

**Note:** Bonus questions are entirely optional. They can get very challenging and/or fall outside the course curriculum. Interested individuals may attempt.

10. This question follows from the bonus question in the previous tutorial. In particular, you may assume  $\text{SUBSET-SUM} \leq_p \text{ZERO-SUBSET-SUM}$  in your proof.

Consider the following problem **SETTLING-DEBTS**:

- **SETTLING-DEBTS**: You and your friends have just returned from a beautiful vacation in the mountains of the Netherlands. You kept all the receipts from the vacation, and wrote down who paid how much for who.

Assuming everyone has enough spare cash to transfer an arbitrary amount of money to another person, you are tasked to figure out the least number of transactions needed to settle the debts (i.e. everyone's balance is 0).

Prove that the decision version of **SETTLING-DEBTS** is NP-Complete.