Note: Bonus questions are entirely optional. They can get very challenging and/or fall outside the course curriculum. Interested individuals may attempt.

9. Consider the following two problems:

   - SUBSET-SUM: Given a set of n non-negative integers $S = \{w_1, \ldots, w_n\}$ and a target W, decide whether there exists a subset $I \subseteq \{1, 2, \ldots, n\}$ such that $\sum_{i \in I} w_i = W$. (In other words, decide whether there exists a subset of S with sum W.)

   - ZERO-SUBSET-SUM: Given a set of n non-negative integers $S = \{w_1, \ldots, w_n\}$, decide whether there exists a subset $I \subseteq \{1, 2, \ldots, n\}$ such that $\sum_{i \in I} w_i = 0$. (In other words, decide whether there exists a subset of S with sum 0.)

   Prove that SUBSET-SUM $\leq_p$ ZERO-SUBSET-SUM.