An Efficient Algorithm for Learning Event-Recording Automata

Shang-Wei Lin[1], Étienne André[1], Jin Song Dong[1], Jun Sun[2], and Yang Liu[2]

[1] School of Computing, National University of Singapore *
{linsw,andre,dongjs}@comp.nus.edu.sg
[2] Singapore University of Technology and Design
{sunjun,liuyang}@sutd.edu.sg

Abstract. In inference of untimed regular languages, given an unknown language to be inferred, an automaton is constructed to accept the unknown language from answers to a set of membership queries each of which asks whether a string is contained in the unknown language. One of the most well-known regular inference algorithms is the L* algorithm, proposed by Angluin in 1987, which can learn a minimal deterministic finite automaton (DFA) to accept the unknown language. In this work, we propose an efficient polynomial time learning algorithm, TL*, for timed regular language accepted by event-recording automata. Given an unknown timed regular language, TL* first learns a DFA accepting the untimed version of the timed language, and then passively refines time constraints on the transitions of the DFA. We prove the correctness, termination, and minimality of the proposed TL* algorithm.

1 Introduction

In formal verification such as model checking [4, 11], system models and properties are assumed to be developed a priori during the verification process. In practice, however, modeling a system appropriately is not an easy task because if the model is too abstract, then it may not describe the exact behavior of the system; if the model is too detailed, it suffers from the state space explosion problem. Thus an automatic inference or construction of abstract model is very helpful for system development.

In 1987, Angluin [3, 12] proposed a learning algorithm, L*, for inference of regular languages. Given an unknown language U to be inferred, the L* algorithm learns a minimal deterministic finite automaton (DFA) to accept U from answers to a set of membership queries each of which asks whether a string is contained in the unknown language U.

After the L* algorithm was proposed, it is widely used in several research fields. The most impressive one is that Cobleigh et al. [5] used the L* algorithm

* This work is supported by Project MOE T2 "Advanced Model Checking Systems" in School of Computing, National University of Singapore.

to automatically generate the assumptions needed in assume-guarantee reasoning (AGR), which can alleviate the state explosion problem of model checking. Another interesting work is that Lin and Hsiung proposed an automatic compositional synthesis framework, CAGS [9], based on the L* algorithm. Given a system modeled by a set of component models and a user given property, if the system does not satisfy the property, CAGS uses the L* algorithm to synthesize each component model such that the refined system satisfies the given property.

However, there were at most no extensions of the learning algorithm to inference timed regular languages until 2004, Grinchtein et al. [7] proposed a learning algorithm for event-recording automata [2] based on the L* algorithm. Grinchtein's learning algorithm, TL^*_{sg}, uses region construction to actively guess all possible time constraints for each untimed word. That is, each original membership query of an untimed word in L* gives rise to several membership queries of timed words with possible time constraints, which increases the number of membership queries exponentially to the largest constant appearing in the time constraints.

In this work, we propose an efficient *polynomial time* learning algorithm TL* for timed regular languages accepted by event-recording automata. *Event-recording automata* (ERA) [2] are a determinizable subclass of timed automata [1] and are sufficiently expressive to model many interesting timed systems. Because of its determinizability, a timed language accepted by an ERA can be classified into finite number of classes (each class can be represented by a location of an ERA). Thus we focus on learning timed languages accepted by ERA. Given a timed regular language U_T accepted by ERA, TL* first learns a DFA M accepting U (the untimed version of U_T) and then passively refines the time constraints on the transitions of M. Thus the number of membership queries required by TL* is much smaller than that of Grinchtein's algorithm. We prove that the TL* algorithm will correctly learn an ERA accepting the unknown language U_T after a finite number of iterations. Further, we also prove the minimality of our TL* algorithm, i.e., the number of locations of the ERA learned by TL* is minimal.

The rest of this paper is organized as follows: Section 2 gives some preliminary knowledge and introduces the L* algorithm. The proposed efficient learning algorithm, TL*, is described in Section 3. The conclusion and future work are given in Section 4.

2 Preliminaries

We give some background knowledge about timed languages and event-recording automata in Section 2.1 and introduce the L* algorithm in Section 2.2.

2.1 Timed Languages and Event-Recording Automata

Let Σ be a finite alphabet. A *timed word* over Σ is a finite sequence $w_t = (a_1, t_1)(a_2, t_2) \ldots (a_n, t_n)$ of symbols $a_i \in \Sigma$ for $i \in \{1, 2, \ldots, n\}$ that are paired with nonnegative real numbers $t_i \in \mathbb{R}^+$ such that the sequence $\bar{t} = t_1 t_2 \dots t_n$ of time-stamps is nondecreasing. Sometimes we denote the timed word w_t by the pair $(\overline{a}, \bar{t})$, where $\overline{a} = a_1 a_2 \dots a_n \in \Sigma^*$ is an untimed word over Σ. For every symbol $a \in \Sigma$, we use x_a to denote the *event-recording clock* of a [2]. Intuitively, x_a records the time elapsed since the last occurrence of the symbol a. We use C_Σ to denote the set of event-recording clocks over Σ, i.e., $C_\Sigma = \{x_a \mid a \in \Sigma\}$. A *clock valuation* $\gamma : C_\Sigma \mapsto \mathbb{R}^+$ assigns a nonnegative real number to an event-recording clock.

A *clocked word* over Σ is a finite sequence $w_c = (a_1, \gamma_1)(a_2, \gamma_2) \dots (a_n, \gamma_n)$ of symbols $a_i \in \Sigma$ for $i \in \{1, 2, \dots, n\}$ that are paired with clock valuations γ_i such that $\gamma_1(x_a) = \gamma_1(x_b)$ for all $a, b \in \Sigma$ and $\gamma_i(x_a) = \gamma_{i-1}(x_a) + \gamma_i(x_{a_{i-1}})$ when $1 < i \leq n$ and $a \neq a_{i-1}$. Each timed word $w_t = (a_1, t_1)(a_2, t_2) \dots (a_n, t_n)$ can be naturally transformed into a clocked word $cw(w_t) = (a_1, \gamma_1)(a_2, \gamma_2) \dots (a_n, \gamma_n)$ where $\gamma_i(x_a) = t_i$ if $a_j \neq a$ for $1 \leq j < i$; $\gamma_i(x_a) = t_i - t_j$ if there exists a_j such that $a_j = a$ for $1 \leq j < i$ and $a_k \neq a$ for $j < k < i$. Sometimes we denote the clocked word w_c by the pair $(\overline{a}, \overline{\gamma})$, where $\overline{a} = a_1 a_2 \dots a_n \in \Sigma^*$ is an untimed word over Σ and $\overline{\gamma} = \gamma_1 \gamma_2 \dots \gamma_n$ is the sequence of clock valuations.

A *clock constraint* φ is a conjunction of atomic constraints of the forms $x_a \sim n$ and $x_a - x_b \sim n$ for $x_a, x_b \in C_\Sigma$, $\sim \in \{<, \leq, \geq, >\}$, and $n \in \mathbb{N}$. A clock constraint φ identifies a $|\Sigma|$-dimensional polyhedron $[\![\varphi]\!] \subseteq (\mathbb{R}^+)^{|\Sigma|}$.

A *clock guard* g is a conjunction of atomic constraints of the form $x_a \sim n$, for $x_a \in C_\Sigma$, $\sim \in \{<, \leq, >, \geq\}$, and $n \in \mathbb{N}$. A clock guard g identifies a $|\Sigma|$-dimensional hypercube $[\![g]\!] \subseteq (\mathbb{R}^+)^{|\Sigma|}$. We use G_Σ to denote the set of clock guards over C_Σ. A *guarded word* is a sequence $w_g = (a_1, g_1)(a_2, g_2) \dots (a_n, g_n)$ where $a_i \in \Sigma$ for $i \in \{1, 2, \dots, n\}$ and $g_i \in G_\Sigma$ is a clock guard. For a clocked word $w_c = (a_1, \gamma_1)(a_2, \gamma_2) \dots (a_n, \gamma_n)$, we use $w_c \models w_g$ to denote $\gamma_i \models g_i$ for all $i \in \{1, 2, \dots, n\}$. Sometimes we denote the guarded word w_g by the pair $(\overline{a}, \overline{g})$, where $\overline{a} = a_1 a_2 \dots a_n \in \Sigma^*$ is an untimed word over Σ and $\overline{g} = g_1 g_2 \dots g_n$ is the sequence of clock guards.

Definition 1. ***(Event-Recording Automata)*** *[2]. An* event-recording automaton *(ERA) $D = (\Sigma, L, l_0, \delta, L^f)$ consists of a finite input alphabet Σ, a finite set L of locations, an initial location $l^0 \in L$, a set L^f of accepting locations, and a transition function $\delta :\subseteq L \times \Sigma \times G_\Sigma \mapsto 2^L$. An ERA is* deterministic *if $\delta(l, a, g)$ is a singleton set when it is defined, and when both $\delta(l, a, g_1)$ and $\delta(l, a, g_2)$ are both defined then $[\![g_1]\!] \cap [\![g_2]\!] = \emptyset$, where $l \in L$, $a \in \Sigma$, and $g_1, g_2 \in G_\Sigma$.*

Note that in ERA each event-recording clock $x_a \in C_\Sigma$ is implicitly and automatically reset when a transition with event a is taken, which gives a good characteristic that each non-deterministic ERA can be determinized by subset construction [2]. Fig. 1 (a) gives a deterministic ERA $\mathcal{A}_1$ accepting the timed word $(a^*, \bar{t}) = (a, t_1)(a, t_2)(a, t_3)(a, t_4) \dots = (a, 1)(a, 4)(a, 5)(a, 8) \dots$, where $t_{2i} = t_{2i-1} + 3$ and $t_{2i+1} = t_{2i} + 1$ for $i \in \mathbb{N}$. We can also use a clocked word $(a^*, \overline{\gamma}) = (a, \gamma_1)(a, \gamma_2)(a, \gamma_3)(a, \gamma_4) \dots$ to represent the timed word $(a^*, \bar{t})$ such that $\gamma_{2i-1}(x_a) = 1$ and $\gamma_{2i}(x_a) = 3$ for $i \in \mathbb{N}$. Or we can use a guarded word $(a^*, \overline{g}) = (a, g_1)(a, g_2)(a, g_3)(a, g_4) \dots$ to represent the timed word $(a^*, \bar{t})$ such that $g_{2i-1} = (x_a = 1)$ and $g_{2i} = (x_a = 3)$ for $i \in \mathbb{N}$.

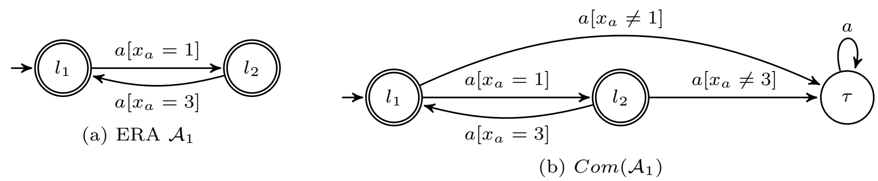


Fig. 1. Timed Language to be Learned

Definition 2. ***(Complete Deterministic ERA)***. *A deterministic ERA* $D = (\Sigma, L, l_0, \delta, L^f)$ *is said to be* complete *if for all* $l \in L$ *and for all* $a \in \Sigma$, $\delta(l, a, g_i)$ *is defined for all* $i \in \{1, 2, \ldots, n\}$ *such that* $[\![g_1]\!] \cup [\![g_2]\!] \cup \ldots \cup [\![g_n]\!] = [\![true]\!]$.

If a deterministic ERA D is not complete, we can always construct a complete deterministic ERA $Com(D) = (\Sigma, L \cup \{\tau\}, l_0, \delta', L^f)$ such that $\mathcal{L}(D) = \mathcal{L}(Com(D))$, where τ is an additional dead state and δ' is defined as follows: $\delta'(l, a, g) = \delta(l, a, g)$ if $\delta(l, a, g)$ is defined; otherwise, $\delta'(l, a, g) = \tau$. Fig. 1 (b) shows the complete deterministic ERA $Com(\mathcal{A}_1)$ of $\mathcal{A}_1$.

2.2 The L* Algorithm

The L* algorithm [3, 12] is a formal method to learn a minimal DFA (with the minimal number of locations) that accepts an unknown language U over an alphabet Σ. During the learning process, the L* algorithm interacts with a *Minimal Adequate Teacher* (Teacher for short) to make two types of queries: the *membership* and *candidate queries*. A *membership query* for a string σ is a function $\mathcal{Q}_m$ such that if $\sigma \in U$, then $\mathcal{Q}_m(\sigma) = 1$; otherwise, $\mathcal{Q}_m(\sigma) = 0$. A *candidate query* for a DFA M is a function $\mathcal{Q}_c$ such that if $\mathcal{L}(M) = U$, then $\mathcal{Q}_c(M) = 1$; otherwise, $\mathcal{Q}_c(M) = 0$. During the learning process, the L* algorithm stores the membership query results in an *observation table* (S, E, T) where $S \subseteq \Sigma^*$ is a set of prefixes, $E \subseteq \Sigma^*$ is a set of suffixes, and $T : (S \cup S \cdot \Sigma) \times E \mapsto \{0, 1\}$ is a mapping function such that if $s \cdot e \in U$, then $T(s, e) = 1$; otherwise, i.e., $s \cdot e \notin U$, then $T(s, e) = 0$, where $s \in (S \cup S \cdot \Sigma)$ and $e \in E$. In the observation table, the L* algorithm categorizes strings based on Myhill-Nerode Congruence [8], as formulated in Definition 3.

Definition 3. ***Myhill-Nerode Congruence***. *For any two strings* $\sigma, \sigma' \in \Sigma^*$, *we say they are* equivalent, *denoted by* $\sigma \equiv \sigma'$, *if* $\sigma \cdot \rho \in U \Leftrightarrow \sigma' \cdot \rho \in U$, *for all* $\rho \in \Sigma^*$. *Under the equivalence relation, we can say* σ *and* σ' *are the representing strings of each other, denoted by* $\sigma = [\sigma']_r$ *and* $\sigma' = [\sigma]_r$.

The L* algorithm will always keep the observation table *closed* and *consistent*. An observation table is *closed* if for all $s \in S$ and $\alpha \in \Sigma$, there always exists $s' \in S$ such that $s \cdot \alpha \equiv s'$. An observation table is *consistent* if for every two elements $s, s' \in S$ such that $s \equiv s'$, then $(s \cdot \alpha) \equiv (s' \cdot \alpha)$ for all $\alpha \in \Sigma$. If the observation table (S, E, T) is closed and consistent, the L* algorithm will construct a corresponding candidate DFA $C = (\Sigma_C, L_C, l_C^0, \delta_C, L_C^f)$ such that

$\Sigma_C = \Sigma$, $L_C = S$, $l_C^0 = \{\lambda\}$, $\delta_C(s,\alpha) = [s \cdot \alpha]_r$ for $s \in S$ and $\alpha \in \Sigma$, and $L_C^f = \{s \in S \mid T(s,\lambda) = 1\}$. Subsequently, the L* algorithm will make a candidate query for C.

If $\mathcal{Q}_C(M) = 0$, i.e., $\mathcal{L}(C) \neq U$, then Teacher will give a counterexample σ_{ce}. The counterexample σ_{ce} is *positive* if $\sigma_{ce} \in \mathcal{L}(U) \setminus \mathcal{L}(C)$; *negative* if $\sigma_{ce} \in \mathcal{L}(C) \setminus \mathcal{L}(U)$. The L* algorithm will analyze the counterexample σ_{ce} to find the witness suffix. For two strings that are classified by L* into an equivalence class, a *witness suffix* is a string that when appended to the two strings provides enough evidence for the two strings to be classified into two different equivalence classes under the Myhill-Nerode Congruence. Given an observation table (S, E, T) and a counterexample σ_{ce} given by Teacher, we define an *i-decomposition query* of σ_{ce}, denoted by $\mathcal{Q}_m^i(\sigma_{ce})$, as follows: $\mathcal{Q}_m^i(\sigma_{ce}) = \mathcal{Q}_m([u_i]_r \cdot v_i)$ where $\sigma_{ce} = u_i \cdot v_i$ is a decomposition of σ_{ce} such that $|u_i| = i$, and $[u_i]_r$ is the representing string of u_i in S. The *witness suffix* of σ_{ce}, denoted by $WS(\sigma_{ce})$, is the suffix v_i of the decomposition of σ_{ce} such that $\mathcal{Q}_m^i(\sigma_{ce}) \neq \mathcal{Q}_m^0(\sigma_{ce})$. Once the witness suffix $WS(\sigma_{ce})$ is obtained, L* uses $WS(\sigma_{ce})$ to refine the candidate DFA C until $\mathcal{L}(C) = \mathcal{L}(U)$. The pseudo-code of the L* algorithm is given in Algorithm 1.

Algorithm 1: L* Algorithm

input : Σ: alphabet
output: a DFA accepting the unknown language U

Let $S = E = \{\lambda\}$;
Update T by $\mathcal{Q}_m(\lambda)$ and $\mathcal{Q}_m(\lambda \cdot \alpha)$, for all $\alpha \in \Sigma$;
while *true* **do**
  while *there exists* $(s \cdot \alpha)$ *such that* $(s \cdot \alpha) \not\equiv s'$ *for all* $s' \in S$ **do**
    $S \longleftarrow S \cup \{s \cdot \alpha\}$;
    Update T by $\mathcal{Q}_m((s \cdot \alpha) \cdot \beta)$, for all $\beta \in \Sigma$;
  Construct candidate DFA M from (S, E, T) ;
  if $\mathcal{Q}_c(M) = 1$ **then return** M ;
  else
    $\sigma_{ce} \longleftarrow$ the counterexample given by Teacher ;
    $v \longleftarrow WS(\sigma_{ce})$;
    $E \longleftarrow E \cup \{v\}$;
    Update T by $\mathcal{Q}_m(s \cdot v)$ and $\mathcal{Q}_m(s \cdot \alpha \cdot v)$, for all $s \in S$ and $\alpha \in \Sigma$;

We use an example to illustrate how the L* algorithm works to learn a minimal DFA accepting an unknown language. Suppose the unknown language $U = (a|b|c) \cdot a^*$ over $\Sigma = \{a, b, c\}$ needs to be learned. Initially, S and E are initialized to $\{\lambda\}$ and then the membership queries of λ, a, b, and c are performed. At this point, the observation table with $S = \{\lambda\}$, $E = \{\lambda\}$ is shown in Fig. 2 (a). The observation table now is not closed because there is not any $s \in S$ such that $a \equiv s$. So, a is added into S, and then the membership queries of aa, ab, and ac are performed respectively. At this point, the observation table with $S = \{\lambda, a\}$,

$E = \{\lambda\}$ is closed as shown in Fig. 2 (b). The corresponding DFA M_1 is shown in Fig. 2 (c). The candidate query of M_1 is performed.

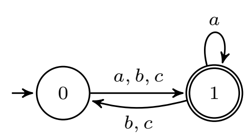
However, Teacher gives a negative counterexample abc that is accepted by M_1 but not in U. The L* algorithm analyzes the negative counterexample abc to get the witness suffix as follows: $\mathcal{Q}^0_m(abc) = 0$. $\mathcal{Q}^1_m(abc) = \mathcal{Q}_m([a]_r \cdot bc) = \mathcal{Q}_m(abc) = 0$, $\mathcal{Q}^2_m(abc) = \mathcal{Q}_m([ab]_r \cdot c) = \mathcal{Q}_m(\lambda \cdot c) = \mathcal{Q}_m(c) = 1 \neq \mathcal{Q}^0_m(abc)$. After analyzing the counterexample abc, the witness suffix is c. So, c is added into E, and the membership queries of c, ac, bc, cc, aac, abc, and acc are performed. The observation table now with $S = \{\lambda, a\}$, $E = \{\lambda, c\}$ is shown in Fig. 3 (a). However, the observation table is not closed because there is no $s \in S$ such that $ab \equiv s$. So, ab is added into S, and then the membership queries of aba, abb, abc, $abac$, $abbc$, and $abcc$ are performed. At this point, the observation table with $S = \{\lambda, a, ab\}$, $E = \{\lambda, c\}$ is closed as shown in Fig. 3 (b). The corresponding DFA M_2 is shown in Fig. 3 (c) and $\mathcal{L}(M_2) = U$.

	λ
λ	0 (s_0)
a	1
b	1
c	1

(a)

	λ
λ	0 (s_0)
a	1 (s_1)
b	1
c	1
$a \cdot a$	1
$a \cdot b$	0
$a \cdot c$	0

(b)



(c)

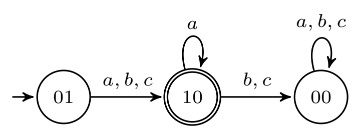
Fig. 2. L* Observation Table and Candidate DFA M_1

	λ	c
λ	0	1 (s_0)
a	1	0 (s_1)
b	1	0
c	1	0
$a \cdot a$	1	0
$a \cdot b$	0	0
$a \cdot c$	0	0

(a)

	λ	c
λ	0	1 (s_0)
a	1	0 (s_1)
b	1	0
c	1	0
$a \cdot a$	1	0
$a \cdot b$	0	0 (s_2)
$a \cdot c$	0	0
$ab \cdot a$	0	0
$ab \cdot b$	0	0
$ab \cdot c$	0	0

(b)



(c)

Fig. 3. L* Observation Table and Candidate DFA M_2

Assume Σ is the alphabet of the unknown regular language U and the number of states of the minimal DFA is n. The L* algorithm needs $n-1$ candidate queries and $O(|\Sigma|n^2 + n \log m)$ membership queries to learn the minimal DFA, where m is the length of the longest counterexample returned by Teacher. Angluin [3] proved that as long as the unknown language U is regular, the L* algorithm will learn a complete minimal DFA M such that $\mathcal{L}(M) = U$ in at most $n-1$ iterations.

3 An Efficient Algorithm for Learning Event-Recording Automata

The intuition behind the L* algorithm is to classify untimed words into finite number of classes by performing membership queries, and each class can be represented by a location of a DFA. Because event-recording automata (ERA) are determinizable, therefore a timed language accepted by an ERA can also be classified into finite number of classes. The proposed TL* algorithm also tries to find the finite number of classes (locations). Section 3.1 introduces the TL* algorithm, and its analysis is given in Section 3.2.

3.1 The TL* Algorithm

Given a timed language U_T, the proposed TL* algorithm interacts with a timed Teacher to make two types of queries: the *timed membership* and *timed candidate queries*. A *timed membership query* for a guarded word w_g is a function $\mathcal{Q}_{m^T}$ such that $\mathcal{Q}_{m^T}(w_g) = 1$ if $w_g \in U_T$; otherwise $\mathcal{Q}_{m^T}(w_g) = 0$. A *timed candidate query* for an ERA M is a function $\mathcal{Q}_{c^T}$ such that $\mathcal{Q}_{c^T}(M) = 1$ if $\mathcal{L}(M) = U_T$; otherwise, $\mathcal{Q}_{c^T}(M) = 0$.

The idea behind the TL* algorithm is to first learn a DFA M accepting $Untime(U_T)$, the untimed language with respect to U_T, and then to refine the time constraints on the transitions of M. Therefore, TL* consists of two phases, namely the *untimed learning* phase and the *timed refinement* phase. Algorithm 2 gives the pseudo-code of TL*. The details of TL* are described as follows.

Untimed Learning. In this phase, the L* algorithm is used to learn a DFA M accepting the untimed language $Untime(U_T)$ with respect to U_T (Line 1). The observation table (S, E, T) constructed in the learning process of L* is reserved for the timed refinement phase (Line 2).

Timed Refinement. In this phase, the TL* algorithm tries to refine the time constraints on the transitions of the DFA M learned in the untimed learning phase. It performs the following steps:

1. Modify the untimed alphabet into timed alphabet, i.e., replace α by $(\alpha, true)$ for each $\alpha \in \Sigma$. Modify all untimed prefixes (rows) and suffixes (columns) in the observation table (S, E, T) into timed versions, i.e., replace s by $(s, true)$, $s{\cdot}\alpha$ by $(s, true)(\alpha, true)$, and e by $(e, true)$ for each $s \in S$, $\alpha \in \Sigma$, and $e \in E$, respectively. (Line 3)
2. Perform the candidate query for the ERA M. If the answer is "yes", then the ERA M accepts the language U_T to be learned, and M is returned. (Line 5)
3. If the answer to the candidate query for M is "no" with a counterexample $(a_1, g_1)(a_2, g_2)\cdots(a_n, g_n)$ given by Teacher, TL* will split prefixes (rows) and suffixes (columns) in the observation table as follows. If a prefix $p \in S \cup (S{\cdot}\Sigma)$ or a suffix $e \in E$ in the observation table has a substring of the form (a_i, g) for some $i \in \{1, 2, \ldots, n\}$ and $[\![g_i]\!] \subset [\![g]\!]$, then $[\![g]\!]$ is partitioned using g_i such that $[\![g]\!] = [\![g_i]\!] \cup G$ where $G = \{\hat{g_1}, \hat{g_2}, \ldots, \hat{g_m}\}$ is obtained by $[\![g]\!] - [\![g_i]\!]$ using DBM subtraction [10]. The prefix p is split into $\{\hat{p_0}, \hat{p_1}, \hat{p_2}, \ldots, \hat{p_m}\}$

Algorithm 2: TL* Algorithm

input : Σ: alphabet, C_Σ: the set of event-recording clocks
output: a deterministic ERA accepting the unknown timed language U_T

1 Use L^* to learn a DFA M accepting $Untime(U_T)$;
2 Let (S, E, T) be the observation table during the L* learning process ;
3 Replace α by $(\alpha, true)$, s by $(s, true)$, and e by $(e, true)$ for each $\alpha \in \Sigma$, $s \in S$ and $e \in E$;
4 **while** *true* **do**
5   **if** $Q_c^T(M) = 1$ **then** **return** M ;
6   **else**
7   Let $(a_1, g_1)(a_2, g_2) \cdots (a_n, g_n)$ be the counterexample given by Teacher ;
8   **foreach** (a_i, g_i), $i \in \{1, 2, \ldots, n\}$ **do**
9   **if** (a_i, g) *is a substring of* p *or* e *for some* $p \in S \cup (S \cdot \Sigma)$ *and* $e \in E$ *such that* $[\![g_i]\!] \subset [\![g]\!]$ **then**
10   Let $G = \{\hat{g_1}, \hat{g_2}, \ldots, \hat{g_m}\}$ obtained by $[\![g]\!] - [\![g_i]\!]$;
11                 $\Sigma = \Sigma \setminus \{(a_i, g)\} \cup \{(a_i, g_i), (a_i, \hat{g_1}), (a_i, \hat{g_2}), \ldots, (a_i, \hat{g_m})\}$;
12   Split p into $\{\hat{p_0}, \hat{p_1}, \hat{p_2}, \ldots, \hat{p_m}\}$ where (a_i, g_i) is a substring of $\hat{p_0}$ and $(a_i, \hat{g_j})$ is a substring of $\hat{p_j}$ for all $j \in \{1, 2, \ldots, m\}$;
13   Split e into $\{\hat{e_0}, \hat{e_1}, \hat{e_2}, \ldots, \hat{e_m}\}$ where (a_i, g_i) is a substring of $\hat{e_0}$ and $(a_i, \hat{g_j})$ is a substring of $\hat{e_j}$ for all $j \in \{1, 2, \ldots, m\}$;
14   Update T by $Q_{m^T}(\hat{p_j} \cdot \hat{e_j})$ for all $j \in \{0, 1, 2, \ldots, m\}$;
15   **while** *there exists* $(s \cdot \alpha)$ *such that* $s \cdot \alpha \not\equiv s'$ *for all* $s' \in S$ **do**
16             $S \longleftarrow S \cup \{s \cdot \alpha\}$;
17   Update T by $Q_{m^T}((s \cdot \alpha) \cdot \beta)$ for all $\beta \in \Sigma$;
18         $v \longleftarrow WS((a_1, g_1)(a_2, g_2) \cdots (a_n, g_n))$;
19   **if** $|v| > 0$ **then**
20             $E \longleftarrow E \cup \{v\}$;
21   Update T by $Q_{m^T}(s \cdot v)$ and $Q_{m^T}(s \cdot \alpha \cdot v)$ for all $s \in S$ and $\alpha \in \Sigma$;
22   Construct candidate M from (S, E, T) ;

where (a_i, g_i) is a substring of $\hat{p_0}$ and $(a_i, \hat{g_j})$ is a substring of $\hat{p_j}$ for all $j \in \{1, 2, \ldots, m\}$. Similarly, the suffix e is also split into $\{\hat{e_0}, \hat{e_1}, \hat{e_2}, \ldots, \hat{e_m}\}$ where (a_i, g_i) is a substring of $\hat{e_0}$ and $(a_i, \hat{g_j})$ is a substring of $\hat{e_j}$ for all $j \in \{1, 2, \ldots, m\}$. Then the observation table is updated by performing the timed membership queries $Q_{m^T}(\hat{p_j} \cdot \hat{e_j})$ for all $j \in \{0, 1, 2, \ldots, m\}$. (Lines 7-14)

4. Check whether the observation table (S, E, T) is closed. If there is a prefix $s \cdot \alpha$ with no $s' \in \Sigma$ such that $s \cdot \alpha \equiv s'$ for some $s \in S$ and $\alpha \in \Sigma$, the observation table is not closed, and $s \cdot \alpha$ is added into the set of prefixes S. Then the observation table is updated by performing the timed membership queries $Q_{m^T}(s \cdot \alpha \cdot \beta)$ for all $\beta \in \Sigma$. (Lines 15-17)
5. Analyze the counterexample to find the witness suffix v. For a counterexample π given by Teacher and the observation table (S, E, T), we also define an i-decomposition query of the guarded word π, denoted by $Q^i_{m^T}(\pi)$, as

follows: $Q^i_{m^T}(\pi) = Q_{m^T}([u_i]_r \cdot v_i)$ where $\pi = u_i \cdot v_i$ is a decomposition of π such that $|u_i| = i$ and $[u_i]_r$ is the representing string of u_i in S. The witness suffix of π, denoted by $WS(\pi)$, is the suffix v_i of the decomposition of π such that $Q^i_{m^T}(\pi) \neq Q^0_{m^T}(\pi)$. If there is a witness suffix v_i, i.e., $|v_i| > 0$, then v_i is added into the set of suffixes E, and the observation table is updated by performing the timed membership queries $Q_{m^T}(s \cdot v_i)$ and $Q_{m^T}(s \cdot \alpha \cdot v_i)$ for each $s \in S$ and $\alpha \in \Sigma$. (Lines 18-21)

6. Construct the ERA $M = (\Sigma_M, L_M, l^0_M, \delta_M, L^f_M)$ from the observation table (S, E, T) such that $\Sigma_M = \Sigma$, $L_M = S$, $l^0_M = \{\lambda\}$, $\delta_M(s, a) = [s \cdot a]_r$ for $s \in S$ and $a \in \Sigma$, and $L^f_M = \{s \in S \mid T(s, \lambda) = 1\}$. Goto Step 3. (Line 22)

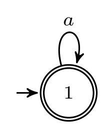
We use an example to illustrate the TL* algorithm. Suppose the timed language U_T to be learned is accepted by the ERA $\mathcal{A}_1$ as shown in Fig. 1 (a). In the untimed learning phase, the L* algorithm is used to learn the DFA M_1, as shown in Fig. 4 (c), accepting the untimed language a^*, and the observation table (S, E, T) obtained by L* is shown in Fig. 4 (a). At this time, $\Sigma = \{a\}$, $S = \{\lambda\}$, and $E = \{\lambda\}$.

	λ
λ	1 (s_0)
a	1

(a) T_1

	λ
λ	1 (s_0)
$(a, true)$	1

(b) T_2



(c) M_1

Fig. 4. Untimed Learning Phase

In the timed refinement phase, TL* first modifies the alphabet and the observation table into timed version. At this time, $\Sigma = \{(a, true)\}$, $S = \{(\lambda, true)\}$, and $E = \{(\lambda, true)\}$. The current timed observation table T_2 is shown in Fig. 4 (b). Then, TL* performs the timed candidate query for the first candidate ERA M_1. However, the answer to the candidate query is "no", and Teacher gives a negative counterexample $(a, x_a < 1) \in \mathcal{L}(M_1) \setminus \mathcal{L}(U_T)$. Because there is a prefix $(a, true)$ in the observation such that $[\![x_a < 1]\!] \subset [\![true]\!]$, the prefix $(a, true)$ is split into $(a, x_a < 1)$ and $(a, x_a \geq 1)$, and the timed membership queries for $(a, x_a < 1)$ and $(a, x_a \geq 1)$ are performed, respectively. The current observation table T_3 is shown in Fig. 5 (a). However, T_3 is not closed because there is $(a, x_a < 1)$ with no $s \in S$ such that $s \equiv (a, x_a < 1)$, so $(a, x_a < 1)$ is added into S and the membership queries for $(a, x_a < 1)(a, x_a < 1)$ and $(a, x_a < 1)(a, x_a \geq 1)$ are performed, respectively. The closed observation table T_4 is shown in Fig. 5 (b), and the corresponding ERA M_2 is constructed as shown in Fig. 5 (c). At this time, $\Sigma = \{(a, x_a < 1), (a, x_a \geq 1)\}$, $S = \{(\lambda, true), (a, x_a < 1)\}$, and $E = \{(\lambda, true)\}$.

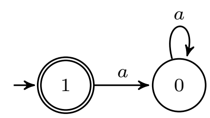
In the second iteration of the timed refinement phase, TL* performs the timed candidate query for M_2. However, the answer is still "no" with a positive counterexample $(a, x_a = 1) \in \mathcal{L}(U_T) \setminus \mathcal{L}(M_2)$. Because there are two prefixes $(a, x_a \geq 1)$ and $(a, x_a < 1)(x_a \geq 1)$ in the observation table (S, E, T) such

	λ
λ	1 (s_0)
$(a, x_a < 1)$	0
$(a, x_a \geq 1)$	0

(a) T_3

	λ
λ	1 (s_0)
$(a, x_a < 1)$	0 (s_1)
$(a, x_a \geq 1)$	0
$(a, x_a < 1)(a, x_a < 1)$	0
$(a, x_a < 1)(a, x_a \geq 1)$	0

(b) T_4



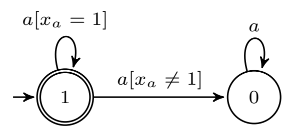
(c) M_2

Fig. 5. Timed Refinement 1

that $[\![x_a = 1]\!] \subset [\![x_a \geq 1]\!]$, the prefix $(a, x_a \geq 1)$ is split into $(a, x_a = 1)$ and $(a, x_a > 1)$, and the prefix $(a, x_a < 1)(x_a \geq 1)$ is split into $(a, x_a < 1)(x_a = 1)$ and $(a, x_a < 1)(x_a > 1)$, respectively. The timed membership queries for the new prefixes are performed. The current closed observation table T_5 is shown in Fig. 6 (a), and the corresponding ERA M_3 is shown in Fig. 6 (b). At this time, $\Sigma = \{(a, x_a < 1), (a, x_a = 1), (a, x_a > 1)\}$, $S = \{(\lambda, true), (a, x_a < 1)\}$, and $E = \{(\lambda, true)\}$.

	λ
λ	1 (s_0)
$(a, x_a < 1)$	0 (s_1)
$(a, x_a = 1)$	1
$(a, x_a > 1)$	0
$(a, x_a < 1)(a, x_a < 1)$	0
$(a, x_a < 1)(a, x_a = 1)$	0
$(a, x_a < 1)(a, x_a > 1)$	0

(a) T_5



(b) M_3

Fig. 6. Timed Refinement 2

In the third iteration of the timed refinement phase, TL* performs the timed candidate query for the ERA M_3. However, the answers is still "no" with a negative counterexample $\pi = (a, x_a = 1)(a, x_a = 1) \in \mathcal{L}(M_3) \setminus \mathcal{L}(U_T)$. This time, no prefix or suffix in the observation table has to be split. TL* analyzes the counterexample as follows. $Q^0_{m^T}(\pi) = Q_{m^T}((a, x_a = 1)(a, x_a = 1)) = 0$. $Q^1_{m^T}(\pi) = Q^1_{m^T}([(a, x_a = 1)]_r(a, x_a = 1)) = Q_{m^T}((a, x_a = 1)) = 1 \neq Q^0_{m^T}(\pi)$. Thus, we have a witness suffix $v = (a, x_a = 1)$, and v is added into the set of suffixes E. Then the timed membership queries for $s \cdot (a, x_a = 1)$ for all $s \in S$ are performed, and the current observation table T_6 is shown in Fig. 7 (a). However, the observation table T_6 is not closed because there is a prefix $(a, x_a = 1)$ with no $s \in S$ such that $s \equiv (a, x_a = 1)$, so $(a, x_a = 1)$ is added into the set of prefixes S and the timed membership queries for $(a, x_a = 1) \cdot \alpha$ for all $\alpha \in \Sigma$ are performed. The closed observation table T_7 is shown in Fig. 7 (b), and the corresponding ERA M_4 is shown in Fig. 7 (c). At this time, $\Sigma = \{(a, x_a < 1), (a, x_a = 1), (a, x_a > 1)\}$, $S = \{(\lambda, true), (a, x_a < 1), (a, x_a = 1)\}$, and $E = \{(\lambda, true), (a, x_a = 1)\}$.

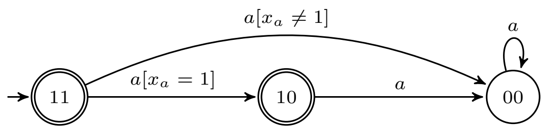
In the fourth iteration of the timed refinement phase, TL* performs the timed candidate query for the ERA M_4 again. However, the answer is still "no" with a positive counterexample $\pi = (a, x_a = 1)(a, x_a = 3) \in \mathcal{L}(U_T) \setminus \mathcal{L}(M_4)$.

	λ	$(a, x_a = 1)$
λ	1	1 (s_0)
$(a, x_a < 1)$	0	0 (s_1)
$(a, x_a = 1)$	1	0
$(a, x_a > 1)$	0	0
$(a, x_a < 1)(a, x_a < 1)$	0	0
$(a, x_a < 1)(a, x_a = 1)$	0	0
$(a, x_a < 1)(a, x_a > 1)$	0	0

(a) T_6

	λ	$(a, x_a = 1)$
λ	1	1 (s_0)
$(a, x_a < 1)$	0	0 (s_1)
$(a, x_a = 1)$	1	0 (s_2)
$(a, x_a > 1)$	0	0
$(a, x_a < 1)(a, x_a < 1)$	0	0
$(a, x_a < 1)(a, x_a = 1)$	0	0
$(a, x_a < 1)(a, x_a > 1)$	0	0
$(a, x_a = 1)(a, x_a < 1)$	0	0
$(a, x_a = 1)(a, x_a = 1)$	0	0
$(a, x_a = 1)(a, x_a > 1)$	0	0

(b) T_7



(c) M_4

Fig. 7. Timed Refinement 3

Because there are three prefixes $(a, x_a > 1)$, $(a, x_a < 1)(a, x_a > 1)$, and $(a, x_a = 1)(a, x_a > 1)$ in the observation table such that $[\![x_a = 3]\!] \subset [\![x_a > 1]\!]$, the prefix $(a, x_a > 1)$ is split into three prefixes $(a, 1 < x_a < 3)$, $(a, x_a = 3)$, and $(a, x_a > 3)$; the prefix $(a, x_a < 1)(a, x_a > 1)$ is split into three prefixes $(a, x_a < 1)(a, 1 < x_a < 3)$, $(a, x_a < 1)(a, x_a = 3)$, and $(a, x_a < 1)(a, x_a > 3)$; the prefix $(a, x_a = 1)(a, x_a > 1)$ is also split into three prefixes $(a, x_a = 1)(a, 1 < x_a < 3)$, $(a, x_a = 1)(a, x_a = 3)$, and $(a, x_a = 1)(a, x_a > 3)$. The timed membership queries for the new split prefixes concatenated with e for all $e \in E$ are performed. Then the TL* algorithm analyzes the counterexample. Since $Q^0_{m^T}(\pi) = Q^1_{m^T}(\pi) = Q^2_{m^T}(\pi)$, therefore there is no witness suffix for π. The closed observation table T_8 is shown in Fig. 8 (a), and it corresponding ERA M_5 is constructed as shown in Fig. 8 (b). At this time, $\Sigma = \{(a, x_a < 1), (a, x_a = 1), (a, 1 < x_a < 3), (a, x_a = 3), (a, x_a > 3)\}$, $E = \{(\lambda, true), (a, x_a < 1), (a, x_a = 1)\}$, and $E = \{(\lambda, true), (a, x_a = 1)\}$.

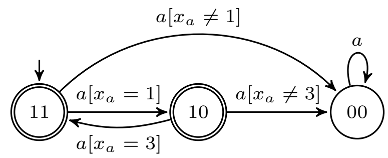
In the fifth iteration of the timed refinement, TL* performs the timed candidate query for M_5. This time, Teacher says that $\mathcal{L}(M_5) = U_T$, and the learning process of TL* is finished.

3.2 Analysis of the TL* Algorithm

The time complexity of TL* is analyzed as follows. Given a timed language U_T accepted by a deterministic ERA $\mathcal{A} = (\Sigma, L, l_0, \delta, L^f)$, the TL* algorithm learns $Com(\mathcal{A})$ to accept U_T. In the learning process of TL*, each untimed word $(\alpha, true)$ for $\alpha \in \Sigma$ might be split into $|G_{\mathcal{A}}|$ timed words, where $G_{\mathcal{A}}$ is the set of clock zones partitioned by the clock guards appearing in $\mathcal{A}$. For example, the clock guards appearing in $\mathcal{A}_1$, as shown in Fig. 1 (a), are $x_a = 1$ and $x_a = 3$, so $G_{\mathcal{A}} = \{x_a < 1, x_a = 1, 1 < x_a < 3, x_a = 3, x_a > 3\}$. Thus, each membership query of untimd word $(a, true)$ gives rise to $|G_{\mathcal{A}}|$ timed membership queries. Totally, TL* needs to perform $O(|\Sigma| \cdot |G_{\mathcal{A}}| \cdot |L|^2 + |L| \log |\pi|)$ membership

	λ	$(a, x_a = 1)$	
λ	1	1	(s_0)
$(a, x_a < 1)$	0	0	(s_1)
$(a, x_a = 1)$	1	0	(s_2)
$(a, 1 < x_a < 3)$	0	0	
$(a, x_a = 3)$	0	0	
$(a, x_a > 3)$	0	0	
$(a, x_a < 1)(a, x_a < 1)$	0	0	
$(a, x_a < 1)(a, x_a = 1)$	0	0	
$(a, x_a < 1)(a, 1 < x_a < 3)$	0	0	
$(a, x_a < 1)(a, x_a = 3)$	0	0	
$(a, x_a < 1)(a, x_a > 3)$	0	0	
$(a, x_a = 1)(a, x_a < 1)$	0	0	
$(a, x_a = 1)(a, x_a = 1)$	0	0	
$(a, x_a = 1)(a, 1 < x_a < 3)$	0	0	
$(a, x_a = 1)(a, x_a = 3)$	1	1	
$(a, x_a = 1)(a, x_a > 3)$	0	0	

(a) T_8



(b) M_5

Fig. 8. Timed Refinement 4

queries to learn $Com(\mathcal{A})$, where π is the counterexamples given by Teacher. After a candidate query of a candidate ERA M, TL* either adds a location into M or splits a clock guard on a transition of M into at least two clock guards. Thus, TL* needs to perform $O(|L| + |\Sigma| \cdot |G_{\mathcal{A}}|)$ to learn $Com(\mathcal{A})$ accepting U_T.

Theorem 1 proves the correctness of the TL* algorithm, and the termination of the TL* algorithm is proved in Theorem 2.

Theorem 1. *The TL* algorithm is correct.*

Proof. Let U_T be the timed language and M be the ERA learned by TL*. We want to prove that $\mathcal{L}(M) = U_T$ holds, i.e., $\mathcal{L}(M) \subseteq U_T$ and $U_T \subseteq \mathcal{L}(M)$. Let's prove $\mathcal{L}(M) \subseteq U_T$ by contradiction. Assume $\mathcal{L}(M) \setminus U_T \neq \emptyset$, which implies $Q_{c^T}(M) = 0$. However, this contradicts to the fact that M is returned by TL* because $Q_{c^T}(M) = 1$. Let's prove $U_T \subseteq \mathcal{L}(M)$ also by contradiction. Assume $U_T \setminus \mathcal{L}(M) \neq \emptyset$, which implies $Q_{c^T}(M) = 0$. However, this also contradicts to the fact that M is returned by TL* because $Q_{c^T}(M) = 1$. $\square$

Theorem 2. *The TL* algorithm terminates.*

Proof. Let U_T be the unknown timed language. Assume that U_T is accepted by a hypothetic deterministic ERA $\mathcal{A} = (\Sigma, L, l_0, \delta, L^f)$. In the learning process, TL* will constructively modify the observation table such that the final observation table is consistent with $Com(\mathcal{A})$. After each timed candidate query of a candidate ERA M, TL* either adds a location into M or splits a guarded word (a, g) on a transition of M into several guarded words $(a, g_1), (a, g_2), \ldots (a, g_n)$ such that $[\![g]\!] = [\![g_1]\!] \cup [\![g_2]\!] \cup \ldots \cup [\![g_n]\!]$. Let $G_{\mathcal{A}}$ be the set of clock zones partitioned by the clock guards appearing in $\mathcal{A}$. At last, each split clock guard g_i will belongs to $G_{\mathcal{A}}$ for all $i \in \{1, 2, \ldots, n\}$. Since both $|L|$ and $|G_{\mathcal{A}}|$ are finite, the TL* algorithm will terminate after $O(|L| + |\Sigma| \cdot |G_{\mathcal{A}}|)$ iterations. $\square$

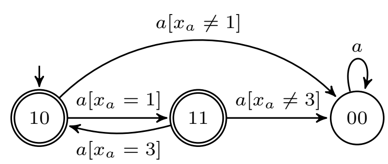
Theorem 3 proves the minimality of the TL* algorithm, i.e., given an unknown timed language U_T, the number of locations of the ERA M learned by the TL* algorithm is minimum among all ERAs M' such that $\mathcal{L}(M') = U_T$.

Theorem 3. *Assume the observation table (S, E, T) is closed and consistent and $M = (\Sigma, L, l^0, \delta, L^f)$ is the ERA constructed from the observation table (S, E, T). If $M' = (\Sigma, L', l^{0'}, \delta', L^{f'})$ is any other ERA consistent with T, then M' has at least $|L|$ locations.*

Proof. Before we prove this theorem, let us first formally define the membership query results of a row in the observation table. If $p \in S \cup (S \cdot \Sigma)$ is a prefix (row) of the observation table, we use $row(p)$ to denote the finite function $f : E \mapsto \{0, 1\}$ defined by $f(e) = T(p \cdot e)$ for $e \in E$. Because M' is consistent with T, therefore for each $p \in S \cup (S \cdot \Sigma)$ and for each $e \in E$, $\delta'(l^{0'}, p \cdot e) \in L^{f'}$ iff $T(p \cdot e) = 1$, which means that $\delta'(\delta'(l^{0'}, p), e) \in L^{f'}$ iff $T(p, e) = 1$, so $row(\delta'(l^{0'}, p))$ is equal to $row(p)$. Since p ranges over L and $row(\delta'(l^{0'}, p))$ ranges over L', so $|L'| \geq |L|$, i.e., M' must have at least $|L|$ locations. $\square$

	λ	$(a, x_a = 3)$
λ	1	0 (s_0)
$(a, x_a = 0)$	0	0 (s_1)
$(a, x_a = 1)$	1	1 (s_2)
$(a, x_a = 2)$	0	0
$(a, x_a = 3)$	0	0
$(a, x_a \geq 3)$	0	0
$(a, 0 \leq x_a \leq 1)$	0	0
$(a, 1 \leq x_a \leq 2)$	0	0
$(a, 2 \leq x_a \leq 3)$	0	0
$(a, 0 \leq x_a \leq 2)$	0	0
$(a, 1 \leq x_a \leq 3)$	0	0
$(a, 0 \leq x_a \leq 3)$	0	0
$(a, x_a = 0)(a, x_a = 0)$	0	0
$(a, x_a = 0)(a, x_a = 1)$	0	0
$(a, x_a = 0)(a, x_a = 2)$	0	0
$(a, x_a = 0)(a, x_a = 3)$	0	0
$(a, x_a = 0)(a, x_a \geq 3)$	0	0
$(a, x_a = 0)(a, 0 \leq x_a \leq 1)$	0	0
$(a, x_a = 0)(a, 1 \leq x_a \leq 2)$	0	0
$(a, x_a = 0)(a, 2 \leq x_a \leq 3)$	0	0
$(a, x_a = 0)(a, 0 \leq x_a \leq 2)$	0	0
$(a, x_a = 0)(a, 1 \leq x_a \leq 3)$	0	0
$(a, x_a = 0)(a, 0 \leq x_a \leq 3)$	0	0
$(a, x_a = 1)(a, x_a = 0)$	0	0
$(a, x_a = 1)(a, x_a = 1)$	0	0
$(a, x_a = 1)(a, x_a = 2)$	0	0
$(a, x_a = 1)(a, x_a = 3)$	1	0
$(a, x_a = 1)(a, x_a \geq 3)$	0	0
$(a, x_a = 1)(a, 0 \leq x_a \leq 1)$	0	0
$(a, x_a = 1)(a, 1 \leq x_a \leq 2)$	0	0
$(a, x_a = 1)(a, 2 \leq x_a \leq 3)$	0	0
$(a, x_a = 1)(a, 0 \leq x_a \leq 2)$	0	0
$(a, x_a = 1)(a, 1 \leq x_a \leq 3)$	0	0
$(a, x_a = 1)(a, 0 \leq x_a \leq 3)$	0	0

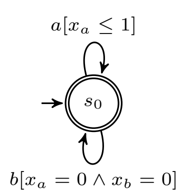
(a)



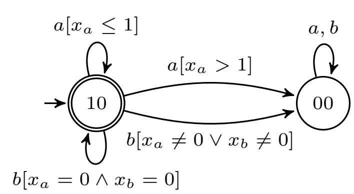
(b)

Fig. 9. Grinchtein's TL^*_{sg} Algorithm

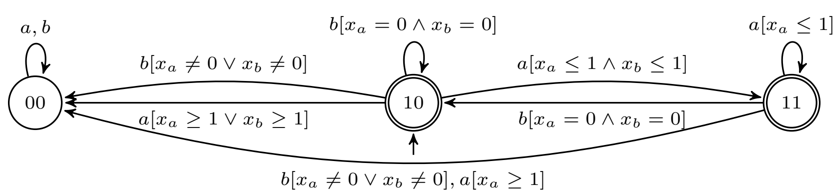
Comparison. Since Grinchtein et al. [6, 7] did not implement the TL^*_{sg} algorithm, to be fair, let us compare our TL^* algorithm with TL^*_{sg} from a theoretical point of view. TL^*_{sg} uses region construction to actively guess all possible time constraints for an untimed word, so an original untimed membership query in L* gives rise to several membership queries of time words. The number of timed membership queries required by the TL^*_{sg} algorithm is $O(|\Sigma \times G_\Sigma| \cdot n^2|\pi| \cdot |w|\binom{|\Sigma|+K}{|\Sigma|})$ where n is the number of locations of the learned ERA, π is the counterexample given by Teacher, w is the longest guarded word queried, and K is the largest constant appearing in the clock guards. We can observe that the number of timed membership queries required by Grinchtein's TL^*_{sg} algorithm increases exponentially to the largest constant K and the number of alphabet $|\Sigma|$. To learn the timed language accepted by $\mathcal{A}_1$, as shown in Fig. 1 (a), Grinchtein's TL^*_{sg} algorithm needs 34 timed membership queries, while our TL* only needs 16 timed membership queries. Fig. 9 (a) shows the observation table obtained by Grinchtein's TL^*_{sg} algorithm, and the final learned ERA is shown in Fig. 9 (b). Note that our TL* algorithm is not affected by the largest constant K appearing in the clock guards. If we change the guarded word $a[x_a = 3]$ in $\mathcal{A}_1$, as shown in Fig. 1 (a), into $a[x_a = 100]$, the number of membership queries required by our TL* algorithm is still 16, while that required by Grinchtein's TL^*_{sg} algorithm increases exponentially. Further, Grinchtein's TL^*_{sg} algorithm cannot guarantee the number of locations of the learned ERA is minimal, while our TL* algorithm can. Fig. 10 (a) gives another unknown timed language U'_T accepted by $\mathcal{A}_2$. Our TL* algorithm learns $Com(\mathcal{A}_2)$ to accept U'_T, whose number of locations is minimal as shown in Fig. 10 (b), while the ERA M' learned by Grinchtein's TL^*_{sg} algorithm is shown in Fig. 10 (c), whose number of locations is not minimal.



(a) $\mathcal{A}_2$



(b) $Com(\mathcal{A}_2)$ learned by TL*



(C) M'

Fig. 10. Difference between TL* and TL^*_{sg}

4 Conclusion and Future Work

We proposed an efficient polynomial time algorithm, TL*, for learning event-recording automata. The TL* algorithm can also be applied to learn other determinizable subclasses of timed automata, such as event-predicting automata (EPA) [2], as long as the timed Teacher answers the timed membership and timed candidate queries correctly. Our future work will implement the TL* algorithm into the PAT model checker [13] such that PAT can automalically generate the assumptions for assume-guarantee reasoning for timed systems.

Acknowledgment. This work benefited from the discussions via e-mails with Olga Grinchtein, the author of [6, 7].

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