CS3230 : Tutorial - 4

Bakh Khoussainov

The deadline is 1pm, 11-Sept-2012.

1. Consider the *Merge Sort Algorithm* from the lecture. The inputs to the algorithm are arrays $A = a_1, \ldots, a_n$ of integers. The number n is the size of the input A. There is one **basic operation** that the algorithm performs. The operation is the comparison operation. Prove that the total number of basic operations needed to obtain the sorted output on input of size n is bounded by $Cn \cdot log(n)$, where C is some constant. What is the value of the constant?

2. You are given n points on real line: $p_1, \ldots, p$. You would like to cover these points by unit intervals (that is intervals of length 1). The problem is to find the smallest number of unit intervals that cover all points. Consider the following two attempts to solving the problem:

   (a) Let I be the interval that covers the most number of points. Add I to the solution set S. Repeat your process until all points are covered. Prove or disprove that the algorithm solves the problem.

   (b) Let p_i be the leftmost point in the list of points. Add the interval $[p_i, p_i + 1]$ o the solution set S. Repeat your process until all the points are covered. Prove or disprove that the algorithm solves the problem

3. Using 8 bits, how many characters can you code in a prefix free way?

4. Suppose the letters you want to code are listed as

   $$A, X, Z, V, T$$

   in decreasing order of their freuency. Construct a Huffman code γ for these letters.

5. Consider the example above. Assume that the frequencies of these letters are 40%, 30%, 20%, 5%, and 5%, respectively. Calculate the ABL for the code γ.

6. Let G be a weighted graph. Recall that by a graph we mean undirected graph with no self-loops. Let us apply Kruskal's algorithm to G. Prove the following:

(a) The algorithm produces a forest F (Recall that a forest is a union of trees).

(b) If F is a union of n trees then G has exactly n components.

(c) Each of $T_1, \ldots, T_n$ is a spanning tree for some of the components of the graph G.

7. Give your analysis of the running times of Prim and Krsukal's Algorithms.

8. Give your analysis of the running time of the algorithm (provided in the lecture) that finds Huffman codes.

9. Prove that

   (a) $\sum_{i=1}^{n} i \log i = \Theta(n^2 \log n)$

   (b) $2^n = O(n!); \quad n! = O(n^n)$

   (c) $n^{n+1} = 2^{(n+1)\log_2 n} \leq 2^{n^2}$

   (d) If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = c$ where $c > 0$ is a real number, then $f(n) = \Theta(g(n))$.

10. Find asymptotic bounds for the following in terms of big O.

   (a) $T(n) \leq 2T(\lfloor n/2 \rfloor) + n^2$

   (b) $T(n) \leq 4T(\lceil n/2 \rceil) + n$

   (c) $T(n) = 2nT(n-1)$

11. Use iteration to solve the following recurrence relations:

   (a) $T(n) = 2T(n-1) + 1, n > 0, T(1) = 1$

   (b) $T(n) = 2^n T(n-1), n > 0, T(0) = 1$

   (c) $T(n) = 2 + \sum_{i=1}^{n-1} T(i), n > 1, T(1) = 1$