

Note: Bonus questions are entirely optional. They can get very challenging and/or fall outside the course curriculum. Interested individuals may attempt.

4. There is an (unknown) labeled tree with N vertices numbered $1, 2, \dots, N$. You are tasked to find the structure of this tree. Luckily, you are given an oracle which gives you a Yes/No answer to the following question (you may specify any X and Y):
- Given two *disjoint* sets X and Y . Does there exist $x \in X$ and $y \in Y$ such that the vertices x and y are connected by an edge?

What is the minimum number of questions (asymptotically) needed to complete this task? Prove both the lower and upper bounds.

(Hint: Search up online for any facts you need.)