National University of Singapore
School of Computing

CS3230 - Design and Analysis of Algorithms
Final Exam

(Semester 2 AY2024/25)

Time Allowed: 120 minutes

INSTRUCTIONS TO STUDENTS:

1. **DO NOT** open this assessment paper until instructed.
2. This paper has **2** sections over **12** pages (cover included).
3. This is an **Open Book** assessment; calculators and electronics are **not allowed**.
4. Use a 2B pencil to write and shade in the box on the right. Please write your Student Number only. Do **NOT** write your name.
5. For Section A (MCQ), fill in the bubbles on page 4 with a 2B pencil. No marks for blank answers.
6. For Section B, answer **ALL** questions in the boxed spaces.
   - Blank answer for a question earns 1 mark.
   - If you write even a single character and it is entirely incorrect, you receive 0 marks.
   - You may use pen or pencil; however, ensure your writing is **legible**.
7. Important tips:
   - Pace yourself; don't linger on one question.
   - Read all questions first; some may be simpler than they appear. Read the questions **carefully**.
8. All logs are base 2 unless stated otherwise.
9. This paper is worth 60 marks, scaled to 40% of the final grade. Each question's marks appear in the right margin.

STUDENT NUMBER

A										
U	○	⓪	⓪	⓪	⓪	⓪	⓪	⓪	Ⓐ	Ⓝ
A	●	①	①	①	①	①	①	①	Ⓑ	Ⓡ
HT	○	②	②	②	②	②	②	②	Ⓔ	Ⓤ
NT	○	③	③	③	③	③	③	③	Ⓗ	Ⓦ
		④	④	④	④	④	④	④	Ⓙ	Ⓧ
		⑤	⑤	⑤	⑤	⑤	⑤	⑤	Ⓛ	Ⓨ
		⑥	⑥	⑥	⑥	⑥	⑥	⑥	Ⓜ	
		⑦	⑦	⑦	⑦	⑦	⑦	⑦		
		⑧	⑧	⑧	⑧	⑧	⑧	⑧		
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Total	/60 marks

A Multiple Choice Questions (20 marks)

A.1. Section A redacted, there were 10 questions that were worth 2 marks each. (20)

B Essay Questions (40 marks)

B.1. A family owns $n \geq 3$ consecutive houses in a row and has three children, Aisha, Bo, and Claire. The parents want to divide the houses among the children in such a way that each child receives a **non-empty** block of **consecutive** houses. Each child has a positive integer value for each house, and a child's value for his/her block is the sum of values for each house in the block. The parents want to divide the houses so that the **sum** of the three children's values for their own block is the highest possible. (10)

[[Example: Suppose $n = 4$, Aisha has value $8, 2, 8, 7$ for the houses in this order, Bo has value $1, 9, 5, 8$, and Claire has value $4, 4, 4, 4$. Then, the highest possible value is 28, achieved by giving the first house to Claire, the second house to Bo, and the third and fourth houses to Aisha.]]

Design and analyze the correctness and running time of an algorithm to output the highest possible value. You should try to optimize the asymptotic running time of the algorithm for the parents, and state this running time in terms of n using the O-notation. (However, you do not need to prove that your running time is optimal.)

B.2. There are four points on a circle, labeled $0, 1, 2, 3$ in clockwise order. Sue starts at point 0, which is already counted as visited. At each time step, she jumps to either the adjacent point clockwise or the adjacent point counterclockwise with probability $1/2$ each. She continues doing this until she has visited all four points. [[Example: A possible sequence is $0 \to 1 \to 2 \to 1 \to 0 \to 3$.]] (10)

What is the probability that point 2 is the **last** point that Sue visits among the four points?

B.3. In the country Wonderland, there are 1-cent, 5-cent, 10-cent, and 25-cent coins (and no other types of coins). Your friend Alice from Wonderland claims that for every positive integer n, she can make n cents using the **smallest** number of coins via the following algorithm: (10)

"Start with an empty set of coins. Take the largest number $r \in \{1, 5, 10, 25\}$ that does not exceed n, add one r-cent coin to the set, and replace n by $n - r$. Repeat this procedure until n becomes 0, and output the final set of coins." [[Example: If $n = 38$, Alice outputs $25 + 10 + 1 + 1 + 1$.]]

Is Alice's claim correct? Justify your answer.

B.4. Don is organizing a sports camp for d days, with s students taking part in the camp. On each day, he needs to decide whether to let the students play tennis or volleyball. Because he has limited budget, all students must play the **same** sport on any given day. Each student has a preference on each day whether he/she is happy to play tennis, volleyball, both, or neither (a student is allowed to have different preferences on different days). The SPORTSCHEDULING problem asks whether it is possible for Don to schedule the sports so that each student is happy on **at least one** day (different students are allowed to be happy on different days). (10)

[[Example: Suppose that $s = 3$, $d = 2$, the first student prefers (tennis, tennis), the second student prefers (neither, volleyball), and the third student prefers (tennis, both). This instance is a YES-instance, since Don can schedule tennis on the first day, which makes the first and third students happy, and volleyball on the second day, which makes the second student happy.]]

Prove that SPORTSCHEDULING is NP-complete.

(You may assume, without proof, that the 3-SAT problem is NP-complete: Given c clauses and v variables $x_1, x_2, \ldots, x_v$, where each clause is a disjunction of three literals, such as $x_1 \vee \overline{x_3} \vee x_6$, decide whether there exists a truth assignment to the variables such that every clause is satisfied. You are also allowed to assume that no clause contains both a variable x_i and its negation $\overline{x_i}$.)

The Answer Sheet

Write your MCQ answers in the **special MCQ answer box below** **for automatic grading.** Answers will not be manually checked. Use a **2B pencil**, shade **one** circle per question fully and clearly, and erase any changes completely. Do not cross out answers.

Q	A.1	A.2	A.3	A.4	A.5	A.6	A.7	A.8	A.9	A.10
A	○	○	○	○	○	○	○	○	○	○
B	○	○	○	○	○	○	○	○	○	○
C	○	○	○	○	○	○	○	○	○	○
D	○	○	○	○	○	○	○	○	○	○
E	○	○	○	○	○	○	○	○	○	○

Q B.1. Design and analyze the correctness and running time of an algorithm to output the highest possible value.

(Leaving a blank answer will result in 1 mark being awarded.)

Q B.2. What is the probability that point 2 is the **last** point that Sue visits among the four points?

(Leaving a blank answer will result in 1 mark being awarded.)

Q B.3. Is Alice's claim correct? Justify your answer.

(Leaving a blank answer will result in 1 mark being awarded.)

Q B.4. Prove that SPORTSCHEDULING is NP-complete.

(Leaving a blank answer will result in 1 mark being awarded.)

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