1. Give an \( \epsilon \)-NFA for the language generated by the following right-linear grammar.

\[
S \rightarrow abA | aaB | \epsilon \\
A \rightarrow baA | bB \\
B \rightarrow aS 
\]

2. The right-linear grammars we studied in class have productions of the form: \( V \rightarrow T^* (V \cup \{ \epsilon \}) \) (that is, the non-terminal on the RHS, if any, is at the right end). A left-linear grammar is one in which the productions are of the form: \( V \rightarrow (V \cup \{ \epsilon \}) T^* \) (that is, the non-terminal on the RHS, if any, is at the left end).

Recall that \( x^R \) denotes the reverse of string \( x \).

(a) Let \( L^R = \{ w^R : w \in L \} \). It was earlier shown that if \( L \) is regular then so is \( L^R \).

(b.1) Suppose \( G \) is a right-linear grammar for \( L \). Show how to produce a left-linear grammar for \( L^R \), using \( G \).

(b.2) Suppose \( G \) is a left-linear grammar for \( L \). Show how to produce a right-linear grammar for \( L^R \), using \( G \).

(c) Using (a) and (b) show that left-linear grammars generate exactly the regular languages.

3. Give context free grammars for the following languages over the alphabet \( \Sigma \):

(a) \( L = \{ cwcw^R : w \in \{ a, b \}^* \} \). \( \Sigma = \{ a, b, c \} \).

(b) \( L = \{ a^n b^m : 2m \geq n \} \). \( \Sigma = \{ a, b \} \).

(c) \( L = \{ w : \text{number of } a\text{'s in } w \text{ is the same as the number of } b\text{'s in } w \} \). \( \Sigma = \{ a, b \} \).

4. Consider the grammar given in the previous question for \( L = \{ w : \text{number of } a\text{'s in } w \text{ is the same as the number of } b\text{'s in } w \} \).

Give a parse tree for \( abbaab \).

5. (a) Show that the following grammar is ambiguous:

\[
S \rightarrow bA | aB \\
A \rightarrow a | aS | bAA \\
B \rightarrow b | bS | aBB 
\]
(b) Find unambiguous grammar for the language generated by the grammar in part (a).

6. Construct NPDA's for the following languages. Let \( \#_a(w) \) denote the number of \( a \)'s in string \( w \), where \( a \in \Sigma \).

(a) \( L = \{ wcw^R : w \in \{ a, b \}^* \} \). \( \Sigma = \{ a, b, c \} \).

(b) \( L = \{ w : \#_a(w) > \#_b(w) \} \). \( \Sigma = \{ a, b \} \).

(c) \( L = \{ a^i b^j c^k : i = j \text{ or } j = k \} \). \( \Sigma = \{ a, b, c \} \).

(d) \( L = \{ w_1cw_2 : w_1, w_2 \in \{ a, b \}^* \text{ and } w_1 \neq w_2^R \} \). \( \Sigma = \{ a, b, c \} \).