UIT2206: The Importance of Being Formal

Assignment Week 08: Midterm Review

Traditional Logic

1. Consider the following syllogism:

   All humans are mortal.
   All Greeks are humans.
   Therefore, all Greeks are mortal.

   Identify all propositions and all categorical terms in this syllogism. Define a suitable universe to give meaning to all terms.

2. Consider the following syllogism:

   All slack track systems are caterpillar systems.
   All Christie suspension systems are slack track systems.
   Therefore, **some** Christie suspension systems are caterpillar systems.

   Give a model in which this syllogism does not hold.

3. Consider the following statements:

   The only articles of food, that my doctor allows me, are such as are not very rich.
   Nothing that agrees with me is unsuitable for supper.
   Wedding cake is always very rich.
   My doctor allows me all articles of food that are suitable for supper.
   Therefore, wedding cakes do not agree with me.

   Identify all categorical terms in these statements. Define a suitable universe to give meaning to all terms.

4. Is the proposition

   `Some humans are humans`

   valid? Use the semantics of categorical propositions in your argument.

Propositional Logic

5. Draw the parse tree for the following formula:

$$((\neg p) \wedge ((\neg q) \wedge ((\neg r) \wedge ((\neg s) \wedge \top))))$$

List all sub-formulas of the expression.

6. According to the operator precedences in Convention 1 (page 7), the following formula has a unique reading.

$$\neg p \wedge q \to \neg r \vee \neg p \to r$$

Indicate this reading by writing all parentheses, according to Definition 1.

7. Consider the special case of propositional logic where the set of atoms A is empty, and where there are no binary operations. Is the set of resulting formulas still infinite? Describe the set of resulting formulas using English or a formal notation such that the reader understands what formulas it contains.

8. Give the result of evaluating the following formula with respect to the valuation $v : v(p) = F, v(q) = T, v(r) = T, v(s) = F$.

- $(\neg q \vee r) \wedge (r \to \neg q)$
- $p \to q \to s$
- $(\bot \to \bot) \wedge (\top \to \top)$

9. Is the following formula valid?

$$(p \wedge \neg q) \to (q \vee \neg p)$$

Show how you arrived at your answer.

Predicate Logic

10. Let P be a predicate symbol of arity 0, Q be a predicate symbol of arity 2, f be a function symbol of arity 2, and g a function symbol of arity 1. Consider the formula

$$\phi = \neg(\exists w((P \vee (\forall v Q(f(x,v), w)))))$$

(a) Draw the parse tree of ϕ.

(b) Indicate the free and bound variables in this parse tree.

(c) Compute $[x \Rightarrow g(f(x,x))]\phi$.

11. Let ϕ be the sentence

$$\forall x \forall y \exists z (R(x, y) \to R(y, z))$$

where R is a predicate symbol of arity 2.

(a) Let $A = \{a, b, c, d\}$ and $R^{\mathcal{M}} = \{(b, c), (b, b), (b, a)\}$. Does $\mathcal{M} \models \phi$ hold? Justify your answer.

(b) Let $A' = \{a, b, c\}$ and $R^{\mathcal{M}'} = \{(b, c), (a, b), (c, b)\}$. Does $\mathcal{M}' \models \phi$ hold? Justify your answer.

12. Show that for any two sentences ϕ and ψ in predicate calculus, we have $\models \phi \to \psi$ whenever $\phi \models \psi$.

13. Is the sentence $\forall x P(x) \vee \forall x Q(x)$ entailed by $\forall x (P(x) \vee Q(x))$? Justify your answer.

14. Consider the sentences

$$\begin{aligned} \phi_1 &::= \forall x P(x, x) \\ \phi_2 &::= \forall x \forall y (P(x, y) \to P(y, x)) \end{aligned}$$

Show $\phi_1 \not\models \phi_2$.