Simplified Cheat Sheet Predicate Logic

Martin Henz, 15/3/2013

Bottom-up

General tactic on bottom-up reasoning: When you are proving a goal, and you think you need and can prove a hypothesis $\phi$, you can add it via `assert $\phi$`. Coq will then ask you to first prove $\phi$. After that, you can work on your original goal, now with the additional hypothesis of $\phi$. This is a special case of $\to e$, see below.

$\top i$: trivial.

$\land i$: split.

$\land e_1$: Let us call the goal $g$. Here is how to use $\land e_1$ bottom-up, in case you would ever need it: `assert $g \land \phi$. prove conjunction, destruct H1. apply H2`. where H1 is the conjunction and H2 is $g$.

$\land e_2$: Let us call the goal $g$. Here is how to use $\land e_2$ bottom-up, in case you would ever need it: `assert $\phi \land g$. prove conjunction, destruct H1. apply H2`. where H1 is the conjunction and H2 is $g$.

$\lor i_1$: left.

$\lor i_2$: right.

$\lor e$: destruct H.

$\to i$: intro.

$\to e$: apply H. (H is the implication)

A variant of the rule allows you to prove a goal $\psi$, by proving first $\phi$, and then $\phi \to \psi$: `assert $\phi$. then prove $\phi$, and finally prove goal $\psi$ using $\phi`.

$\neg e$: assert $\phi \land \neg \phi$. split. prove $\phi$ and $\neg \phi$ separately, then use destruct H1. contradiction H2., where H1 is the asserted conjunction, and H2 is one part of it.

$\neg i$: unfold not. intro.

$\bot e$: exfalso.

$\neg \neg e$: Let us call the goal $g$. Here is how to use $\neg \neg e$ bottom-up, in case you would ever need it: `assert ($\neg \neg g$). prove $\neg \neg g$. Now use: tauto. equality H from right to left`.

Derived rule: LEM $\lor e$: LEM ($\phi$).

Top-down

Coq allows you to apply some rules within the hypotheses, which makes many proofs a lot shorter. Here are some common uses of top-down reasoning:

$\to e$: spec H1 H2. (H1 is the implication)

$\neg i$: unfold not in H.

$\land i$: destruct H.