CS4234: Optimisation Algorithms	Tutorial 7
Graph-Matching	
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Discussion Points

Q1: Back in Lecture 1, we have learned about the Min-Vertex-Cover (MVC) problem. In T03, we also have learned about the Max-Independent-Set (MIS) problem. We haven't discuss the special cases of these two problems if they are asked on *Bipartite Graph*, so we will do it now.

Given a Bipartite Graph $G = (V_L, V_R), E$ of approximately 100 vertices and up to 2 500 edges, show how to find the MVC and MIS on G by reducing those problems into Bipartite Matching and further reduce them into Max Flow problems.

Follow up question: What if the MVC/MIS problems asked are the weighted variants?

Q2: Find the underlying (bipartite, that is a major hint) graph in this (graph matching, that is another major hint) problem: `https://uva.onlinejudge.org/external/6/670.pdf`

Q3: Mr. Kwan is a postman who wants to deliver a bag of mails to houses in an unnamed Chinese city. The houses in that city are located along the streets and no house is located at a junction. Streets in that city are one-way and Kwan is a very law-abiding citizen and will never walk that one-way street in the opposite direction. Mr. Kwan wants to design a route whereby he starts from a starting junction 0 (his favorite landmark in the city), go through *every* one-way street in that city *at least once*, and returns to that starting junction 0 again to deliver the mails. Kwan knows that his government has designed the city well enough so that there will always be at least one such route. Kwan just want to know what is *the minimum number of streets* that he has to traverse to accomplish his objective. There are V junctions and E streets in that Chinese city (graph G).

Complete Search

To kick start your understanding of this Combinatorial Optimization Problem (COP), please do a complete search on the small instance of this problem shown in Figure 1.

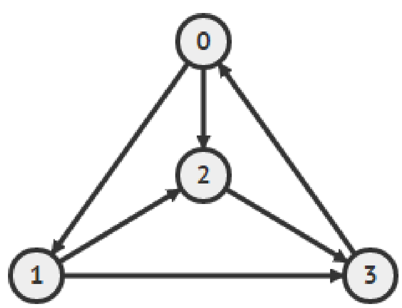

Figure 1: Small instance of this problem.

The minimum number of streets that Mr. Kwan has to traverse to accomplish his objective is optimal route taken by Mr. Kwan is ________ streets/edges.
Just to ensure that you do not put random answer, please write down the optimal route here:
$0 \rightarrow$ ____ $\rightarrow 0$.

Complexity Class

Is this COP NP-hard? (circle one)

YES	NO

Special Case

Suppose that the city has one more additional property, i.e. the number of streets that goes into a junction is *always equal* to the number of streets that goes out from that junction, i.e. there is a "flow conservation" property. For example, imagine that there is no junction 2 (and all edges associated with it) in Figure 1 above and all the other 3 junctions $\{0, 1, 2\}$ will each have 1 street that goes in/out of those junctions so Mr Kwan's route will be something like $0 \to 1 \to 3 \to 0$.

Does this additional property simplify the COP?
If yes (Obviously the answer is a yes :) otherwise the questions in this section are meaningless), design a polynomial time algorithm to solve this problem!

What is the time complexity of your polynomial time algorithm?

General Case

Now solve the general case of this COP using *any algorithm that you have learned in class* and analyze the time complexity of your solution!

Postscript

In the last few tutorials: we will go back to NP-hard optimization problems but will use different search paradigm: Stochastic Local Search.