformation that spatially align the corresponding points of the two images.

Stabilization
Given a temporal image sequence in which the objects of interest are changing location, changing shape or both, apply deformation to each image frame in the sequence such that after the deformation the object of interest in the sequence does not change location, change shape or both.

1.3 Organization of the Paper
Some existing work has been done on temporal medical image stabilization, and some of them are based on image registration algorithms. Therefore, Section 2.1 discusses existing registration algorithms, whereas Section 2.2 discusses temporal image stabilization algorithms. Based on the literature review in Chapter 2, one possible research topic is given in Chapter 3, followed by preliminary work and experimental results in Chapter 4. Chapter 5 concludes this paper.
Chapter 2
Existing Work

Registration techniques are widely employed to solve the medical image sequence stabilization problem, including the rigid registration and non-rigid registration. Therefore, registration algorithms will be discussed first, followed by temporal image stabilization algorithms.

2.1 Image Registration Algorithms

Image registration is the process of spatially aligning two or more images of the same scene taken at different times, from different viewpoints or by different sensors. It geometrically transforms one image to spatially align its pixels to the corresponding pixels in the other image to get the best fit. Survey of general medical image registrations algorithms is given in [54].

Image registration can be roughly classified into two categories by its geometrical properties [54], namely rigid registration and non-rigid registration. If the transformation is rigid, such as the similarity transformation which handles image rotation, scaling and translation, the registration approach is called rigid registration. If the transformation is non-rigid, such as warping and shape change, the registration approach is called non-rigid registration.

Rigid registration algorithms cannot perform well by themselves in most medical image applications because they are not sophisticated enough. However, they can be used as parts of more sophisticated algorithms such as non-rigid registration algorithms. Therefore, in this section, non-rigid registration is mainly focused on.
2.1.1 Parametric Transformation

Non-rigid registration with known correspondence is a well-posed problem. It can be solved using polynomial registration algorithms such as affine transformation and polynomial transformation.

Affine Registration

Affine transformation \( \text{[14]} \) has six degrees of freedom, and it can account for rotation, scaling, translation, reflection and shearing. It is represented by affine matrix \( \mathbf{M} \):

\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix} = \mathbf{M}
\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]

where \((x, y)\) denotes a point in the model or template image, and \((x', y')\) denotes the corresponding point in the target image, and \( \mathbf{M} \) is the affine matrix given by:

\[
\mathbf{M} = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  0 & 0 & 1
\end{bmatrix}
\]

where \( a_{ij} \) are the 6 parameters to be determined. Usually, more than 3 pairs of corresponding points are used, which means the parameters are determined by a set of over-constrained equations. Therefore, The aim of the problem is to find the transformation matrix that minimizes the sum-squared error \( E \) \[27\]:

\[
E = \sum_{i=1}^{n} \| \mathbf{M} p_i - p'_i \|^2
\] (2.1)

A sample registration result using affine transformation is shown in Figure 2.1.

Polynomial Registration

Polynomial registration is another generic parametric non-rigid registration algorithm. Polynomial registration allows more deformation than the affine registration. The polynomial transformation is represented as follows:

\[
x' = \sum_{m} \sum_{n} a_{mn} x^m y^n, \quad 0 \leq m + n \leq \Omega
\] (2.2)

\[
y' = \sum_{m} \sum_{n} b_{mn} x^m y^n, \quad 0 \leq m + n \leq \Omega
\] (2.3)
where $a_{mn}$ and $b_{mn}$ are the parameters of the polynomial transformation, and $\Omega$ is the order of the polynomial. The higher the order, the more complex is the deformation that the transformation represents. The parameters can be determined in the same way as the affine transformation since the correspondence is known.

### 2.1.2 Iterative Closest Points and Its Variations

The main idea of iterative closest points algorithm (ICP) [5] is based on the fact that the features on an object should be close to the corresponding features on the other object when the two objects are registered. These features can be points, curves, surfaces, etc. ICP iteratively makes educated guess about the correspondence, and then determines the transformation by the estimated correspondence. ICP algorithm can solve the registration of two objects with unknown correspondence between their feature points.

#### Iterative Closest Points

The ICP algorithm consists of three steps. The first step is to associate features by the closest feature criteria. This step allows the algorithm to guess the correspondence between the points of the two objects. The distance between the estimated corresponding points is measured by the Euclidean distance. The second step is to estimate the transformation parameters using the parametric transformation discussed in Section 2.1.1. In Besl’s algorithm [5], the similarity transformation is applied. The transformation could also be the affine transformation, the quadric transformation or some other polynomial transformation, depending on how complex is the deformation exists between the two objects. The third step is to transform the points using the estimated transformation parameters. These steps are repeated until con-
Figure 2.2: An example of registering two images using ICP. (a) The overlapped images before registration. (b) The overlapped images after registration using ICP.

ICP algorithm has a disadvantage that it associates points only by the closest point criterion. If one point in an object has no corresponding points in the other object, the association procedure may go wrong.

Robust ICP is proposed to overcome this disadvantage. Robust ICP algorithm uses robust methods to remove false correspondences. There are three kinds of robust methods: outlier thresholding, median estimation, and M-estimation. Outlier thresholding is based on estimating the distribution of the distance between corresponding points and removing the corresponding points which larger than multiples of the standard deviation. Median estimation uses the median of the error as a threshold to remove the outliers. M-estimation is based on estimating the probability that two points are the correct corresponding points.

ICP algorithm can give a coarse registration result but may not produce accurate result in some cases due to the mismatch of some corresponding points. Figure 2.3 shows the misalignment of robust ICP. In the small area the registration result is accurate, but it fails in the global area.
Iterative Multiple Close Features (IMCF) algorithm \[44\] and dual-bootstrap ICP algorithm \[45\], which will be discussed later.

**Iterative Multiple Close Features**

The most crucial modification that Iterative Multiple Close Features (IMCF) algorithm makes to ICP is the mapping function \[44\]. IMCF allows multiple matches per point, while the original ICP algorithm associates the corresponding points by a 1-to-1 mapping function.

The IMCF algorithm consists of three steps. In the first step, IMCF looks for multiple matches for each point \(p\). Let \(p'\) denote the transformation of a point \(p\) in the template image. Just as in ICP, the closest point to \(p'\) is found (\(q_1\) in Figure 2.4). In addition, other feature points in a circular region surrounding \(p'\) are identified. These closest points are used to form additional matches with \(p'\). The search radius is a small multiple of the alignment error. In the second step, a parametric transformation is estimated to minimize the weighted sum-square error of there corresponding points. The weighted function is defined according to the similarity (such as pixel intensity or gradient) between match points. The third step is to transform the points using the estimated transformation. Same as ICP, these steps are repeated until convergence. In Figure 2.5, the example shows that IMCF can give a more accurate registration result than robust ICP.
Dual-Bootstrap ICP

Another variation of ICP algorithm is called dual-bootstrap ICP algorithm [45], which can overcome the drawback of ICP’s misalignment due to mismatch. Its main idea is to begin the registration from a local region, which is referred to as the bootstrap region, then iteratively expand the bootstrap region and refine the transformation. Dual-bootstrap ICP requires only one known correspondence to start with.

The dual-bootstrap ICP algorithm works as follows:

1. In the initialization step, feature points are extracted and initial correspondences are found.
2. For each initial correspondence, the algorithm starts from the lowest order transformation, and iterate the following steps until convergence:

- Estimate transformation parameters using robust ICP.
- Select appropriate transformation and estimate parameters using statistical model selection techniques.
- Expand the bootstrap region based on the uncertainty of transformation estimated.
- Algorithm terminates when transformation is accurate enough.

3. If no more initial estimates are available, the algorithm terminates with failure.

The appropriate transformation in the iteration is chosen using statistical model selection technique[6, 47]:

- Choose the model that optimizes a tradeoff between the fitting accuracy of high-order models and the stability of low-order models.
- Stability is measured in terms of the covariance matrix of the parameters:
  - Covariance matrix with larger eigenvalues is more stable
  - Covariance matrices of higher-order transformations generally have smaller eigenvalues then those of lower-order transformations

Bootstrapping region is another important issue in the iteration. In Dual-Bootstrap ICP algorithm, region expansion is controlled by the uncertainty in the transformation estimate, and the uncertainty in mapping of point locations is computed from the covariance of the transformation parameter estimate [21].

Figure 2.6 shows the final registration result of the retinal blood vessels using dual-bootstrap ICP algorithm. Compared with the previous result produced by robust ICP, dual-bootstrap ICP algorithm produces a more accurate registration result. Dual-bootstrap ICP algorithm can also be applied to register general images under varying lighting conditions such as different intensity and different contrast [52].
Figure 2.6: Registration procedure using Dual-Bootstrap ICP. Square region is bootstrap region [27].
2.1.3 Variational Approach

Different from ICP algorithms in which the deformation are determined by a set of parameters, the variational approach uses non-parametric transformations. The deformation in variational approach is represented by a displacement \( u(x) \) of point \( x \), which means a point \( x \) in the template image will become \( x + u(x) \) in the deformed template image.

The problem of variational approach can be formulated as follows: Given a fixed reference image \( R \), a deformable template image \( T \), a difference measure \( D \), and a regularizer \( S \), find the displacements \( u(x) \) of points \( x \) in \( T \) that minimize the error \( E \):

\[
E = D(R, T; u) + \lambda S(u)
\]

where \( \lambda \) is a regularization parameter. Applying variational calculus method [27], Equation 2.4 can be rearranged into an iterative expression:

\[
u_{(k+1)} = (I + \gamma \lambda A)^{-1} [u_k - \gamma F(u_k)]
\]

where \( Au \) is obtained from \( A(u) \), \( A(u) \) is obtained from the regularizer \( S(u) \) and the external force \( F \) is obtained from difference measure \( D \).

There are several types of difference measures [1]: sum-squared difference [25], gradient information [11], cross correlation [7, 26], ratio image uniformity [22, 23] and mutual information [10]. Sum-squared difference are usually applied when images have similar intensity and contrast [15, 24]. Mutual Information can be used for registering images with different intensities and contrasts, such as registering MR images to CT images [40]. A detailed survey of mutual information for medical image registration can be found in [39].

Variational approach can be classified into four classes based on the regularizer (smoothing function \( S(u) \)) used: elastic registration, fluid registration, diffusion registration, curvature registration.

### Elastic Registration

Elastic registration [3] [15] [31] is based on the physical properties of elastic body. A elastic regularizer \( S \) represents an elastic body’s ability to stretch or shrink without tearing. Elastic regularizer gives the equation:

\[
A(u) = \mu \nabla^2 u + (\lambda + \mu) \nabla \nabla \cdot u
\]
where $\nabla$ is the gradient operator, and $\nabla^2$ is the Laplacian operator.

A sample elastic registration result is shown in Figure 2.7. As can be seen, compared with affine registration, elastic registration produces more deformation and a closer alignment than does the affine registration. Therefore, elastic registration algorithm has been widely applied in recent medical image registration applications.

**Fluid Registration**

Different from elastic registration which focuses on the spatial smoothing of displacement field ($u$), the fluid registration \[27, 33\] focuses on the spatial smoothing of velocity field ($\partial u / \partial t$). Fluid regularizer gives the equation:

$$A(u) = \mu \nabla^2 \frac{\partial u}{\partial t} + (\lambda + \mu) \nabla \nabla \cdot \frac{\partial u}{\partial t}$$ \hspace{1cm} (2.7)

A example of fluid registration is shown in Figure 2.8. Fluid registration can produce more deformation than elastic registration because the constraint on shape deformation is less than that of elastic registration. But this property will lead to a trade-off. On the one hand, fluid registration can register images with more significant shape change than does elastic registration. On the other hand, fluid registration may cause wrong deformation due to the flexibility of fluid model. To overcome this drawback, a linear pre-registration, such as the affine registration, is usually applied to normalize the size, position and orientation before applying the fluid registration procedure.

**Diffusion Registration**

Diffusion registration \[27, 33\] is another registration method that is similar to elastic registration. However, they are based on different models. Elastic registration is based on physical model but diffusion registration is based on intensity gradient model. Diffusion regularizer gives the equation:

$$A(u) = \nabla^2 u$$ \hspace{1cm} (2.8)

Compared with elastic registration, diffusion registration can be regarded as a special case of elastic registration without the $\nabla \nabla \cdot u$ term. So it allows for less shape change. But diffusion registration can also be extended to velocity-based method, which is similar to fluid registration. Under this condition, the diffusion registration allows for more significant shape change,
Figure 2.7: Elastic registration produces a closer alignment than affine registration [31]. (a) Reference image. (b) Template image. (c) Registration result using affine transformation. (d) Registration result using elastic registration.
Figure 2.8: Fluid registration of hand image [31]. (a) Fluid registration only. (b) Affine registration followed by fluid registration.

but pre-registration such as affine registration is required. An example of diffusion registration is shown in Figure 2.9.

Curvature Registration

As mentioned above, elastic, fluid, and diffusion registrations are flexible but sensitive to noise, and an affine pre-registration is required to roughly align images before non-rigid registration [27, 33]. Curvature registration [27] can partially overcome this drawback. It is less dependent on the initial configuration of the reference and template images. Curvature regularizer gives the equation:

\[ A(u) = \nabla^2 \nabla^2 u \quad (2.9) \]

Curvature registration focuses on minimizing the curvature of the components of the displacement \( u(\mathbf{x}) \). As can be seen in Figure 2.10, curvature registration is less sensitive to the initial rough alignment of the images.

2.1.4 Differential Elastic Registration

Similar to the variational approach, differential elastic registration [35, 36, 37, 38] also uses difference measure \( D \) and regularizer \( S \) to form the error function. However, it does not use \( u(\mathbf{x}) \) to represent the displacement of each
Figure 2.9: Diffusion registration of hand image [31]. (a) Elastically registered. (b) Diffusion registration with affine pre-registration.

Figure 2.10: Curvature registration of hand image [31]. (a) Curvature registration only. (b) Curvature registration with affine pre-registration.
Let $f(x,y,t)$ denote the image intensity function, $R$ denote the reference image and $T$ denote the template image. Then, the problem of differential elastic registration can be formulated as follows: Given $R$ and $T$, difference measure $E_b$, smoothness constraint $E_s$, the parameter weighting matrix $L$, find the possible affine transformation $m$ that minimizes the error $E$:

$$E = E_b(m) + E_s(m, L)$$  \hspace{1cm} (2.10)

where $E_b$ and $E_s$ are defined as:

$$E_b(m) = \left[ k - c^T m \right]^2$$  \hspace{1cm} (2.11)

$$E_s(m, L) = \sum_{i=1}^{6} \lambda_i \left[ \left( \frac{\partial m_i}{\partial x} \right)^2 + \left( \frac{\partial m_i}{\partial y} \right)^2 \right]$$  \hspace{1cm} (2.12)

where $m$ is the vector formed by the transformation parameters, $\lambda_1, \ldots, \lambda_6$ are the weight of each transformation parameter and $L$ is a diagonal matrix with diagonal elements equaling $\lambda_1, \ldots, \lambda_6$. $f_x = \partial f / \partial x$, $f_y = \partial f / \partial y$, $k = f_x + x f_x + y f_y$, $c^T = [x f_x \ y f_x \ x f_y \ y f_y \ f_x \ f_y]$.

Equation (2.10) can be rearranged into an iterative expression [36]:

$$m = \left( cc^T + L \right)^{-1} \left( c k + L \bar{m} \right)$$  \hspace{1cm} (2.13)

where $\bar{m}$ is the initial estimate of $m$. This process is repeated and on each iteration a new estimate of $\bar{m}$ is generated.

Differential elastic registration algorithm is widely applied in medical image applications for the following two advantages. First, differential elastic registration algorithm has the ability to align images with significant shape change, as shown in Figure 2.11 and Figure 2.12. Second, differential elastic registration is image-intensity-based, and it can work well with images which landmark-based algorithms cannot solve precisely.

However, differential elastic registration algorithm also has two drawbacks. First, differential elastic registration cannot properly register images with intensity changes because it uses the sum-squared error as the difference measure. A normalization procedure must be applied to account for the intensity variations. Second, differential elastic registration is often cited as being ineffective due to its high computational cost [4] [32].
Figure 2.11: Registration using differential elastic registration [36]. (a) The template image. (b) The reference image. (c) The deformed template image.

Figure 2.12: Another example of registration using differential elastic registration [36]. (a) The template image. (b) The reference image. (c) The deformed template image.
2.1.5 Demons Registration

Demons registration \([46]\) algorithm is related to the diffusion model. The template image is considered as a deformable grid of particles that diffuse into the reference image. Demons are placed at some locations of the reference image. They act locally to push the pixels or voxels in template image inside the reference image if its corresponding point in template image is outside, and outside if its corresponding point is inside \([41]\). (See Figure 2.13.)

In demons registration, optical flow is employed to compute the required displacement between demons and its corresponding point in template image, and the direction from inside to outside is generally based on the gradient. Low-pass filtering such as Gaussian filtering is usually applied to produce a smooth solution. The algorithm runs iteratively until convergence. An registration result is shown in Figure 2.14.

Demons may be placed at different locations in the reference image. They can be placed on all the pixels in the image, or just on the contours of the anatomical parts of interest (See Figure 2.14).

Demons algorithm requires that the anatomical parts in the template image must overlap with the corresponding parts in the reference image. It can only work properly in the cases with small displacement between images because optical flow is applied to compute the displacement. Usually, a possible way to extend the algorithm to the case with large displacement is to
Figure 2.14: An example of demons registration on MR head images [46].
(a) Reference image. (b) Deformed image. (c) Deformation corrected with
demons located at all pixels of the reference image. (d) Deformation corrected
with demons placed on the contours of anatomical parts only.

apply the algorithm top-down in an image pyramid.

2.1.6 Summary

In this section, some non-rigid registration algorithms are reviewed, including
general parametric algorithm, ICP and its variations, variational approaches,
differential elastic algorithm and demons algorithm.

General parametric algorithms, including affine and polynomial transfor-
mations, can solve registration problems with known correspondence. Usu-
ally they are applied in other sophisticated algorithms such as ICP and its
variations. ICP and its variations, variational approaches, differential elastic
and demons algorithm can solve registration problems with unknown cor-
respondence. ICP and its variations need to extract feature points while
variational approaches and differential elastic registration algorithms use the
intensity of each pixel instead of extracting feature points.

Non-rigid registration algorithms can be categorized into parametric algo-
rithms and non-parametric algorithms. General parametric registration, dif-
ferential elastic algorithm, ICP and its variations are parametric algorithms.
They work by estimating the parameters of the transformation function to
get the best fit of the images. Variational approaches and demons algorithm
are non-parametric algorithms. They represent the transformation function
as the displacement of every pixel. Therefore, they can handle more flexible
deformation than parametric algorithms. Usually a regularizer is defined to
constrain the deformation of the image, which is also referred as the smoothing
function.
2.2 Image Sequence Stabilization Algorithms

In a temporal image sequence, the differences between each pair of continuous images may arise for a wide variety of factors, such as sensor motion, global illumination variation, object motion and object spatial change. Before analyzing the influence of some particular factors on the temporal image sequence, all differences caused by other factors, which are referred to as the unwanted difference, should be eliminated.

Temporal image sequence stabilization is the procedure of selectively eliminating the differences caused by some of the factors mentioned above. It applies suitable deformation to the images in the sequence so that after the deformation, the scene in the image does not move, the objects of interest in the image do not move or change shape over time.

In most medical applications, image sequences are used to analyze the temporal change of organ. So doctors hope that the organs can keep still while the image sequences are taken. However, it is not realistic because of human physiological processes. For example, the heart cannot stop beating and breath cannot be held for a long time while the image sequence is taken, which may cause significant motion of organs in abdomen. Therefore the main purpose of medical image sequence stabilization is to eliminate the spatial change of organs.

The existing stabilization algorithms can be categorized into three types based on: Fourier registration and motion model. They will be discussed in detail in this section.

2.2.1 Stabilization by Fourier Transform

Stabilization algorithms by Fourier transform focuses on eliminating the differences in positions and orientations of the objects in the imaged sequence \cite{13}. The main idea is to translate the consecutive image frames into Fourier domain, and analyze the phase correlation to obtain the displacement of objects in the image frames. The proposed algorithm \cite{13} consists of two stages, namely phase correlation surface building and correlation surface peaks locating.
Let $R$ denote the reference image, and $T$ denote the template image, and let $(d_x, d_y)$ denote the motion vector of the object from $T$ to $R$, that in all pixels that the object locates this equation holds:

$$T(x, y) = \alpha \times R(x - d_x, y - d_y)$$ (2.14)

where $\alpha$ represents the intensity difference between $R$ and $T$.

In the first stage, $R$ and $T$ are transformed into Fourier domain using the two-dimensional discrete Fourier transform. Let $R_f$ and $T_f$ denote $R$ and $T$ after the transformation respectively. For each spatial frequency component $(u, v)$, the normalized cross power spectrum that retains phase difference information is given as [13]:

$$e^{j(\phi_1(u,v) - \phi_2(u,v)}) = \frac{Rf(u,v) \times T_f^*(u,v)}{|Rf(u,v) \times T_f^*(u,v)|}$$ (2.15)

where $*$ denotes the complex conjugate.

Then the phase correlation surface $P(x, y)$ is given by the inverse discrete Fourier transform (IDFT) of the normalized cross power spectrum [13]:

$$P(x, y) = IDFT(e^{j(\phi_1(u,v) - \phi_2(u,v))}) = \delta(x + d_x, y + d_y)$$ (2.16)

where IDFT is the inverse discrete Fourier transform operator and $\delta$ is the impulse function.

In the ideal case where all pixels in $T$ have the consistent motion, $P(x, y)$ is zero everywhere, except for a $\delta$-function peak at the location $(d_x, d_y)$. However, this assumption is not hold in real cases, and in real cases there are many small peaks in the surface. Therefore we should adopt some peak selection technique to choose the required peaks.

In the second stage, peaks are picked up from the phase correlation surface, each corresponding to a motion vector representing the object movement between two images. Note that the peaks are not ideal $\delta$-function, so the correct motion vectors cannot be obtained by taking the locations where local maximum values exists. The phase correlation surface amplitude is normalized and a thresholding value is set to eliminate those peaks with small amplitude on the phase correlation surface.

Stabilization algorithms by Fourier transform have two advantages. First, it has the ability to stabilize the objects having large displacement with different sub-pixel accuracy. Second, it is not sensitive to the global illumination.
However, Stabilization algorithms by Fourier transform also have one limitation. It stabilizes the image sequence by directly shifting images, so it cannot stabilize the image sequence in which there are several objects with non-consistent motions.

### 2.2.2 Registration-Based Stabilization

The main idea of registration based object motion stabilization algorithms is to register each image frames in the sequence to the first image so that the objects of interest is immobilized in the whole sequence after the stabilization [19].

A stabilization algorithm based on image texture analysis is proposed by Flores [16]. In this algorithm, homographic transformation is selected as the deformation, and corresponding points are determined by analyzing the texture. In the first step, feature points in each image are extracted using Harris detector [20], which is based in the first derivates of the images. The textures that appear in the image generate enough edges that can be detected by the Harris detector. Correspondences are associated to there initial points. In the second step, homographic transformation matrix $H$ is determined in the following way using the sets of feature points in both images. An initial correlation matching is determined by the similarity around the points in the first two image. Normalized cross correlation is selected as the similarity measure. Then the algorithm uses a randomized search algorithm to determine the homographic transformation matrix. It starts by choosing a small subset of points to obtain an initial homographic matrix. The process is repeated several times on different subsets in order to find the best matrix that maximizes the number of correspondence points. After that the homographic transformation parameters are calculated by minimizing the squared error of all remaining corresponding points.

In Wollny’s stabilization procedure [51], an registration algorithm proposed by Christensen [8, 9] is selected in the registration procedure. In this algorithm, the sum squared difference is used as the similarity measure, and the deforming force is given as the first order derivative due to the fluid dynamics. By applying the variational calculus method, the registration procedure is given by the iterative expression below:

$$u(x, t_{i+1}) = u(x, t_i) + \Delta t \cdot [v(x, t_i) - \nabla u(x, t_i)v(x, t_i)]$$  \hspace{1cm} (2.17)

where $u(x, t)$ is the displacement field representing the displacement of any
point \( x \) at time \( t \), \( v \) is the velocity field \( v = \frac{\partial u}{\partial t} \), and \( \Delta t \) is the time step.

Wollny’s stabilization procedure has two advantages. First, it can stabilize image sequence in which the object has large deformation. Second, it requires only minimal pre-processing such as the intensity normalization between image frames. But it is also reported to have a high computational cost.

Another algorithm is proposed by Song [43], which is based on the mesh deformable model. The mesh model is defined as a set of control nodes, which define polygon elements (patches) in the image (See Figure 2.15). The displacement of control nodes represents the object motion in the whole stabilization procedure. Generally the algorithm consists of four steps. In the first step, affine registration is applied to align each frame with the first frame roughly. In the second step the mesh nodes are defined in the first image. These nodes can be defined either manually or automatically. In the third step, the positions of these control nodes in the following frames are estimated using a Bayesian framework. Let \( I_1 \) and \( I_2 \) denote the first and second image frame, and \( x_{C1}^1, \ldots, x_{Ck}^1 \) and \( x_{C1}^2, \ldots, x_{Ck}^2 \) denote the coordinates of the \( K \) control nodes in \( I_1 \) and \( I_2 \) respectively, and the stabilization problem is to find the the location of all nodes \( x_{C2}^1, \ldots, x_{C2}^k \) and deform all image elements in \( I_2 \) to match the corresponding image elements in \( I_1 \).

The parameters \( x_{C1}^1, \ldots, x_{C2}^k \) can be estimated by maximizing the posterior probability of \( P(x_{C1}^2, \ldots, x_{C2}^k | I_1, I_2; x_{C1}^1, \ldots, x_{C2}^k) \). The posterior probability consists of two parts: one is the likelihood term \( P(I_2 | x_{C1}^2, \ldots, x_{C2}^k; x_{C1}^1, \ldots, x_{C2}^k; I_1) \) and the other is the prior term \( P(x_{C2}^1, \ldots, x_{C2}^k | x_{C1}^1, \ldots, x_{C2}^k) \), which is related not on the intensity but on the nodes location. An energy function is defined as the negative log form of the posterior probability, and the algorithm is to find the \( x_{C2}^1, \ldots, x_{C2}^k \) that maximize the energy. The energy function is given by

\[
E(x_{C1}^2, \ldots, x_{C2}^k) = \sum_m (I_2(B^m) - I_1(B^m))^T \sum_{B^m} (I_2(B^m) - I_1(B^m)) \\
+ \frac{\alpha}{2} (x_C^2 - x_C^1)^T \sum_C (x_C^2 - x_C^1) \\
+ \frac{\beta}{2} \sum_{j,n} ||x_{Cj}^2 - x_{Cn}^2||^2 \tag{2.18}
\]

where \( B^m \) denotes the \( m \)th polygon element (patch) in the image, \( \alpha \) and \( \beta \) are two parameters controlling the strength of prior terms in Bayesian networks. In the fourth step, the image frame is deformed, where the displacement
Figure 2.15: A quadrilateral mesh [49]. Number 1, 2, \ldots, 25 denote the control nodes and $D_1, D_2, \ldots, D_6$ denote the quadrilateral patches.

of each interior point in each image element is interpolated from the corresponding nodal displacements. Step two, three and four can be executed iteratively until convergence. Image pyramid can be built to give a better performance. An example of mesh model based image sequence stabilization is given in Figure 2.16.

Song’s algorithm is reported as effective in stabilizing the medical image sequences such as iris [43]. First, the motion field over the entire sequence is described by the displacements of the nodes, which makes it possible to reproduce very complex motion field if sufficient number of nodes are given. Second, it enables continuous tracking of the same set of nodes over consecutive frames, which is important for stabilization of image sequences. However, this algorithm has two main disadvantages. First, the topological structure of the mesh cannot be changes. Second, the image intensity should not change significantly during the whole sequence.

Another stabilization procedure proposed by Sarrut [42] is to stabilize the 2D lung CT slice image sequence in which the lung changes shape due to patient’s breathing. Demon’s algorithm [46] (Section 2.1.5) is selected for the registration procedure. All pixels in the reference image (first image frame) are considered as the demons.
Figure 2.16: An example of mesh model based stabilization algorithm. Tho image frames of an iris image sequence is selected \[43\]. (a) Frame 1. (b) Frame 30. (c) The absolute intensity difference between the two frames before registration. (d) The estimated deformation field found by the algorithm. (e) The registration result when image (b) is aligned with image (a). (f) The absolute intensity difference between the two frames after image (b) is aligned with image (a).
2.2.3 Motion Model-Based Stabilization

Typically the physiological motion can be categorized into two groups: non-periodical motion and periodical motion. There are two main sources of periodical motion inside the body, namely respiration and heartbeat [17], which yields large cyclic displacements of several organs in the image sequence, mainly in the chest and abdomen. Motion model based stabilization algorithms are proposed to stabilize the periodical organ motion caused by these physiological processes.

A breathing motion model for radiotherapy was proposed by Low in 2005 [28], which is used to stabilize the lung cancer tissues in CT scans. In this breathing model, a breathing cycle is divided into many phases, and the motion of the lung cancer tissues in each image frame has a relationship with the current phase in the breathing cycle, current lung volume, and the lung airflow, which can be obtained from the temporal derivative of the lung volume in sequence. Thus, the displacement vector from a reference position at any point of time \( t \) is represented as follows [28]:

\[
\mathbf{x}_p(t) = \mathbf{x}_v(t) + \mathbf{x}_f(t)
\]

(2.19)

where \( \mathbf{x}_p(t) \) represents the displacement vector. \( \mathbf{x}_v(t) \) and \( \mathbf{x}_f(t) \) are linear functions, representing the displacement vectors caused by lung volume and lung airflow respectively. The parameters of \( \mathbf{x}_v(t) \) and \( \mathbf{x}_f(t) \) are determined by fitting from measured positions using a template matching algorithm, where a template of contoured tumor volume was tracked at different breathing cycle phases.

2.2.4 Summary

In this section, some image stabilization algorithms are reviewed, including Fourier transform-based, registration-based and motion model-based algorithms. They are applied to temporal medical image sequence analysis. However, the stabilization purpose varies in different medical applications. Therefore, the stabilization algorithms should be carefully chosen for each medical image applications.

Fourier transform-based stabilization algorithms can stabilize the objects that have large displacement while retaining the sub-pixel accuracy. But it is not sophisticated enough to stabilize several objects with non-consistent motions.
The main idea of registration-based stabilization algorithms is to select one image frame as the reference image (usually the first frame), then deform each subsegment image frame to align to the first image frame using some registration algorithms. Non-rigid registration algorithms including ICP and its variations, variational approach and demons registration are applied, because they are sophisticated enough to register images with complex deformation. Registration-based algorithms are more accurate than Fourier transform-based algorithms, but they usually have a high computation cost.

Motion model-based stabilization algorithms focuses on modeling the organ’s periodical motion caused by periodical physiological process such as respiration and heartbeat. They deform each image frame using the estimated motion. They make good use of the information between consecutive image frames, thus they have a higher efficiency than pure registration-based algorithms. However, the stabilization result is strongly dependent on the motion model. Therefore the motion model should be carefully chosen for each application.
Chapter 3

Possible Research Topics

Based on the literature review in Chapter 2, stabilization of periodical motion is the most complex and important one in medical image stabilization. Some possible research topics on medical image sequence stabilization are proposed below.

3.1 Stabilization of Blood Vessels in a Temporal Sequence of Cardiac Angiogram

Blood vessels change location, intensity and shape over time. In angiogram, the blood vessels change location, intensity and shape due to the blood flowing inside and the pumping of the heart through the blood vessels. Figure 3.1 shows 4 image frames in a temporal angiogram image sequence. As can be seen, the blood vessels change location, intensity and shape significantly. The blood vessels may overlap with each other sometimes, and in some image frames they become blurred. Moreover, it is reported that the heart motion consists of two non-harmonic periodical movement [17], which makes it hard to model.

To get a correct and precise stabilization result, it is necessary to track the movement of the blood vessels between different slices taken at different time. Existing algorithms discussed in Section 2 cannot work well on temporal angiogram image sequences.
Figure 3.1: 4 image frames in a 2-D angiogram temporal image sequence. Blood vessels change position, intensity and shape over time. (a)-(c). The blood vessels overlap with each other. (d). The blood vessels become blurred.
Figure 3.2: 4 image frames in a 4D Kidney MR sequence. These 2D slides are taken at the same depth. But the kidney motion and intensity changes significantly in these slides.

3.2 Stabilization of Sequential Kidney Slices in a 4-D MR Sequence

Most of the algorithms discussed in Section 2 must guarantee that the image intensity does not change much, but this precondition does not hold all the time. In the sequential MR kidney sequence, the MR sensor captures images of the kidney at different depth at different time. The image intensity and kidney motion between any two image frames at the same depth always changes significantly due to the contrast agent injection and patient’s physiological processes (See Figure 3.2). Existing algorithms discussed in Section 2 cannot work well on it.
Chapter 4

Preliminary Work

We will describe some preliminary work which has been done in this chapter. The objective is to find an automated method to stabilize a temporal image sequence of 2-D MR images, in which the abdomen and the organs in it change locations and shapes over time due to the breathing of the patient. Registration based stabilization algorithm is employed here, registering all the images to the first image of the sequence using differential elastic image registration.

4.1 Characteristics of Input Image Sequence

The input images are 2-D MR temporal image sequence of the frontal view of abdomen. As can be seen in Figure 4.1, the abdomen has a clear boundary with the black background, but the liver doesn’t have a clear boundary with other tissues. Comparing images in the sequence with each other (Figure 4.1), it can be seen that the image intensity doesn’t change much when the image sequence is taken, but the location and shape of the tissues change a lot, especially the liver, heart and blood vessels.

4.2 Problem Description

The task is to stabilize the temporal MR image sequence of abdomen (Figure 4.1). The regions of interest, such as the liver on the middle-left part of the abdomen and the main blood vessel, are required to be stabilized as exactly as possible. And for the sake of efficiency, the stabilization procedure should be as automatical as possible.
The problem of automatic stabilization of temporal MR image sequence can be considered to deform each image frame in the sequence to the first image frame of the sequence. An appropriate non-rigid registration algorithm should be applied to register the image frames as precisely as possible.

For this problem, I used the matlab implementation proposed by Hany Farid, which can be downloaded from "http://www.cs.dartmouth.edu/farid/reserch/registration.html". The implementation consists of two main stages: a 3-level image pyramid building and differential elastic registration.

### 4.3 Experiment and Discussion

Experiment was conducted on a temporal abdomen MR image sequence to test the algorithm. Qualitative and quantitative evaluations of the performance of the stabilization algorithm were carried out.

The test temporal image sequence consists of a series of 60 image frames of a patient. Each image frame has a resolution of 224 × 256 pixels.

In our experiment, the parameters of the program are set as follows:

1. In the image pyramid building stage, the 3-level Gaussian pyramid size is set to 72 × 72, 144 × 144 and 288 × 288 respectively.
Table 4.1: Comparison of error before and after registration.

<table>
<thead>
<tr>
<th>Image</th>
<th>error before Reg</th>
<th>error after Reg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Image 20</td>
<td>11.6229</td>
<td>6.8491</td>
</tr>
<tr>
<td>Target Image 30</td>
<td>11.1313</td>
<td>7.1601</td>
</tr>
<tr>
<td>Target Image 40</td>
<td>19.5915</td>
<td>8.0104</td>
</tr>
<tr>
<td>Target Image 50</td>
<td>12.6153</td>
<td>5.8167</td>
</tr>
<tr>
<td>Target Image 59</td>
<td>24.7307</td>
<td>10.4490</td>
</tr>
<tr>
<td>Average</td>
<td>13.0843</td>
<td>6.5546</td>
</tr>
</tbody>
</table>

2. In the differential elastic registration stage, the local area size is set to $72 \times 72$, and the iteration times is fixed to 40.

Figure 4.2 shows the detailed result of one image frame.

Both qualitative and quantitative evaluation were conducted to evaluate the performance of the algorithm.

We randomly picked five image frames from the temporal image sequence with frame number 20, 30, 40, 50 and 59, and calculate the difference between the reference image frame and the stabilized image frames. Sum-squared error was selected as the difference measure. As a comparison, the difference between the reference image frame and input image frames will also be calculated. As can be seen from Table 4.1, the average error is significantly reduced after the stabilization procedure.

More comparisons can be seen in Figure 4.3. The red line represents the sum-squared error before stabilization, and the blue line represents the sum-squared error after stabilization.

However, seen from Table 4.1 and Figure 4.3, some error still occurs, which is mainly because of the overall intensity change and the limit of the images resolutions. Some future improvement can be done by registering pixel to pixel by mutual information method, since the sum-squared distance require the overall intensity to remain unchanged.
Figure 4.2: The final result of image 20. (a) The reference image. (b) The template image. (c) The geometric map representing the deformation. (d) The registration result.
Figure 4.3: Sum-Squared Error. The upper line represents the sum-squared error before stabilization, and the lower line represents the sum-squared error after stabilization.
Chapter 5

Conclusion

This paper introduced the importance of medical image stabilization in temporal image analysis, and then reviewed some existing work on registration algorithms and stabilization algorithms.

Non-rigid registration algorithms are crucial techniques in medical image sequence stabilization. General parametric registration algorithms, including affine transformation and polynomial transformation, can solve registration problems with known correspondence. Sophisticated algorithms such as ICP, Iterative Multiple Close Features, dual-bootstrap ICP, variational approaches, differential elastic registration and demons algorithm can solve registration problems with unknown correspondence.

Temporal image sequence stabilization algorithms are categorized into three categories, namely Fourier transform-based, registration-based and motion model-based stabilization algorithms. They are important in medical image applications. Fourier transform-based algorithms is effective and tolerant with image intensity change, but can only stabilize single object’s motion. Registration-based algorithms can accurately stabilize the objects with complex motion, but they are usually high in computational expense. Motion model-based algorithms are more effective than the registration based algorithms, but the motion model should be carefully chosen.

Based on the literature review, two possible research topics was proposed in Chapter 3. One is to stabilize the blood vessels in a temporal sequence of cardiac angiogram. The other is to stabilize the kidney in a 4D MR medical sequence. None of the existing methods can solve these problem satisfactorily.

A preliminary work was presented in Chapter 4. Differential elastic regis-
oration algorithm is applied to stabilize the temporal 2D MR image sequence of the human abdomen. Test results show that the algorithm can successfully stabilize the image sequence. Therefore, this algorithm can make contribution to the area of temporal medical image stabilization.
Bibliography


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