Note: Bonus questions are entirely optional. They can get very challenging and/or fall outside the course curriculum. Interested individuals may attempt.

8. You have encountered this problem in Tutorial 4 bonus: There is an (unknown) labeled tree with N vertices numbered $1, 2, \ldots, N$. Your are tasked to find the structure of this tree. Luckily, you are given an oracle which gives you a Yes/No answer to the following question (you may specify any X and Y):

   - Given two *disjoint* sets X and Y. Does there exist $x \in X$ and $y \in Y$ such that the vertices x and y are connected by an edge?

   It was proven that the best algorithm uses $\Theta(n \log n)$ questions to recover the tree.

   Assume you are now given one pre-order traversal of the tree to start with (for free!), what is the minimum number of questions (asymptotically) needed to complete this task? Prove both the lower and upper bounds.