Graph Visualization

Input: a graph \( G \) with \( n \) nodes and \( m \) edges
Output: a 2D position matrix \( X \)

Drawing:
- Position each node \( v_i \) at its coordinate \( X[i] \)
- Link two endpoints of each edge with a straight segment

Aesthetic criteria:
- Evaluate the readability of \( X \)
- Node distribution (ND):
  - Measure the distribution evenness of the nodes on the screen
- Uniform Length Coefficient Variance (ULCV):
  - Measure the length skewness of edge segments on the screen

PPRviz Framework

Idea: interactively show the partial view of \( G \) level by level
Offline: supergraph hierarchy construction
- Build a hierarchy \( H \) for \( G \) by Louvain with balanced size

Online #1: node distance computation
- Propose a new distance measure PDist
- Compute PDist matrix \( \Delta \) for children in \( S \) by our Tau-Push

Online #2: node position embedding
- Compute \( X \) by \( \Delta \)
- Make node pair’s Euclidean distance resemble its PDist

PPDist for Leaf Nodes

Personalized PageRank (PPR)
- Input: a source \( v_i \), a target \( v_j \), a stopping probability \( \alpha \)
- Random walk with restart (RWR) from \( v_i \):
  - At each step, stops with probability \( \alpha \) at the current node,
  - With \( 1 - \alpha \) probability randomly jumps to one of neighbors
- PPR from \( v_i \) to \( v_j \):
  \[ \pi(v_i, v_j) = \mathbb{P}[\text{RWR from } v_i \text{ stops at } v_j] \]

PPDist between any nodes \( v_i, v_j \):
- Degree-normalized PPR (DPPR):
  \[ \pi_d(v_i, v_j) = \pi(v_i, v_j) \cdot d(v_i) \]
- Convert DPPR to a distance:
  \[ 1 - \log(\pi_d(v_i, v_j) + \pi_d(v_j, v_i)) \]
- Pros:
  - Preserve high-order information
  - Guarantee visualization quality in terms of ND and ULCV

Tau-Push for Leaf Nodes

Step #1: Tau value computation
- Compute \( \tau_j \) for each \( v_j \): \( \tau_j = \frac{1}{\log(2)} \sum_i \pi_d(v_i, v_j) \)
- Identify \( v_j \) with \( \tau_j \) larger than a pre-defined \( \tau \)

Step #2: Forward Push
- A deterministic version of RWR
- Estimate \( \pi_d(v_i, v_j) \) from \( v_i \) to \( v_j \) with \( \tau_j < \tau \)

Step #3: Backward Push
- A reversed deterministic version of RWR
- Estimate \( \pi_d(v_j, v_i) \) from \( v_j \) to \( v_i \) with \( \tau_j \geq \tau \)

Index: \( \tau_j \) in step #1 and \( \pi_d(v_i, v_j) \) in step #3
Result: For any \( v_i, v_j \in S \) and \( v_i \neq v_j \):
  \[ |\pi_d(v_i, v_j) - \pi_d(v_j, v_i)| \leq \epsilon \cdot \delta \]

Datasets and competitors

- 12 real-world graphs from different fields
- 11 single-level and 2 multi-level competitors

Effectiveness

PPRviz outperforms all competitors in terms of aesthetic criteria and visualization results

Efficiency

PPRviz outperforms all competitors in terms of preprocessing and response time

Experiments

Visualization results of PPRviz (left) and the best competitor FR (right) on FilmTrust

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