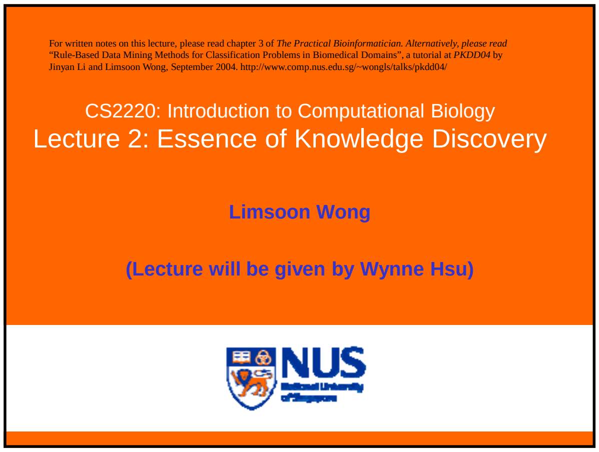
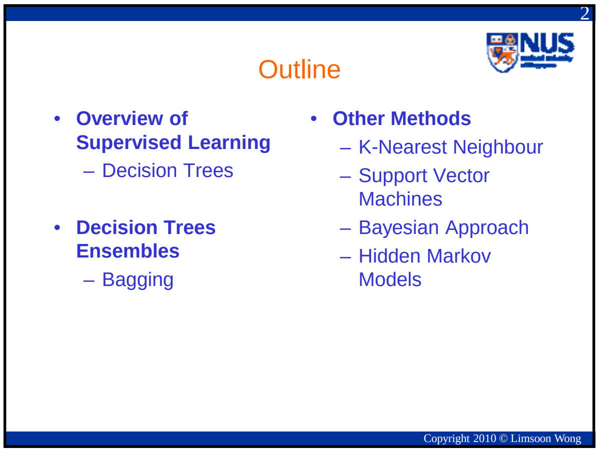


For written notes on this lecture, please read chapter 3 of *The Practical Bioinformatician. Alternatively, please read* “Rule-Based Data Mining Methods for Classification Problems in Biomedical Domains”, a tutorial at *PKDD04* by Jinyan Li and Limsoon Wong, September 2004. http://www.comp.nus.edu.sg/~wongls/talks/pkdd04/

CS2220: Introduction to Computational Biology
Lecture 2: Essence of Knowledge Discovery

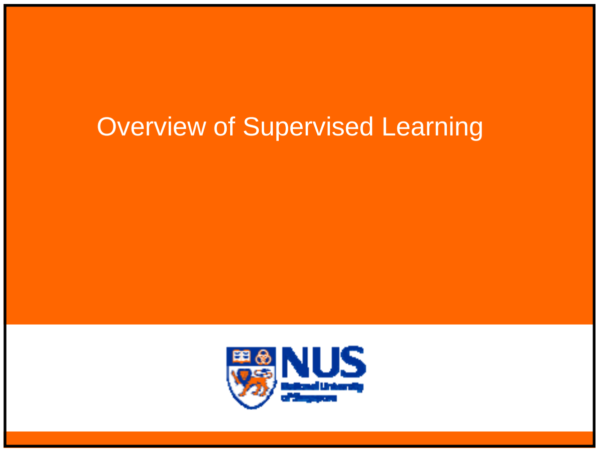
Limsoon Wong

(Lecture will be given by Wynne Hsu)

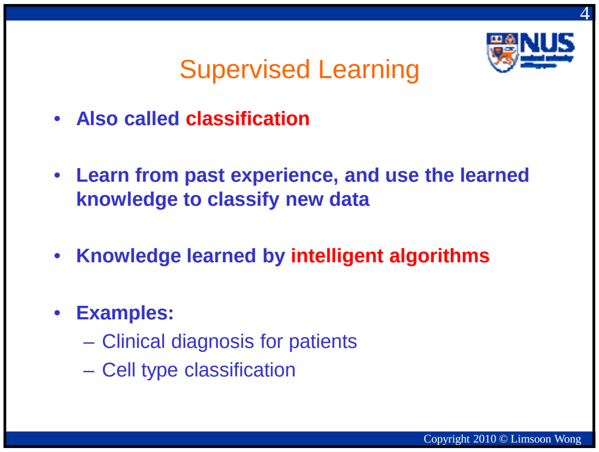


Outline

- **Overview of Supervised Learning**
  - Decision Trees
- **Decision Trees Ensembles**
  - Bagging
- **Other Methods**
  - K-Nearest Neighbour
  - Support Vector Machines
  - Bayesian Approach
  - Hidden Markov Models

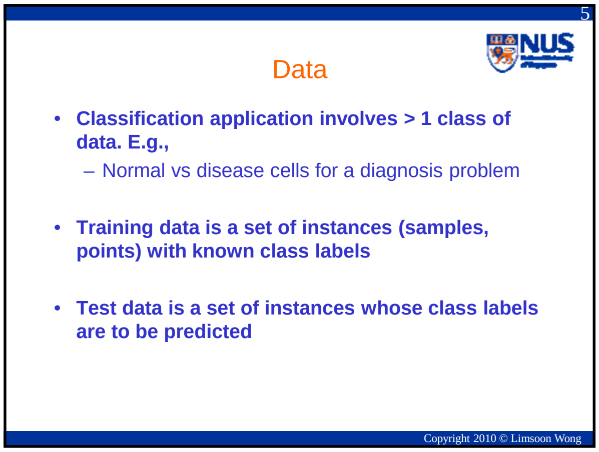


Overview of Supervised Learning



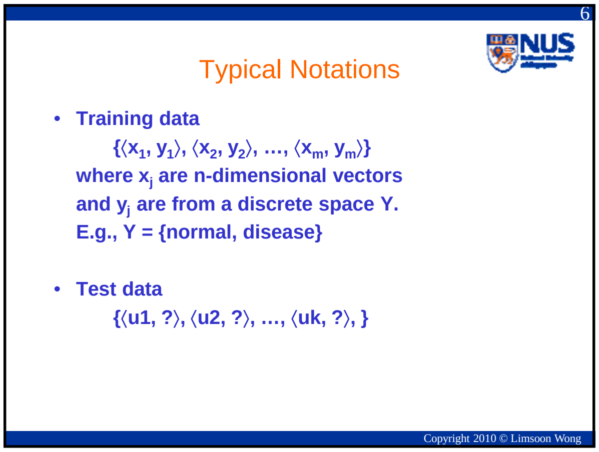
Supervised Learning

- **Also called classification**
- **Learn from past experience, and use the learned knowledge to classify new data**
- **Knowledge learned by intelligent algorithms**
- **Examples:**
  - Clinical diagnosis for patients
  - Cell type classification



Data

- **Classification application involves > 1 class of data. E.g.,**
  - Normal vs disease cells for a diagnosis problem
- **Training data is a set of instances (samples, points) with known class labels**
- **Test data is a set of instances whose class labels are to be predicted**



Typical Notations

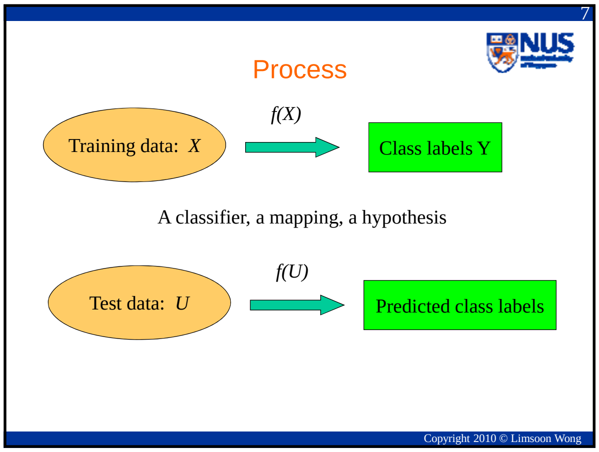
- **Training data**

  $\{\langle x_1, y_1\rangle, \langle x_2, y_2\rangle, \ldots, \langle x_m, y_m\rangle\}$

  where x_j are n-dimensional vectors and y_j are from a discrete space Y. E.g., Y = {normal, disease}

- **Test data**

  $\{\langle u1, ?\rangle, \langle u2, ?\rangle, \ldots, \langle uk, ?\rangle, \}$

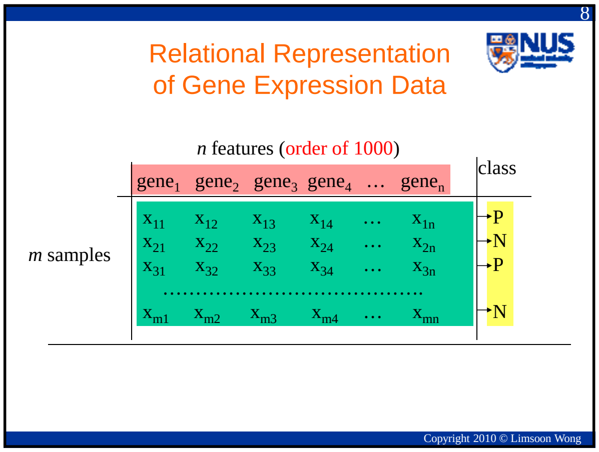


Process

Training data: *X* $\xrightarrow{f(X)}$ Class labels Y

A classifier, a mapping, a hypothesis

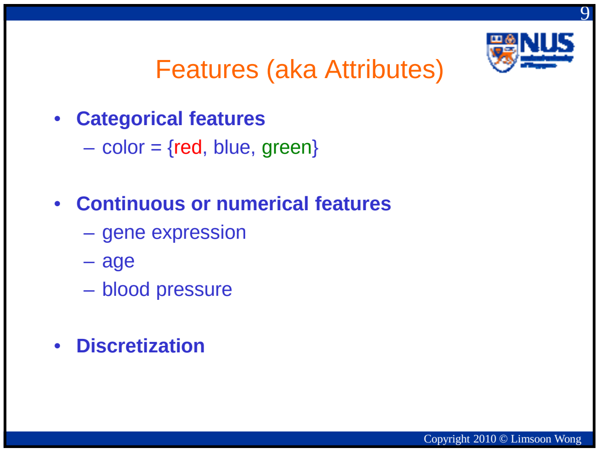
Test data: *U* $\xrightarrow{f(U)}$ Predicted class labels



Relational Representation of Gene Expression Data

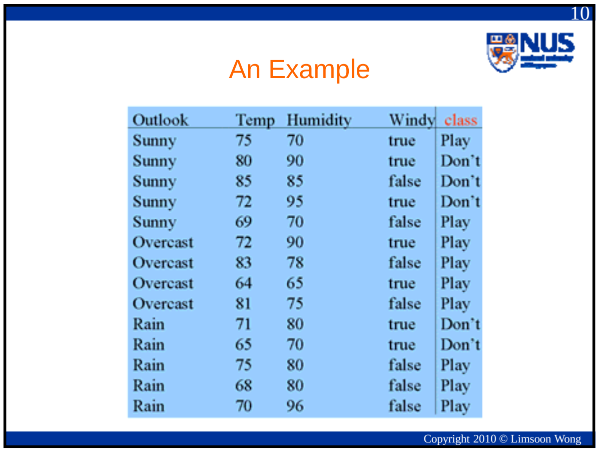
n features (order of 1000)

$$\begin{array}{c|cccccc|c}
 & gene_1 & gene_2 & gene_3 & gene_4 & \ldots & gene_n & \text{class} \\
\hline
 & x_{11} & x_{12} & x_{13} & x_{14} & \ldots & x_{1n} & \rightarrow P \\
 & x_{21} & x_{22} & x_{23} & x_{24} & \ldots & x_{2n} & \rightarrow N \\
m \text{ samples} & x_{31} & x_{32} & x_{33} & x_{34} & \ldots & x_{3n} & \rightarrow P \\
 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \\
 & x_{m1} & x_{m2} & x_{m3} & x_{m4} & \ldots & x_{mn} & \rightarrow N
\end{array}$$



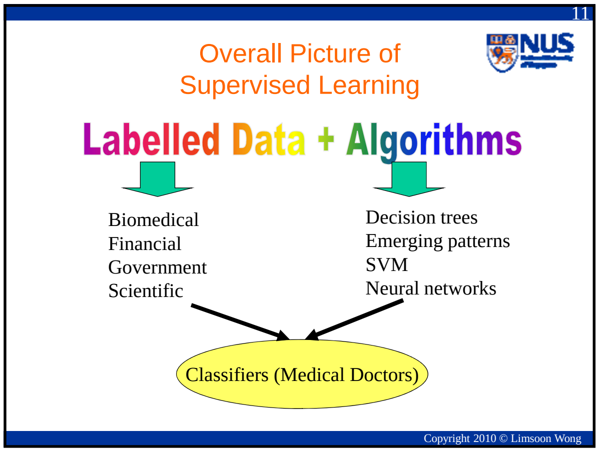
Features (aka Attributes)

- **Categorical features**
  - color = {red, blue, green}
- **Continuous or numerical features**
  - gene expression
  - age
  - blood pressure
- **Discretization**



An Example

Outlook	Temp	Humidity	Windy	class
Sunny	75	70	true	Play
Sunny	80	90	true	Don't
Sunny	85	85	false	Don't
Sunny	72	95	true	Don't
Sunny	69	70	false	Play
Overcast	72	90	true	Play
Overcast	83	78	false	Play
Overcast	64	65	true	Play
Overcast	81	75	false	Play
Rain	71	80	true	Don't
Rain	65	70	true	Don't
Rain	75	80	false	Play
Rain	68	80	false	Play
Rain	70	96	false	Play



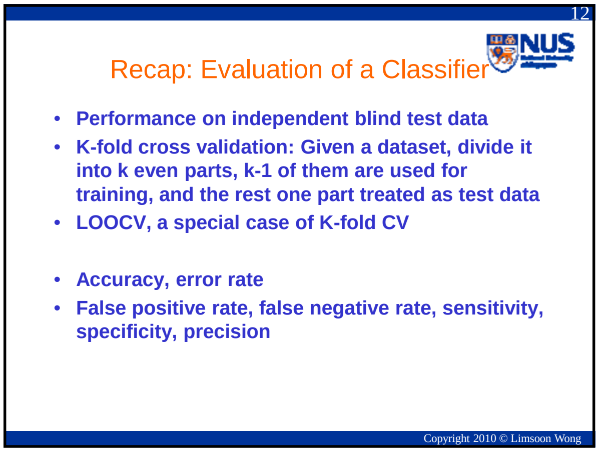
Overall Picture of Supervised Learning

Labelled Data + Algorithms

Biomedical
Financial
Government
Scientific

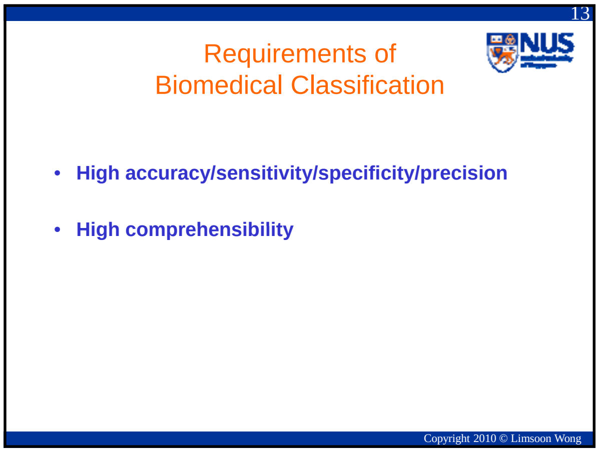
Decision trees
Emerging patterns
SVM
Neural networks

Classifiers (Medical Doctors)



Recap: Evaluation of a Classifier

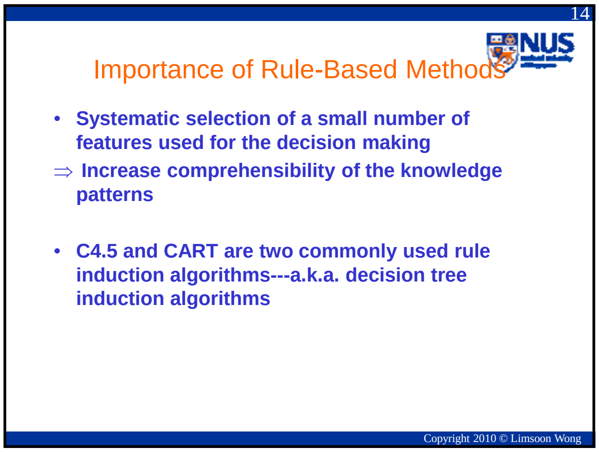
- **Performance on independent blind test data**
- **K-fold cross validation: Given a dataset, divide it into k even parts, k-1 of them are used for training, and the rest one part treated as test data**
- **LOOCV, a special case of K-fold CV**
- **Accuracy, error rate**
- **False positive rate, false negative rate, sensitivity, specificity, precision**



13

Requirements of Biomedical Classification

- High accuracy/sensitivity/specificity/precision
- High comprehensibility



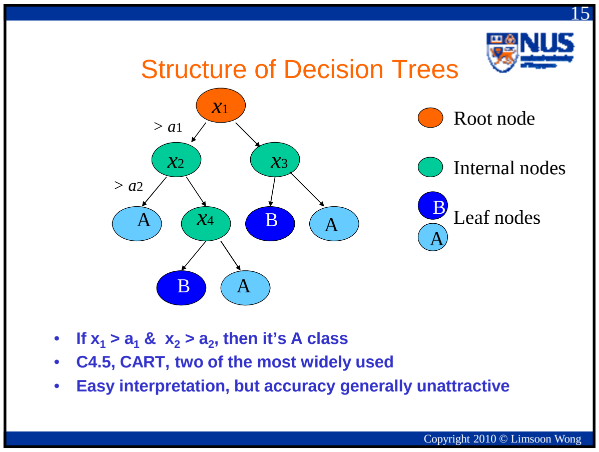
14

Importance of Rule-Based Methods

- Systematic selection of a small number of features used for the decision making

⇒ Increase comprehensibility of the knowledge patterns

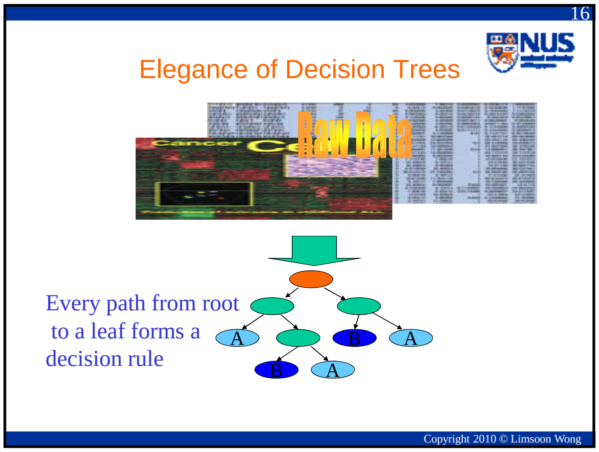
- C4.5 and CART are two commonly used rule induction algorithms---a.k.a. decision tree induction algorithms



15

Structure of Decision Trees

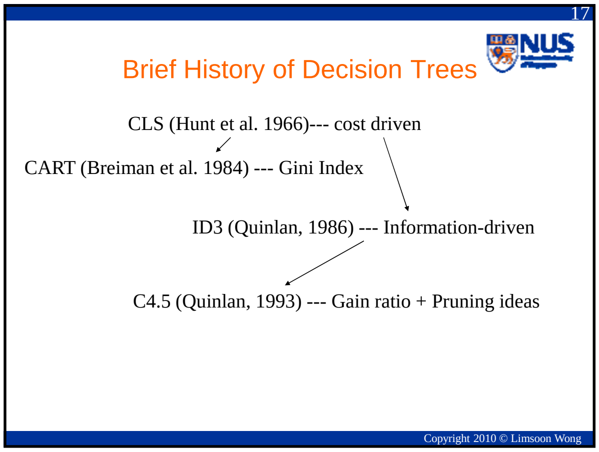
- If $x_1 > a_1$ & $x_2 > a_2$, then it's A class
- C4.5, CART, two of the most widely used
- Easy interpretation, but accuracy generally unattractive



16

Elegance of Decision Trees

Every path from root to a leaf forms a decision rule



17

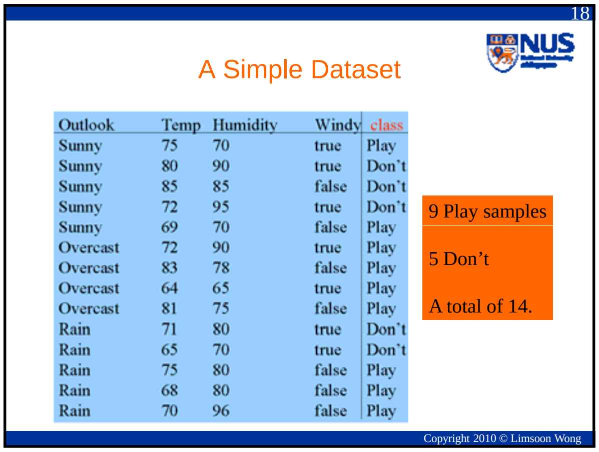
Brief History of Decision Trees

CLS (Hunt et al. 1966)--- cost driven

CART (Breiman et al. 1984) --- Gini Index

ID3 (Quinlan, 1986) --- Information-driven

C4.5 (Quinlan, 1993) --- Gain ratio + Pruning ideas



18

A Simple Dataset

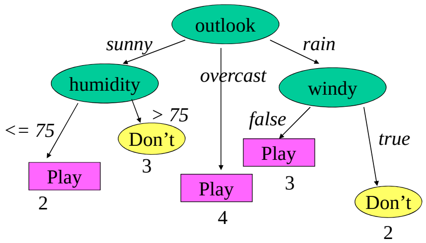
Outlook	Temp	Humidity	Windy	class
Sunny	75	70	true	Play
Sunny	80	90	true	Don't
Sunny	85	85	false	Don't
Sunny	72	95	true	Don't
Sunny	69	70	false	Play
Overcast	72	90	true	Play
Overcast	83	78	false	Play
Overcast	64	65	true	Play
Overcast	81	75	false	Play
Rain	71	80	true	Don't
Rain	65	70	true	Don't
Rain	75	80	false	Play
Rain	68	80	false	Play
Rain	70	96	false	Play

9 Play samples

5 Don’t

A total of 14.

A Decision Tree

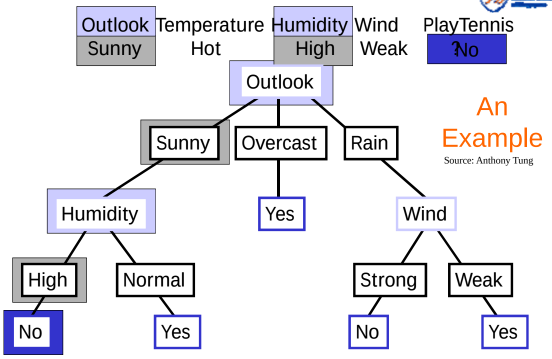


- **Construction of a tree is equiv to determination of root node of the tree and root nodes of its sub-trees**

Exercise: What is the accuracy of this tree?

An Example

Source: Anthony Tung



Most Discriminatory Feature

- **Every feature can be used to partition the training data**
- **If the partitions contain a pure class of training instances, then this feature is most discriminatory**

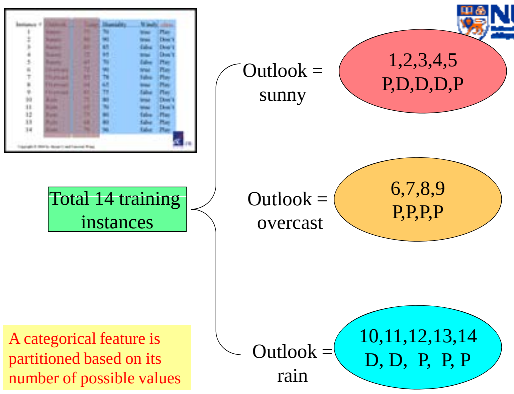
Example of Partitions

- **Categorical feature**
  - Number of partitions of the training data is equal to the number of values of this feature
- **Numerical feature**
  - Two partitions

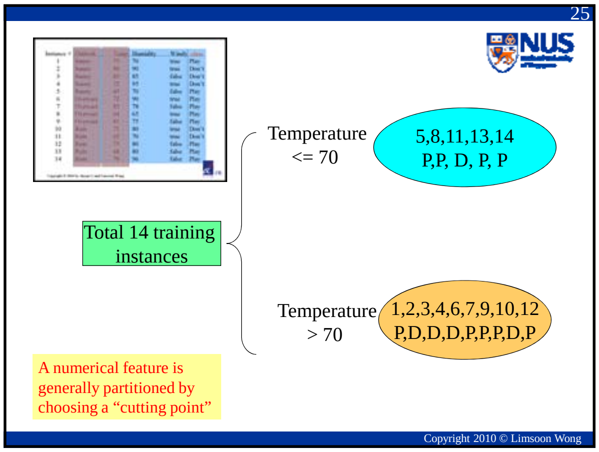
Categorical feature

Numerical feature

Instance #	Outlook	Temp	Humidity	Windy	class
1	Sunny	75	70	true	Play
2	Sunny	80	90	true	Don't
3	Sunny	85	85	false	Don't
4	Sunny	72	95	true	Don't
5	Sunny	69	70	false	Play
6	Overcast	72	90	true	Play
7	Overcast	83	78	false	Play
8	Overcast	64	65	true	Play
9	Overcast	81	75	false	Play
10	Rain	71	80	true	Don't
11	Rain	65	70	true	Don't
12	Rain	75	80	false	Play
13	Rain	68	80	false	Play
14	Rain	70	96	false	Play



A categorical feature is partitioned based on its number of possible values



25

Total 14 training instances

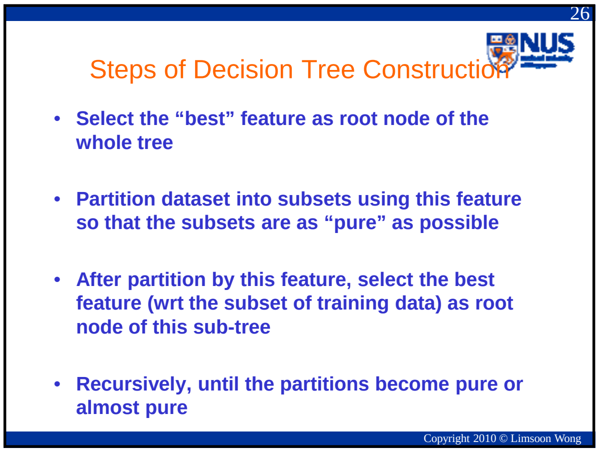
Temperature <= 70

5,8,11,13,14
P,P, D, P, P

Temperature > 70

1,2,3,4,6,7,9,10,12
P,D,D,D,P,P,P,D,P

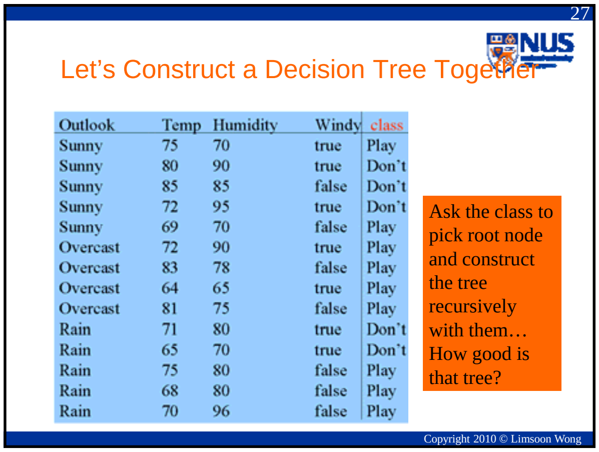
A numerical feature is generally partitioned by choosing a "cutting point"



26

Steps of Decision Tree Construction

- Select the "best" feature as root node of the whole tree
- Partition dataset into subsets using this feature so that the subsets are as "pure" as possible
- After partition by this feature, select the best feature (wrt the subset of training data) as root node of this sub-tree
- Recursively, until the partitions become pure or almost pure

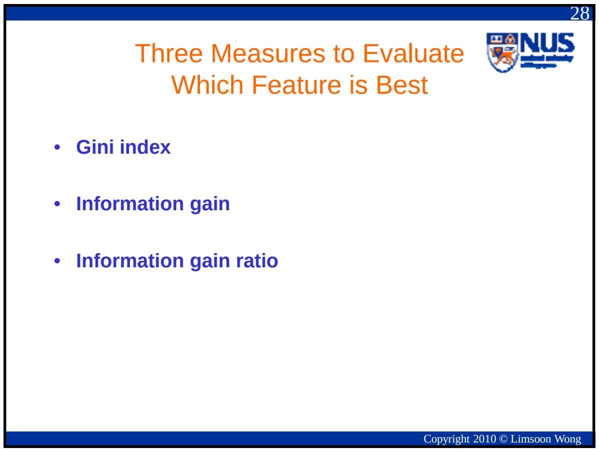


27

Let's Construct a Decision Tree Together

Outlook	Temp	Humidity	Windy	class
Sunny	75	70	true	Play
Sunny	80	90	true	Don't
Sunny	85	85	false	Don't
Sunny	72	95	true	Don't
Sunny	69	70	false	Play
Overcast	72	90	true	Play
Overcast	83	78	false	Play
Overcast	64	65	true	Play
Overcast	81	75	false	Play
Rain	71	80	true	Don't
Rain	65	70	true	Don't
Rain	75	80	false	Play
Rain	68	80	false	Play
Rain	70	96	false	Play

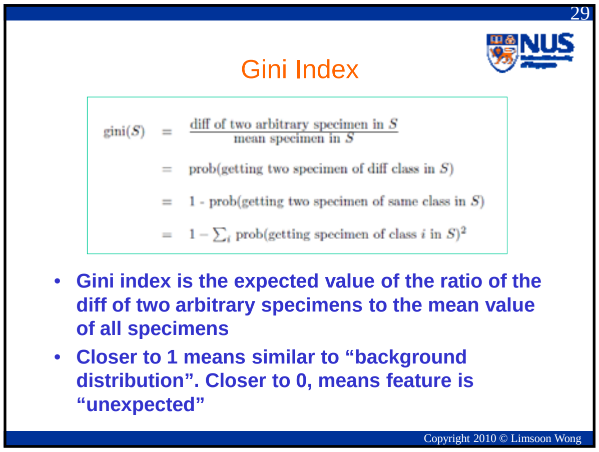
Ask the class to pick root node and construct the tree recursively with them… How good is that tree?



28

Three Measures to Evaluate Which Feature is Best

- Gini index
- Information gain
- Information gain ratio

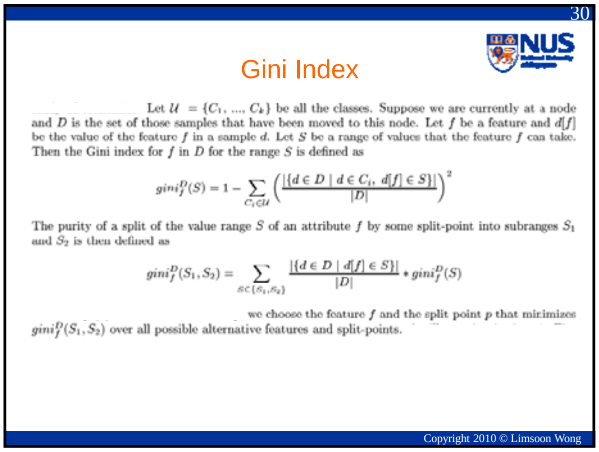


29

Gini Index

$$\begin{aligned}\text{gini}(S) &= \frac{\text{diff of two arbitrary specimen in } S}{\text{mean specimen in } S}\\ &= \text{prob(getting two specimen of diff class in } S)\\ &= 1 \text{ - prob(getting two specimen of same class in } S)\\ &= 1 - \textstyle\sum_i \text{prob(getting specimen of class } i \text{ in } S)^2\end{aligned}$$

- Gini index is the expected value of the ratio of the diff of two arbitrary specimens to the mean value of all specimens
- Closer to 1 means similar to "background distribution". Closer to 0, means feature is "unexpected"



30

Gini Index

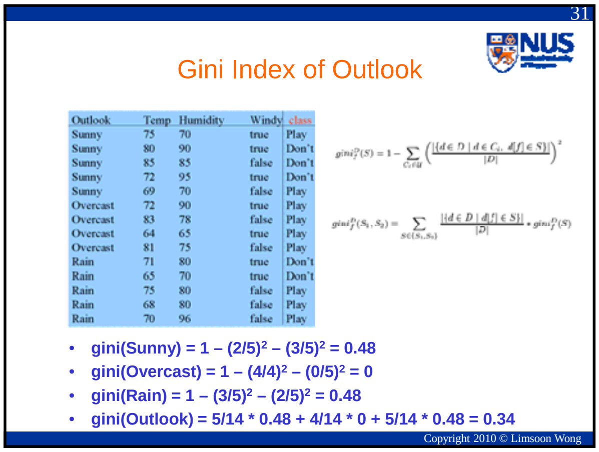
Let $\mathcal{U} = \{C_1, ..., C_k\}$ be all the classes. Suppose we are currently at a node and D is the set of those samples that have been moved to this node. Let f be a feature and $d[f]$ be the value of the feature f in a sample d. Let S be a range of values that the feature f can take. Then the Gini index for f in D for the range S is defined as

$$gini_f^D(S) = 1 - \sum_{C_i \in \mathcal{U}} \left(\frac{|\{d \in D \mid d \in C_i,\ d[f] \in S\}|}{|D|} \right)^2$$

The purity of a split of the value range S of an attribute f by some split-point into subranges S_1 and S_2 is then defined as

$$gini_f^D(S_1, S_2) = \sum_{S \in \{S_1, S_2\}} \frac{|\{d \in D \mid d[f] \in S\}|}{|D|} * gini_f^D(S)$$

we choose the feature f and the split point p that minimizes $gini_f^D(S_1, S_2)$ over all possible alternative features and split-points.



31

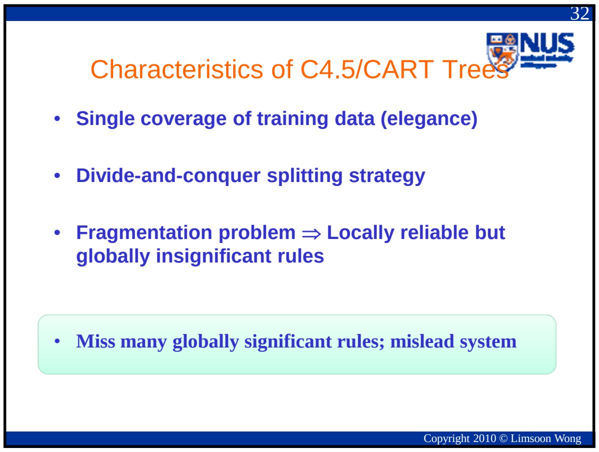
Gini Index of Outlook

Outlook	Temp	Humidity	Windy	class
Sunny	75	70	true	Play
Sunny	80	90	true	Don't
Sunny	85	85	false	Don't
Sunny	72	95	true	Don't
Sunny	69	70	false	Play
Overcast	72	90	true	Play
Overcast	83	78	false	Play
Overcast	64	65	true	Play
Overcast	81	75	false	Play
Rain	71	80	true	Don't
Rain	65	70	true	Don't
Rain	75	80	false	Play
Rain	68	80	false	Play
Rain	70	96	false	Play

$$gini_f^D(S) = 1 - \sum_{C_i \in U} \left(\frac{|\{d \in D \mid d \in C_i, d[f] \in S\}|}{|D|} \right)^2$$

$$gini_f^D(S_1, S_2) = \sum_{S \in \{S_1, S_2\}} \frac{|\{d \in D \mid d[f] \in S\}|}{|D|} * gini_f^D(S)$$

- gini(Sunny) = 1 – $(2/5)^2$ – $(3/5)^2$ = 0.48
- gini(Overcast) = 1 – $(4/4)^2$ – $(0/5)^2$ = 0
- gini(Rain) = 1 – $(3/5)^2$ – $(2/5)^2$ = 0.48
- gini(Outlook) = 5/14 * 0.48 + 4/14 * 0 + 5/14 * 0.48 = 0.34

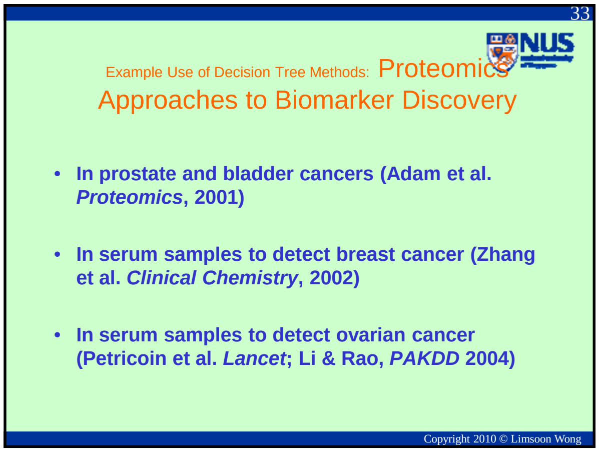


32

Characteristics of C4.5/CART Trees

- **Single coverage of training data (elegance)**
- **Divide-and-conquer splitting strategy**
- **Fragmentation problem ⇒ Locally reliable but globally insignificant rules**

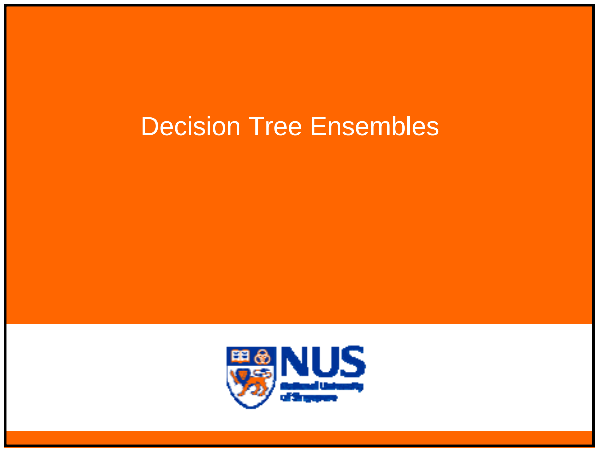
- **Miss many globally significant rules; mislead system**



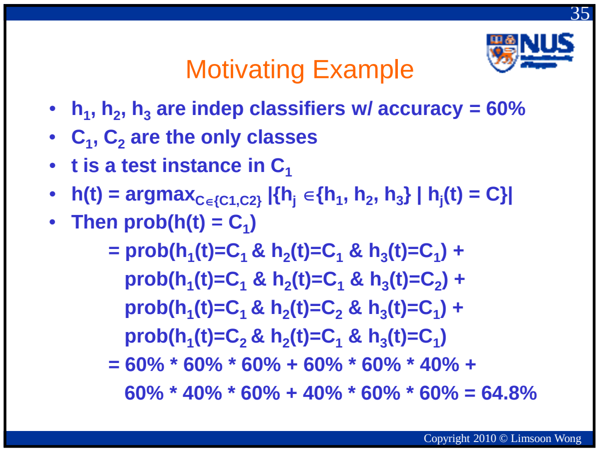
33

Example Use of Decision Tree Methods: Proteomics Approaches to Biomarker Discovery

- **In prostate and bladder cancers (Adam et al. *Proteomics*, 2001)**
- **In serum samples to detect breast cancer (Zhang et al. *Clinical Chemistry*, 2002)**
- **In serum samples to detect ovarian cancer (Petricoin et al. *Lancet*; Li & Rao, *PAKDD* 2004)**



Decision Tree Ensembles



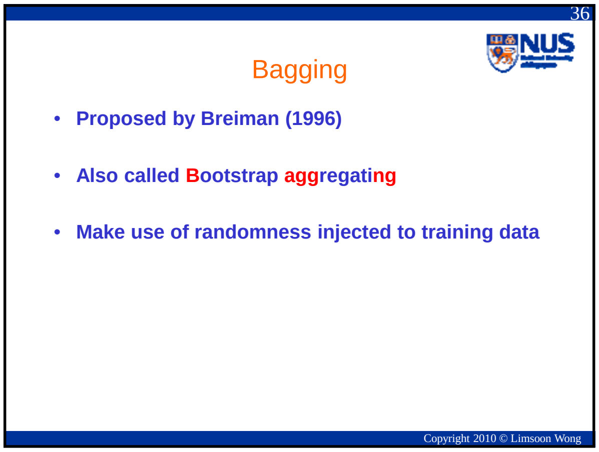
35

Motivating Example

- **h_1, h_2, h_3 are indep classifiers w/ accuracy = 60%**
- **C_1, C_2 are the only classes**
- **t is a test instance in C_1**
- **h(t) = argmax$_{C \in \{C1,C2\}}$ $|\{h_j \in \{h_1, h_2, h_3\} \mid h_j(t) = C\}|$**
- **Then prob(h(t) = C_1)**

  = prob($h_1(t)=C_1$ & $h_2(t)=C_1$ & $h_3(t)=C_1$) +
  prob($h_1(t)=C_1$ & $h_2(t)=C_1$ & $h_3(t)=C_2$) +
  prob($h_1(t)=C_1$ & $h_2(t)=C_2$ & $h_3(t)=C_1$) +
  prob($h_1(t)=C_2$ & $h_2(t)=C_1$ & $h_3(t)=C_1$)

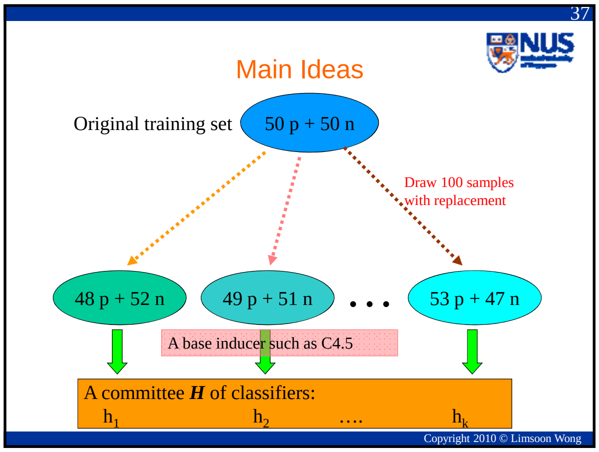
  = 60% * 60% * 60% + 60% * 60% * 40% +
  60% * 40% * 60% + 40% * 60% * 60% = 64.8%



36

Bagging

- **Proposed by Breiman (1996)**
- **Also called Bootstrap aggregating**
- **Make use of randomness injected to training data**



37

Main Ideas

Original training set 50 p + 50 n

Draw 100 samples with replacement

48 p + 52 n

49 p + 51 n

. . .

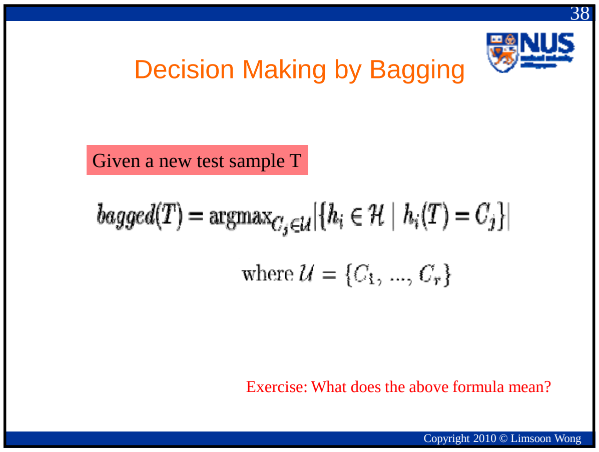
53 p + 47 n

A base inducer such as C4.5

A committee ***H*** of classifiers:

h_1 h_2 …. h_k

Copyright 2010 © Limsoon Wong



38

Decision Making by Bagging

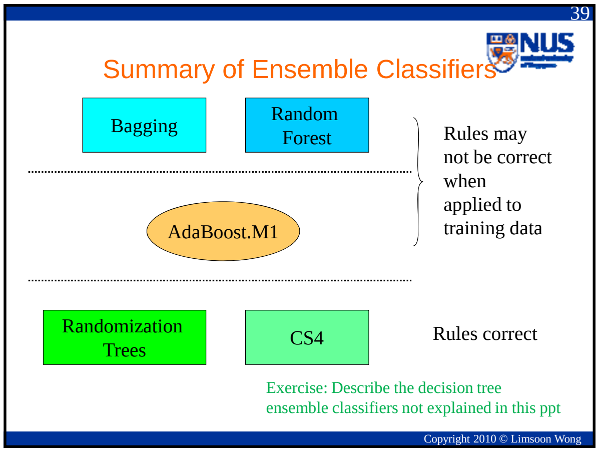
Given a new test sample T

$$bagged(T) = \text{argmax}_{C_j \in \mathcal{U}} |\{h_i \in \mathcal{H} \mid h_i(T) = C_j\}|$$

$$\text{where } \mathcal{U} = \{C_1, \ldots, C_r\}$$

Exercise: What does the above formula mean?

Copyright 2010 © Limsoon Wong



39

Summary of Ensemble Classifiers

Bagging

Random Forest

AdaBoost.M1

Rules may not be correct when applied to training data

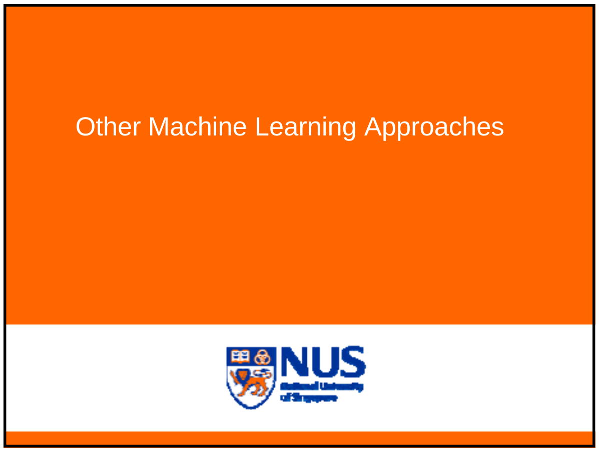
Randomization Trees

CS4

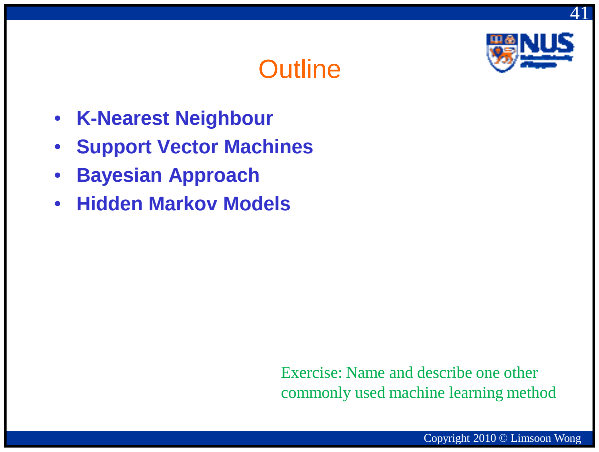
Rules correct

Exercise: Describe the decision tree ensemble classifiers not explained in this ppt

Copyright 2010 © Limsoon Wong



Other Machine Learning Approaches



41

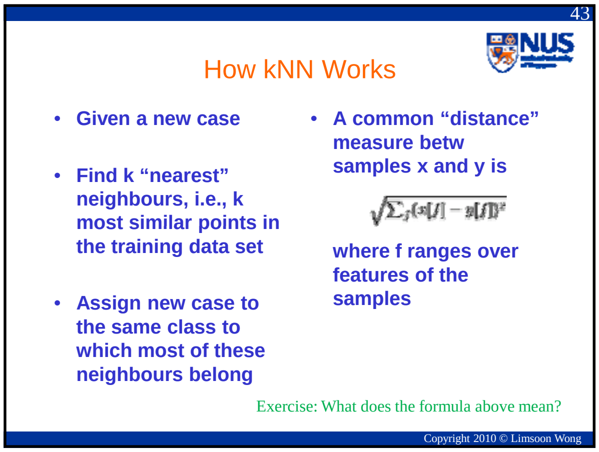
Outline

- K-Nearest Neighbour
- Support Vector Machines
- Bayesian Approach
- Hidden Markov Models

Exercise: Name and describe one other commonly used machine learning method

Copyright 2010 © Limsoon Wong

K-Nearest Neighbours



43

How kNN Works

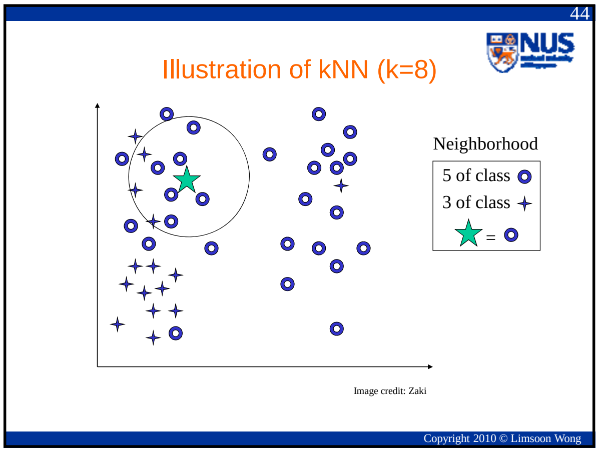
- Given a new case
- Find k "nearest" neighbours, i.e., k most similar points in the training data set
- Assign new case to the same class to which most of these neighbours belong
- A common "distance" measure betw samples x and y is

$$\sqrt{\Sigma_f (x[f] - y[f])^2}$$

where f ranges over features of the samples

Exercise: What does the formula above mean?

Copyright 2010 © Limsoon Wong

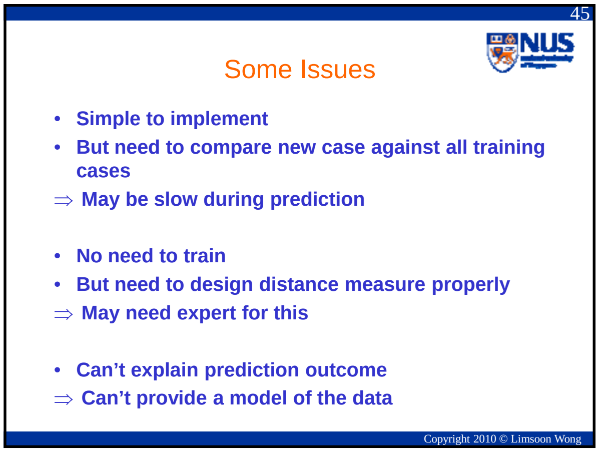


44

Illustration of kNN (k=8)

Image credit: Zaki

Copyright 2010 © Limsoon Wong



45

Some Issues

- Simple to implement
- But need to compare new case against all training cases

⇒ May be slow during prediction

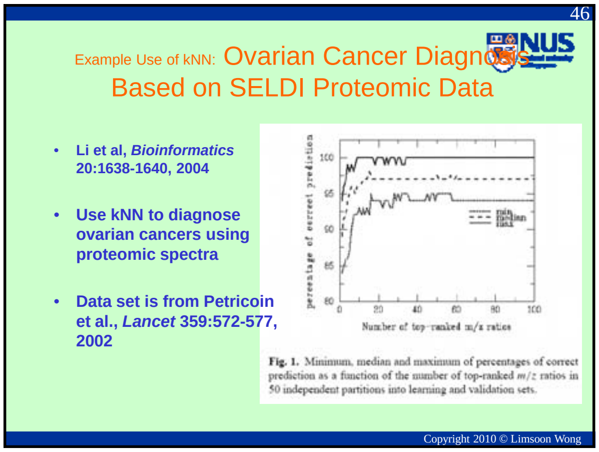
- No need to train
- But need to design distance measure properly

⇒ May need expert for this

- Can't explain prediction outcome

⇒ Can't provide a model of the data

Copyright 2010 © Limsoon Wong



46

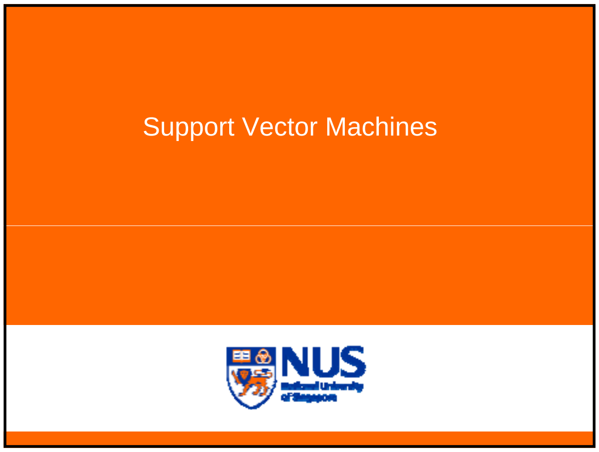
Example Use of kNN: Ovarian Cancer Diagnosis Based on SELDI Proteomic Data

- Li et al, *Bioinformatics* 20:1638-1640, 2004
- Use kNN to diagnose ovarian cancers using proteomic spectra
- Data set is from Petricoin et al., *Lancet* 359:572-577, 2002

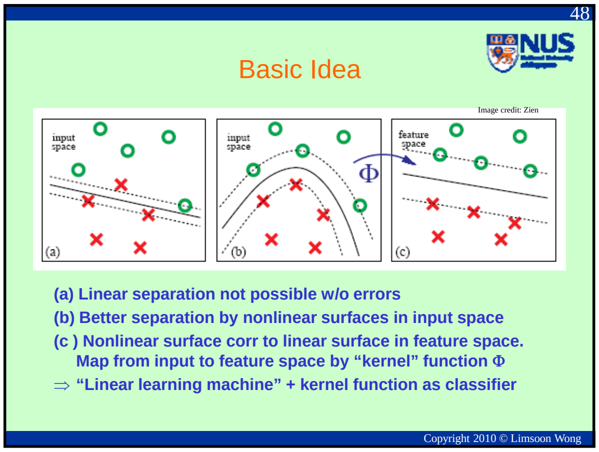
percentage of correct prediction
100
95
90
85
80
0
20
40
60
80
100
Number of top-ranked m/z ratios
min
median
max

Fig. 1. Minimum, median and maximum of percentages of correct prediction as a function of the number of top-ranked *m/z* ratios in 50 independent partitions into learning and validation sets.

Copyright 2010 © Limsoon Wong



Support Vector Machines



48

Basic Idea

Image credit: Zien

(a) Linear separation not possible w/o errors
(b) Better separation by nonlinear surfaces in input space
(c) Nonlinear surface corr to linear surface in feature space. Map from input to feature space by "kernel" function Φ

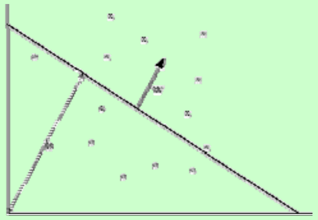
⇒ "Linear learning machine" + kernel function as classifier

Copyright 2010 © Limsoon Wong

Linear Learning Machines

- Hyperplane separating the x's and o's points is given by $(W \bullet X) + b = 0$, with $(W \bullet X) = \Sigma_j W[j]*X[j]$

$\Rightarrow$ Decision function is $llm(X) = sign((W \bullet X) + b))$



Linear Learning Machines

- Solution is a linear combination of training points X_k with labels Y_k

  $W[j] = \Sigma_k \alpha_k * Y_k * X_k[j]$,

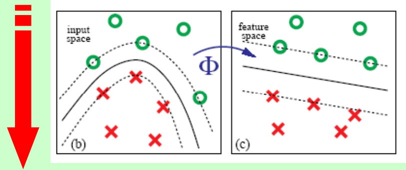
  with $\alpha_k > 0$, and $Y_k = \pm 1$

$\Rightarrow$ $llm(X) = sign(\Sigma_k \alpha_k * Y_k * (X_k \bullet X) + b)$

"data" appears only in dot product!

Kernel Function

- $llm(X) = sign(\Sigma_k \alpha_k * Y_k * (X_k \bullet X) + b)$



- $svm(X) = sign(\Sigma_k \alpha_k * Y_k * (\Phi X_k \bullet \Phi X) + b)$

$\Rightarrow$ $svm(X) = sign(\Sigma_k \alpha_k * Y_k * K(X_k,X) + b)$

where $K(X_k,X) = (\Phi X_k \bullet \Phi X)$

Kernel Function

- $svm(X) = sign(\Sigma_k \alpha_k * Y_k * K(X_k,X) + b)$

$\Rightarrow$ $K(A,B)$ can be computed w/o computing Φ

- In fact replace it w/ lots of more "powerful" kernels besides $(A \bullet B)$. E.g.,
  - $K(A,B) = (A \bullet B)^d$
  - $K(A,B) = \exp(- || A\ B||^2 / (2*\sigma))$, ...

How SVM Works

- $svm(X) = sign(\Sigma_k \alpha_k * Y_k * K(X_k,X) + b)$

- To find α_k is a quadratic programming problem

  max: $\Sigma_k \alpha_k - 0.5 * \Sigma_k \Sigma_h \alpha_k * \alpha_h Y_k * Y_h * K(X_k,X_h)$

  subject to: $\Sigma_k \alpha_k * Y_k = 0$

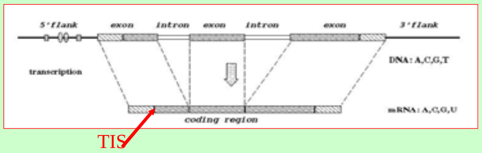
  and for all α_k , $C \geq \alpha_k \geq 0$

- To find b, estimate by averaging

  $Y_h - \Sigma_k \alpha_k * Y_k * K(X_h,X_k)$

  for all $\alpha_h \geq 0$

Example Use of SVM: Recognition of Protein Translation Initiation Sites



- Zien et al., *Bioinformatics* 16:799-807, 2000
- Use SVM to recognize protein translation initiation sites from genomic sequences
- Raw data set is same as Liu & Wong, *JBCB* 1:139-168, 2003

Bayesian Approach

Bayes Theorem

$$P(h|d) = \frac{P(d|h) * P(h)}{P(d)}$$

- P(h) = prior prob that hypothesis h holds
- P(d|h) = prob of observing data d given h holds
- P(h|d) = posterior prob that h holds given observed data d

Bayesian Approach

- Let *H* be all possible classes. Given a test instance w/ feature vector {$f_1 = v_1$, …, $f_n = v_n$}, the most probable classification is given by

$$\operatorname{argmax}_{h_j \in H} P(h_j | f_1 = v_1, \ldots, f_n = v_n)$$

- Using Bayes Theorem, rewrites to

$$\operatorname{argmax}_{h_j \in H} \frac{P(f_1 = v_1, \ldots, f_n = v_n | h_j) * P(h_j)}{P(f_1 = v_1, \ldots, f_n = v_n)}$$

- Since denominator is independent of h_j, this simplifies to

$$\operatorname{argmax}_{h_j \in H} P(f_1 = v_1, \ldots, f_n = v_n | h_j) * P(h_j)$$

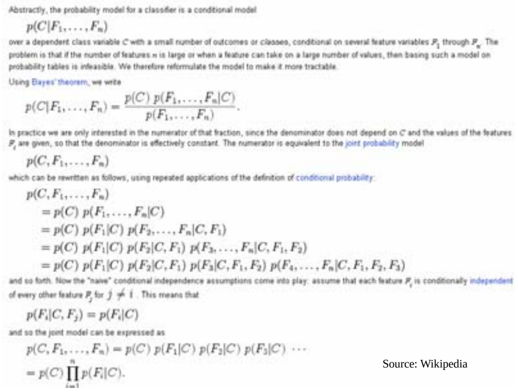
Naïve Bayes

- But estimating $P(f_1{=}v_1, \ldots, f_n{=}v_n|h_j)$ accurately may not be feasible unless training data set is large
- "Solved" by assuming $f_1, \ldots, f_n$ are conditionally independent of each other
- Then $\operatorname{argmax}_{h_j \in H} P(f_1 = v_1, \ldots, f_n = v_n | h_j) * P(h_j)$

$$= \operatorname{argmax}_{h_j \in H} \prod_i P(f_i = v_i | h_j) * P(h_j)$$

- where $P(h_j)$ and $P(f_i{=}v_i|h_j)$ can often be estimated reliably from typical training data set

Exercise: How do you estimate $P(h_j)$ and $P(f_j{=}v_j|h_j)$?



Abstractly, the probability model for a classifier is a conditional model

$$p(C|F_1, \ldots, F_n)$$

over a dependent class variable C with a small number of outcomes or classes, conditional on several feature variables F_1 through F_n. The problem is that if the number of features n is large or when a feature can take on a large number of values, then basing such a model on probability tables is infeasible. We therefore reformulate the model to make it more tractable.

Using Bayes' theorem, we write

$$p(C|F_1, \ldots, F_n) = \frac{p(C)\ p(F_1, \ldots, F_n|C)}{p(F_1, \ldots, F_n)}.$$

In practice we are only interested in the numerator of that fraction, since the denominator does not depend on C and the values of the features F_i are given, so that the denominator is effectively constant. The numerator is equivalent to the joint probability model

$$p(C, F_1, \ldots, F_n)$$

which can be rewritten as follows, using repeated applications of the definition of conditional probability:

$$\begin{aligned} & p(C, F_1, \ldots, F_n) \\ &= p(C)\ p(F_1, \ldots, F_n|C) \\ &= p(C)\ p(F_1|C)\ p(F_2, \ldots, F_n|C, F_1) \\ &= p(C)\ p(F_1|C)\ p(F_2|C, F_1)\ p(F_3, \ldots, F_n|C, F_1, F_2) \\ &= p(C)\ p(F_1|C)\ p(F_2|C, F_1)\ p(F_3|C, F_1, F_2)\ p(F_4, \ldots, F_n|C, F_1, F_2, F_3) \end{aligned}$$

and so forth. Now the "naive" conditional independence assumptions come into play: assume that each feature F_i is conditionally independent of every other feature F_j for $j \neq i$. This means that

$$p(F_i|C, F_j) = p(F_i|C)$$

and so the joint model can be expressed as

$$\begin{aligned} p(C, F_1, \ldots, F_n) &= p(C)\ p(F_1|C)\ p(F_2|C)\ p(F_3|C) \cdots \\ &= p(C) \prod_{i=1}^{n} p(F_i|C). \end{aligned}$$

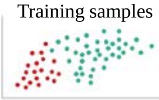
Source: Wikipedia

Independence vs Conditional Independence

- Independence: P(A,B) = P(A) * P(B)
- Conditional Independence: P(A,B|C) = P(A|C) * P(B|C)
- Indep does not imply conditional indep
  - Consider tossing a fair coin twice
    - A is event of getting head in 1st toss
    - B is event of getting head in 2nd toss
    - C is event of getting exactly one head
  - Then A={HT, HH}, B={HH, TH} and C={HT, TH}
  - P(A,B|C) =P({HH}|C)=0
  - P(A|C) = P(A,C)/P(C) =P({HT})/P(C)=(1/4)/(1/2) =1/2
  - Similarly, P(B|C) =1/2

An Example

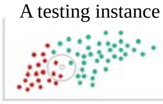
Training samples



$$\text{Prior probability for GREEN} \propto \frac{\text{Number of GREEN objects}}{\text{Total number of objects}} = 40/60$$

$$\text{Prior probability for RED} \propto \frac{\text{Number of RED objects}}{\text{Total number of objects}} = 20/60$$

A testing instance X



$$\text{Likelihood of } X \text{ given GREEN} \propto \frac{\text{Number of GREEN in the vicinity of } X}{\text{Total number of GREEN cases}} = 1/40$$

$$\text{Likelihood of } X \text{ given RED} \propto \frac{\text{Number of RED in the vicinity of } X}{\text{Total number of RED cases}} = 3/20$$

$$\text{Posterior probability of } X \text{ being GREEN} \propto \text{Prior probability of GREEN} \times \text{Likelihood of } X \text{ given GREEN} = \frac{4}{6} \times \frac{1}{40} = \frac{1}{60}$$

$$\text{Posterior probability of } X \text{ being RED} \propto \text{Prior probability of RED} \times \text{Likelihood of } X \text{ given RED} = \frac{2}{6} \times \frac{3}{20} = \frac{1}{20}$$

we classify X as RED since its class membership achieves the largest posterior probability

Source: http://www.statsoft.com/textbook/stnaiveb.html

Example Use of Bayesian: Design of Screens for Macromolecular Crystallization

- Hennessy et al., *Acta Cryst* D56:817-827, 2000
- Xtallization of proteins requires search of expt settings to find right conditions for diffraction-quality xtals
- BMCD is a db of known xtallization conditions
- Use Bayes to determine prob of success of a set of expt conditions based on BMCD

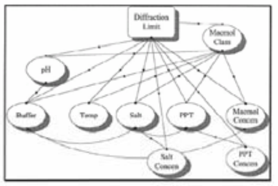
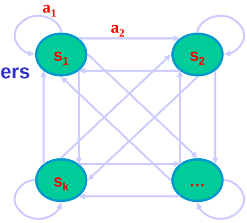


Figure 1
Crystallization parameter dependency graph. The graph represents the parameters included in the calculation of the estimated probability of success and their dependencies. A connecting arc from pH to buffer indicates that the probability distribution for the buffer may depend on the value of the pH. The lack of a connecting arc between two parameters reflects conditional independence (the probability distribution for a parameter is independent of the value of the other parameter).

Hidden Markov Models

What is a HMM

- HMM is a stochastic generative model for seqs
- Defined by model parameters
  - finite set of states S
  - finite alphabet A
  - transition prob matrix T
  - emission prob matrix E
- Move from state to state as per T while emitting symbols as per E



Order of a HMM

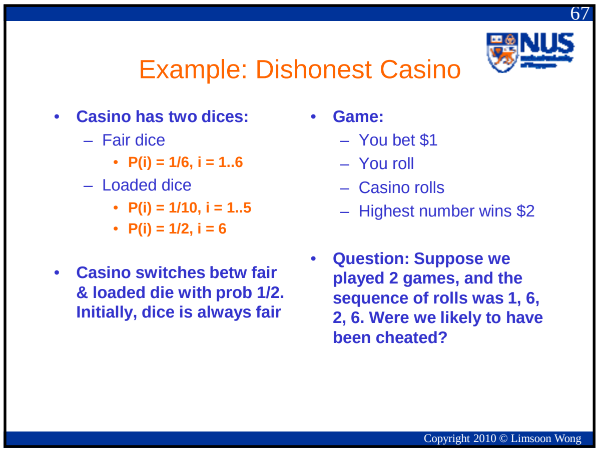
- In *n*th order HMM, *T* & *E* depend on all *n* previous states
- E.g., for 1st order HMM, given emissions $X = x_1, x_2, \ldots$, & states $S = s_1, s_2, \ldots$, the prob of this seq is

$$Prob(X,S) = \prod_i Prob(x_i|s_i) = \prod_i E(x_i|s_i) * T(s_{i-1}, s_i)$$

Using HMM

- Given the model parameters, compute the probability of a particular output sequence. Solved by the **forward algorithm**
- Given the model parameters, find the most likely sequence of (hidden) states which could have generated a given output sequence. Solved by the **Viterbi algorithm**
- Given an output sequence, find the most likely set of state transition and output probabilities. Solved by the **Baum-Welch algorithm**

Exercise: Describe these algorithms

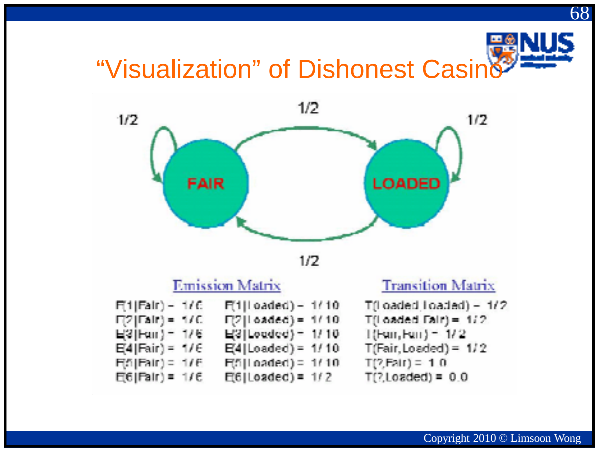


67

Example: Dishonest Casino

- **Casino has two dices:**
  - Fair dice
    - **P(i) = 1/6, i = 1..6**
  - Loaded dice
    - **P(i) = 1/10, i = 1..5**
    - **P(i) = 1/2, i = 6**
- **Casino switches betw fair & loaded die with prob 1/2. Initially, dice is always fair**

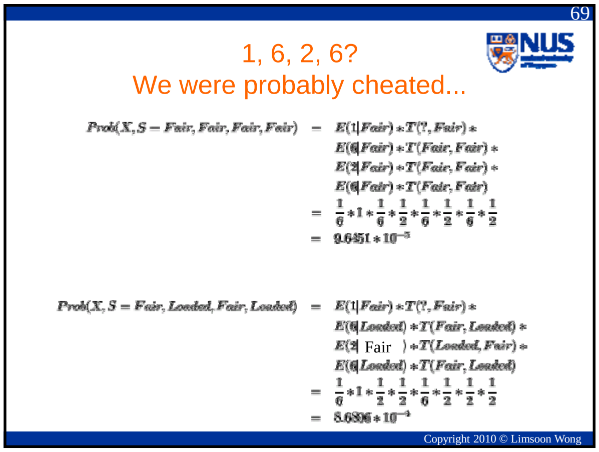
- **Game:**
  - You bet $1
  - You roll
  - Casino rolls
  - Highest number wins $2
- **Question: Suppose we played 2 games, and the sequence of rolls was 1, 6, 2, 6. Were we likely to have been cheated?**



68

“Visualization” of Dishonest Casino

Emission Matrix		Transition Matrix
E(1|Fair) = 1/6	E(1|Loaded) = 1/10	T(Loaded,Loaded) = 1/2
E(2|Fair) = 1/6	E(2|Loaded) = 1/10	T(Loaded,Fair) = 1/2
E(3|Fair) = 1/6	E(3|Loaded) = 1/10	T(Fair,Fair) = 1/2
E(4|Fair) = 1/6	E(4|Loaded) = 1/10	T(Fair,Loaded) = 1/2
E(5|Fair) = 1/6	E(5|Loaded) = 1/10	T(?,Fair) = 1.0
E(6|Fair) = 1/6	E(6|Loaded) = 1/2	T(?,Loaded) = 0.0

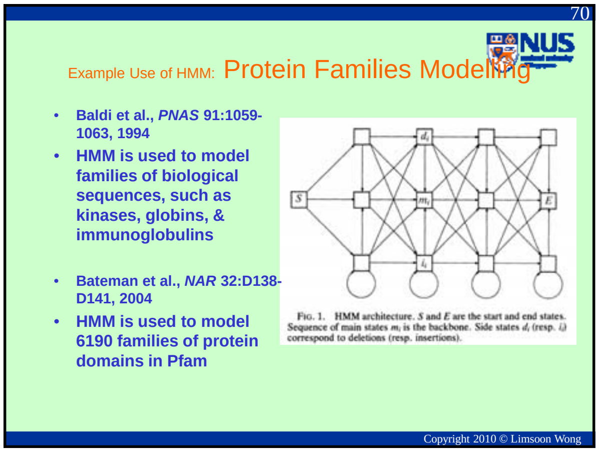


69

1, 6, 2, 6?
We were probably cheated...

$$\begin{aligned}
Prob(X, S = Fair, Fair, Fair, Fair) &= E(1|Fair) * T(?, Fair) * \\
&\quad E(6|Fair) * T(Fair, Fair) * \\
&\quad E(2|Fair) * T(Fair, Fair) * \\
&\quad E(6|Fair) * T(Fair, Fair) \\
&= \frac{1}{6} * 1 * \frac{1}{6} * \frac{1}{2} * \frac{1}{6} * \frac{1}{2} * \frac{1}{6} * \frac{1}{2} \\
&= 9.6451 * 10^{-5}
\end{aligned}$$

$$\begin{aligned}
Prob(X, S = Fair, Loaded, Fair, Loaded) &= E(1|Fair) * T(?, Fair) * \\
&\quad E(6|Loaded) * T(Fair, Loaded) * \\
&\quad E(2|\text{ Fair }) * T(Loaded, Fair) * \\
&\quad E(6|Loaded) * T(Fair, Loaded) \\
&= \frac{1}{6} * 1 * \frac{1}{2} * \frac{1}{2} * \frac{1}{6} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} \\
&= 8.6806 * 10^{-4}
\end{aligned}$$



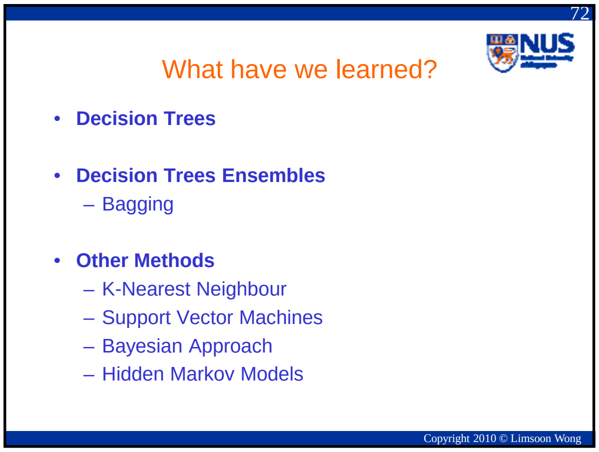
70

Example Use of HMM: Protein Families Modelling

- **Baldi et al., *PNAS* 91:1059-1063, 1994**
- **HMM is used to model families of biological sequences, such as kinases, globins, & immunoglobulins**
- **Bateman et al., *NAR* 32:D138-D141, 2004**
- **HMM is used to model 6190 families of protein domains in Pfam**

FIG. 1. HMM architecture. S and E are the start and end states. Sequence of main states m_i is the backbone. Side states d_i (resp. i_i) correspond to deletions (resp. insertions).

Concluding Remarks…

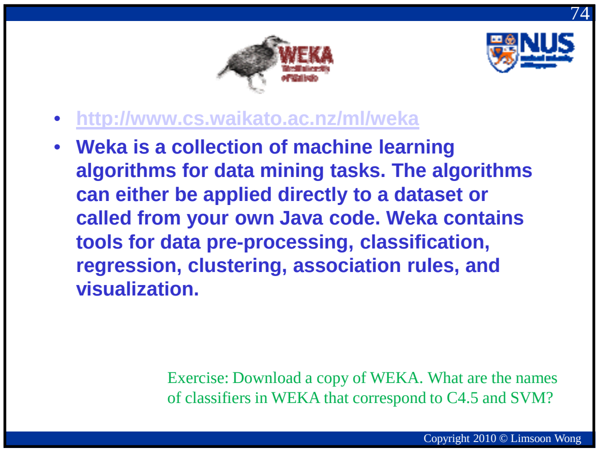


72

What have we learned?

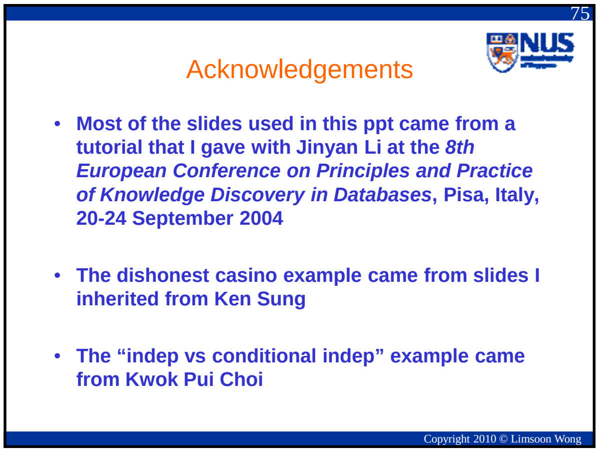
- **Decision Trees**
- **Decision Trees Ensembles**
  - Bagging
- **Other Methods**
  - K-Nearest Neighbour
  - Support Vector Machines
  - Bayesian Approach
  - Hidden Markov Models

Any Question?



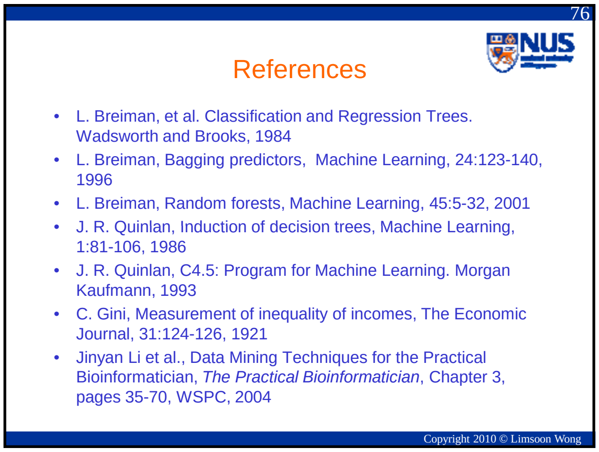
- http://www.cs.waikato.ac.nz/ml/weka
- **Weka is a collection of machine learning algorithms for data mining tasks. The algorithms can either be applied directly to a dataset or called from your own Java code. Weka contains tools for data pre-processing, classification, regression, clustering, association rules, and visualization.**

Exercise: Download a copy of WEKA. What are the names of classifiers in WEKA that correspond to C4.5 and SVM?



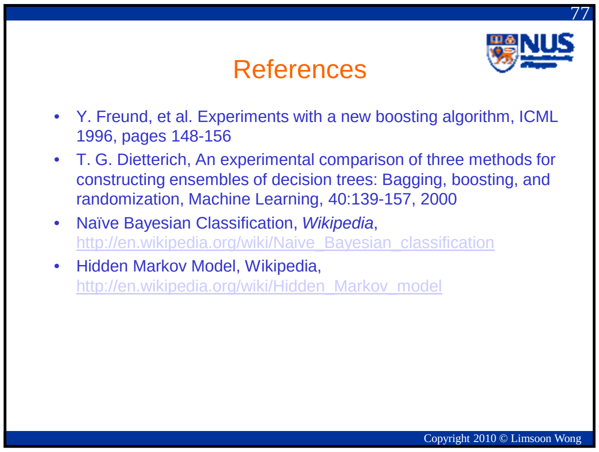
Acknowledgements

- **Most of the slides used in this ppt came from a tutorial that I gave with Jinyan Li at the *8th European Conference on Principles and Practice of Knowledge Discovery in Databases*, Pisa, Italy, 20-24 September 2004**
- **The dishonest casino example came from slides I inherited from Ken Sung**
- **The "indep vs conditional indep" example came from Kwok Pui Choi**



References

- L. Breiman, et al. Classification and Regression Trees. Wadsworth and Brooks, 1984
- L. Breiman, Bagging predictors, Machine Learning, 24:123-140, 1996
- L. Breiman, Random forests, Machine Learning, 45:5-32, 2001
- J. R. Quinlan, Induction of decision trees, Machine Learning, 1:81-106, 1986
- J. R. Quinlan, C4.5: Program for Machine Learning. Morgan Kaufmann, 1993
- C. Gini, Measurement of inequality of incomes, The Economic Journal, 31:124-126, 1921
- Jinyan Li et al., Data Mining Techniques for the Practical Bioinformatician, *The Practical Bioinformatician*, Chapter 3, pages 35-70, WSPC, 2004



References

- Y. Freund, et al. Experiments with a new boosting algorithm, ICML 1996, pages 148-156
- T. G. Dietterich, An experimental comparison of three methods for constructing ensembles of decision trees: Bagging, boosting, and randomization, Machine Learning, 40:139-157, 2000
- Naïve Bayesian Classification, *Wikipedia*, http://en.wikipedia.org/wiki/Naive_Bayesian_classification
- Hidden Markov Model, Wikipedia, http://en.wikipedia.org/wiki/Hidden_Markov_model