CS4234: Optimisation Algorithms Tutorial 5

More Max-Flow Application; Push-Relabel

V1.4: Steven Halim *September 28, 2020*

Discussion Points

Q1: Please read `https://onlinejudge.org/external/128/12873.pdf` and try to reduce this problem into a max flow problem, solve it using $O(n^2 \times m)$ Dinic's algorithm (assuming that you have such implementation ready), and analyze its time complexity.

Q2: Please perform a manual execution of a basic $O(n^2m)$ Push-Relabel algorithm works on the small flow network shown in Figure 1 (there is no VisuAlgo visualization on Push-Relabel algorithm yet but you can use other people's tool, like `http://www.adrian-haarbach.de/idp-graph-algorithms/implementation/maxflow-push-relabel/index_en.html`). For this question, you are allowed to perform the push or relabel actions *in any order* (but do not use the strategy mentioned in Q3 yet). Tutor may change the graph for the actual tutorial.

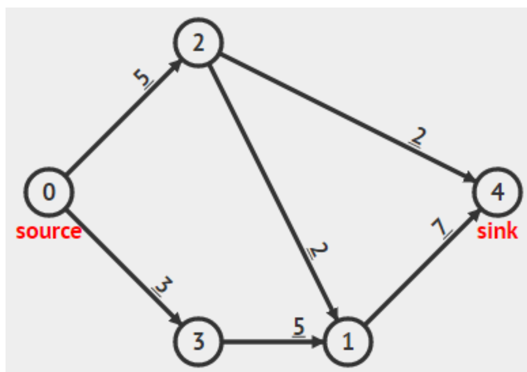

Figure 1: The initial flow graph

Q3: The 'slowest' part of a basic $O(n^2m)$ Push-Relabel algorithm is due to the $2n^2 + 4n^2m$ possible non-saturating push operations. We can make this bound tighter to $O(n^3)$ by doing this strategy: "If at each step, we choose the vertex with excess *at maximum height* (or in another word, we discharge *all* excess flow from that vertex first), then the number of non-saturating push operations throughout the algorithm is at most $4n^3$", thus giving rise to the tighter $O(n^3)$ Push-Relabel algorithm. In CLRS, this strategy is called the Relabel-To-Front version of Push-Relabel algorithm.

Now perform this strategy on the same Figure 1 and give a short sketch on why this is faster.

Q4: Someone suggests that we can optimize the performance of Push-Relabel algorithm for Max-Flow problem by *not* processing vertices that still have excess (no more push or relabel operation on those vertices) when their heights are $\geq n$ *if we only need the s-t* Max-Flow *value* of the flow graph. Show that this idea is actually correct by explaining succinctly on what will happen to the excess flow in those non-processed vertices (i.e. vertices with heights $\geq n$) if we run Push-Relabel algorithm as per normal (i.e. until all vertices have no more excess flow)?

Past Paper Discussions

Time permitting, the tutor will then discuss some relevant questions from past papers.
This is not crucial on Week 07, but it (past paper discussion) is the main topic for T06 (that has no new question).

Important

Please do not forget to hand in your 5 pages report for PS3 by this Sat, 03 Oct 2020, 07.59am.