

For written notes on this lecture, please read chapter 3 of *The Practical Bioinformatician*. Alternatively, please read “Rule-Based Data Mining Methods for Classification Problems in Biomedical Domains”, a tutorial at *PKDD04* by Jinyan Li and Limsoon Wong, September 2004. <http://www.comp.nus.edu.sg/~wongls/talks/pkdd04/>

# CS2220: Introduction to Computational Biology

## Lecture 2: Essence of Knowledge Discovery

**Limsoon Wong**



# Outline

- **Overview of Supervised Learning**
  - Decision Trees
- **Decision Trees Ensembles**
  - Bagging
- **Other Methods**
  - K-Nearest Neighbour
  - Support Vector Machines
  - Bayesian Approach
  - Hidden Markov Models

# Overview of Supervised Learning



# Supervised Learning

- Also called **classification**
- Learn from past experience, and use the learned knowledge to classify new data
- Knowledge learned by **intelligent algorithms**
- **Examples:**
  - Clinical diagnosis for patients
  - Cell type classification

# Data

- **Classification application involves  $> 1$  class of data. E.g.,**
  - Normal vs disease cells for a diagnosis problem
- **Training data is a set of instances (samples, points) with known class labels**
- **Test data is a set of instances whose class labels are to be predicted**

# Typical Notations

- **Training data**

$$\{\langle \mathbf{x}_1, y_1 \rangle, \langle \mathbf{x}_2, y_2 \rangle, \dots, \langle \mathbf{x}_m, y_m \rangle\}$$

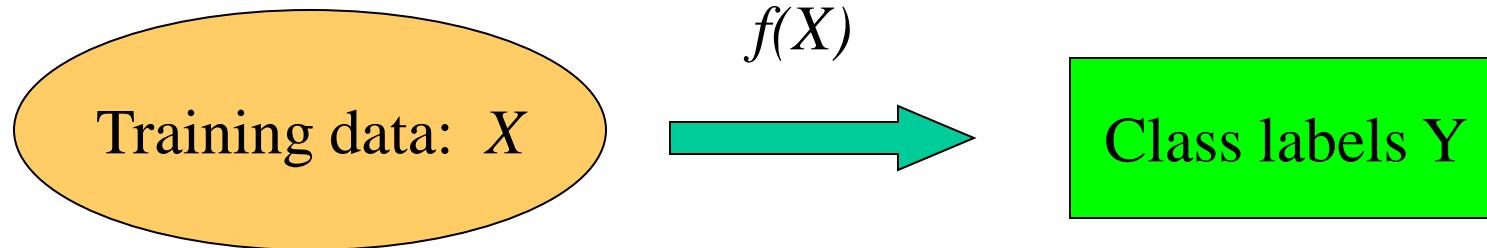
where  $\mathbf{x}_j$  are n-dimensional vectors  
and  $y_j$  are from a discrete space  $Y$ .

E.g.,  $Y = \{\text{normal, disease}\}$

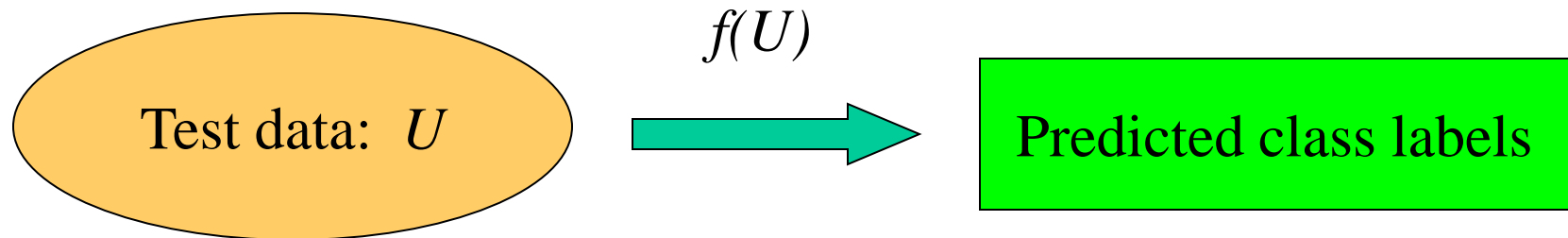
- **Test data**

$$\{\langle \mathbf{u}_1, ? \rangle, \langle \mathbf{u}_2, ? \rangle, \dots, \langle \mathbf{u}_k, ? \rangle, \}$$

# Process



A classifier, a mapping, a hypothesis



# Relational Representation of Gene Expression Data

$n$  features (order of 1000)

$m$  samples

gene <sub>1</sub> gene <sub>2</sub> gene <sub>3</sub> gene <sub>4</sub> ... gene <sub>n</sub>						class
X <sub>11</sub>	X <sub>12</sub>	X <sub>13</sub>	X <sub>14</sub>	...	X <sub>1n</sub>	→ P
X <sub>21</sub>	X <sub>22</sub>	X <sub>23</sub>	X <sub>24</sub>	...	X <sub>2n</sub>	→ N
X <sub>31</sub>	X <sub>32</sub>	X <sub>33</sub>	X <sub>34</sub>	...	X <sub>3n</sub>	→ P
.....						
X <sub>m1</sub>	X <sub>m2</sub>	X <sub>m3</sub>	X <sub>m4</sub>	...	X <sub>mn</sub>	→ N



# Features (aka Attributes)

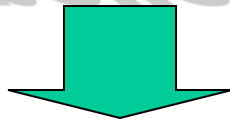
- **Categorical features**
  - color = {red, blue, green}
- **Continuous or numerical features**
  - gene expression
  - age
  - blood pressure
- **Discretization**

# An Example

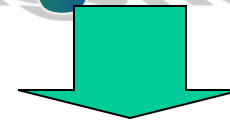
Outlook	Temp	Humidity	Windy	class
Sunny	75	70	true	Play
Sunny	80	90	true	Don't
Sunny	85	85	false	Don't
Sunny	72	95	true	Don't
Sunny	69	70	false	Play
Overcast	72	90	true	Play
Overcast	83	78	false	Play
Overcast	64	65	true	Play
Overcast	81	75	false	Play
Rain	71	80	true	Don't
Rain	65	70	true	Don't
Rain	75	80	false	Play
Rain	68	80	false	Play
Rain	70	96	false	Play

# Overall Picture of Supervised Learning

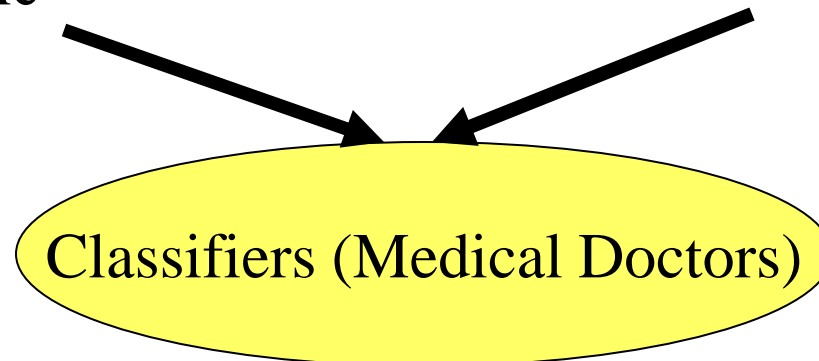
## Labelled Data + Algorithms



Biomedical  
Financial  
Government  
Scientific



Decision trees  
Emerging patterns  
SVM  
Neural networks



# Recap: Evaluation of a Classifier

- Performance on independent blind test data
- K-fold cross validation: Given a dataset, divide it into  $k$  even parts,  $k-1$  of them are used for training, and the rest one part treated as test data
- LOOCV, a special case of K-fold CV
- Accuracy, error rate
- False positive rate, false negative rate, sensitivity, specificity, precision

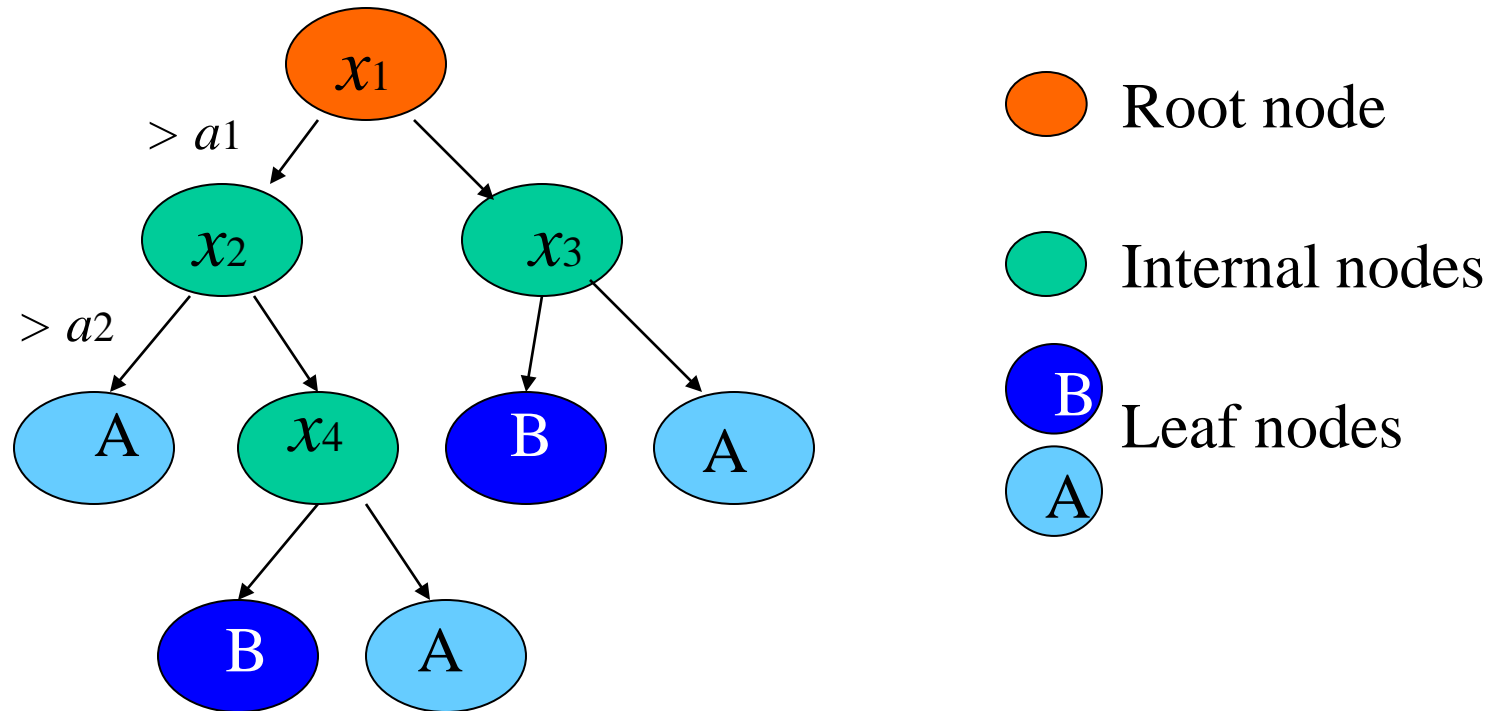
# Requirements of Biomedical Classification

- **High accuracy/sensitivity/specificity/precision**
- **High comprehensibility**

# Importance of Rule-Based Methods

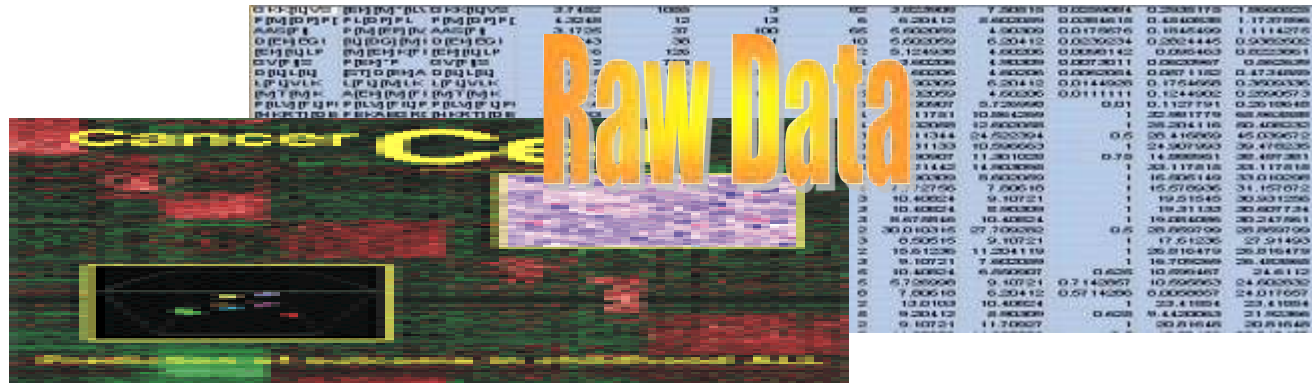
- **Systematic selection of a small number of features used for the decision making**  
⇒ **Increase comprehensibility of the knowledge patterns**
- **C4.5 and CART are two commonly used rule induction algorithms---a.k.a. decision tree induction algorithms**

# Structure of Decision Trees

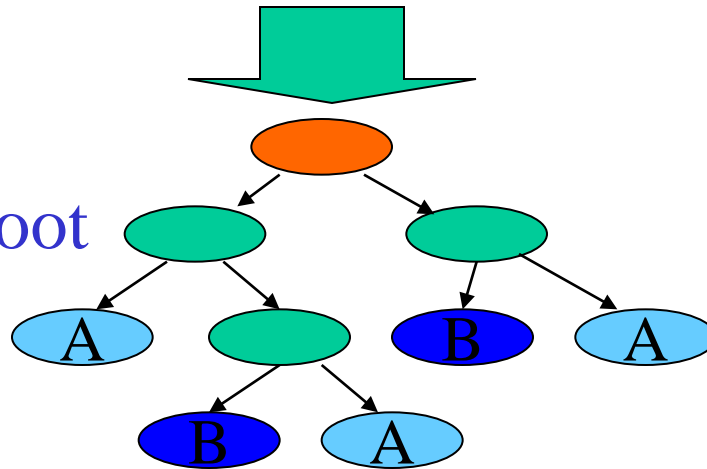


- If  $x_1 > a_1$  &  $x_2 > a_2$ , then it's A class
- C4.5, CART, two of the most widely used
- Easy interpretation, but accuracy generally unattractive

# Elegance of Decision Trees



Every path from root to a leaf forms a decision rule





# Brief History of Decision Trees

CLS (Hunt et al. 1966) --- cost driven

CART (Breiman et al. 1984) --- Gini Index

ID3 (Quinlan, 1986) --- Information-driven

C4.5 (Quinlan, 1993) --- Gain ratio + Pruning ideas

# A Simple Dataset

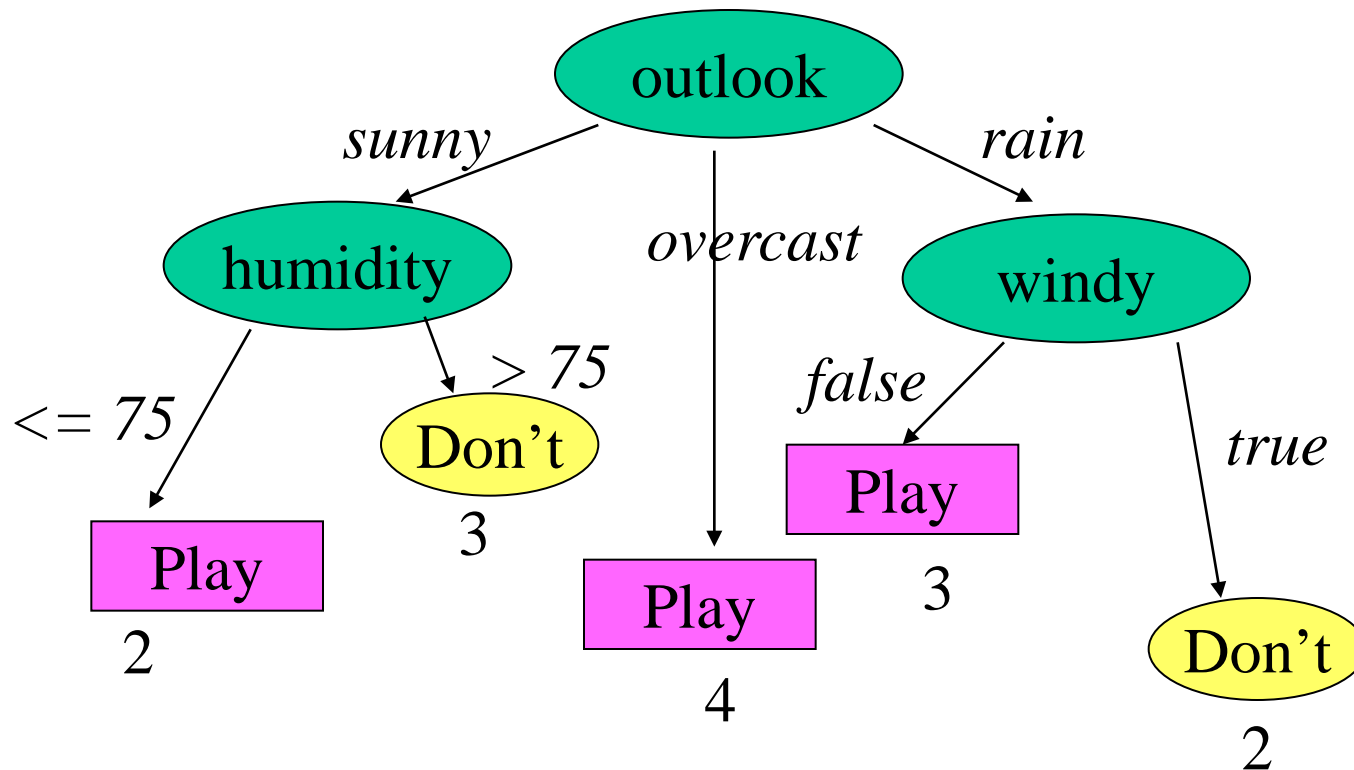
Outlook	Temp	Humidity	Windy	class
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Sunny	80	90	true	Don't
Sunny	85	85	false	Don't
Sunny	72	95	true	Don't
Sunny	69	70	false	Play
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Overcast	83	78	false	Play
Overcast	64	65	true	Play
Overcast	81	75	false	Play
Rain	71	80	true	Don't
Rain	65	70	true	Don't
Rain	75	80	false	Play
Rain	68	80	false	Play
Rain	70	96	false	Play

9 Play samples

5 Don't

A total of 14.

# A Decision Tree



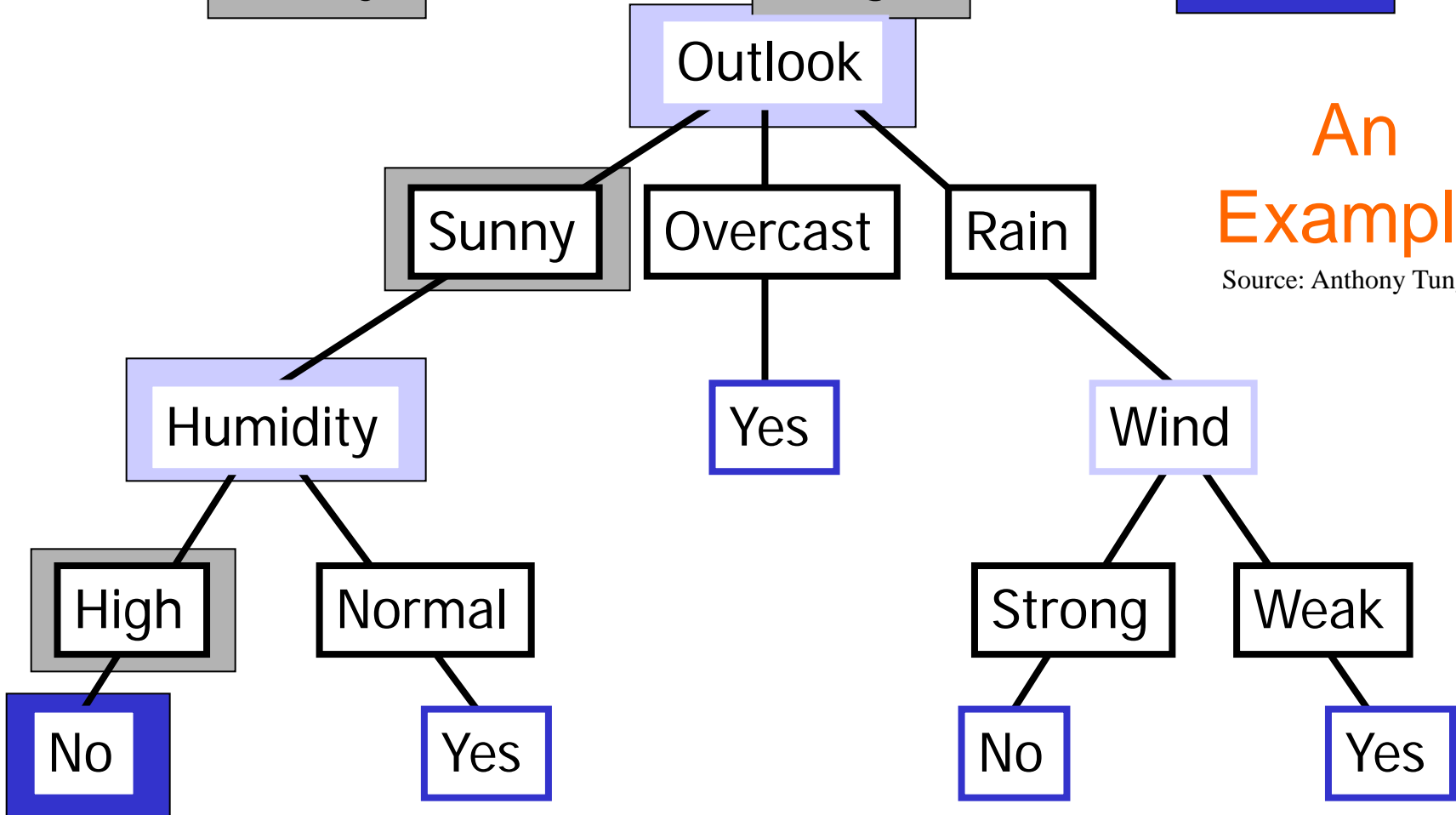
- **Construction of a tree is equiv to determination of root node of the tree and root nodes of its sub-trees**

Exercise: What is the accuracy of this tree?

Outlook	Temperature	Humidity	Wind	PlayTennis
Sunny	Hot	High	Weak	No

# An Example

Source: Anthony Tung



# Most Discriminatory Feature

- **Every feature can be used to partition the training data**
- **If the partitions contain a pure class of training instances, then this feature is most discriminatory**

# Example of Partitions

- **Categorical feature**
  - Number of partitions of the training data is equal to the number of values of this feature
- **Numerical feature**
  - Two partitions

Categorical feature

Numerical feature

Instance #	Outlook	Temp	Humidity	Windy	class
1	Sunny	75	70	true	Play
2	Sunny	80	90	true	Don't
3	Sunny	85	85	false	Don't
4	Sunny	72	95	true	Don't
5	Sunny	69	70	false	Play
6	Overcast	72	90	true	Play
7	Overcast	83	78	false	Play
8	Overcast	64	65	true	Play
9	Overcast	81	75	false	Play
10	Rain	71	80	true	Don't
11	Rain	65	70	true	Don't
12	Rain	75	80	false	Play
13	Rain	68	80	false	Play
14	Rain	70	96	false	Play

Instance #	Outlook	Temp	Humidity	Windy	class
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8	Overcast	64	65	true	Play
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12	Rain	75	80	false	Play
13	Rain	68	80	false	Play
14	Rain	70	96	false	Play

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Total 14 training instances

Outlook = sunny

1,2,3,4,5  
P,D,D,D,P

Outlook = overcast

6,7,8,9  
P,P,P,P

Outlook = rain

10,11,12,13,14  
D, D, P, P, P

A categorical feature is partitioned based on its number of possible values



Instance #	Outlook	Temp	Humidity	Windy	class
1	Sunny	75	70	true	Play
2	Sunny	80	90	true	Don't
3	Sunny	85	85	false	Don't
4	Sunny	72	95	true	Don't
5	Sunny	69	70	false	Play
6	Overcast	72	90	true	Play
7	Overcast	83	78	false	Play
8	Overcast	64	65	true	Play
9	Overcast	81	75	false	Play
10	Rain	71	80	true	Don't
11	Rain	65	70	true	Don't
12	Rain	75	80	false	Play
13	Rain	68	80	false	Play
14	Rain	70	96	false	Play

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Total 14 training instances

Temperature  $\leq 70$

5,8,11,13,14  
P,P, D, P, P

Temperature  $> 70$

1,2,3,4,6,7,9,10,12  
P,D,D,D,P,P,P,D,P

A numerical feature is generally partitioned by choosing a “cutting point”

# Steps of Decision Tree Construction

- **Select the “best” feature as root node of the whole tree**
- **Partition dataset into subsets using this feature so that the subsets are as “pure” as possible**
- **After partition by this feature, select the best feature (wrt the subset of training data) as root node of this sub-tree**
- **Recursively, until the partitions become pure or almost pure**

# Let's Construct a Decision Tree Together

Outlook	Temp	Humidity	Windy	class
Sunny	75	70	true	Play
Sunny	80	90	true	Don't
Sunny	85	85	false	Don't
Sunny	72	95	true	Don't
Sunny	69	70	false	Play
Overcast	72	90	true	Play
Overcast	83	78	false	Play
Overcast	64	65	true	Play
Overcast	81	75	false	Play
Rain	71	80	true	Don't
Rain	65	70	true	Don't
Rain	75	80	false	Play
Rain	68	80	false	Play
Rain	70	96	false	Play

Ask the class to pick root node and construct the tree recursively with them... How good is that tree?

# Three Measures to Evaluate Which Feature is Best

- **Gini index**
- **Information gain**
- **Information gain ratio**

# Gini Index

$$\begin{aligned} \text{gini}(S) &= \frac{\text{diff of two arbitrary specimen in } S}{\text{mean specimen in } S} \\ &= \text{prob}(\text{getting two specimen of diff class in } S) \\ &= 1 - \text{prob}(\text{getting two specimen of same class in } S) \\ &= 1 - \sum_i \text{prob}(\text{getting specimen of class } i \text{ in } S)^2 \end{aligned}$$

- **Gini index is the expected value of the ratio of the diff of two arbitrary specimens to the mean value of all specimens**
- **Closer to 1 means similar to “background distribution”. Closer to 0, means feature is “unexpected”**

# Gini Index

Let  $\mathcal{U} = \{C_1, \dots, C_k\}$  be all the classes. Suppose we are currently at a node and  $D$  is the set of those samples that have been moved to this node. Let  $f$  be a feature and  $d[f]$  be the value of the feature  $f$  in a sample  $d$ . Let  $S$  be a range of values that the feature  $f$  can take. Then the Gini index for  $f$  in  $D$  for the range  $S$  is defined as

$$gini_f^D(S) = 1 - \sum_{C_i \in \mathcal{U}} \left( \frac{|\{d \in D \mid d \in C_i, d[f] \in S\}|}{|D|} \right)^2$$

The purity of a split of the value range  $S$  of an attribute  $f$  by some split-point into subranges  $S_1$  and  $S_2$  is then defined as

$$gini_f^D(S_1, S_2) = \sum_{S \in \{S_1, S_2\}} \frac{|\{d \in D \mid d[f] \in S\}|}{|D|} * gini_f^D(S)$$

we choose the feature  $f$  and the split-point  $p$  that minimizes  $gini_f^D(S_1, S_2)$  over all possible alternative features and split-points.

# Gini Index of Outlook

Outlook	Temp	Humidity	Windy	class
Sunny	75	70	true	Play
Sunny	80	90	true	Don't
Sunny	85	85	false	Don't
Sunny	72	95	true	Don't
Sunny	69	70	false	Play
Overcast	72	90	true	Play
Overcast	83	78	false	Play
Overcast	64	65	true	Play
Overcast	81	75	false	Play
Rain	71	80	true	Don't
Rain	65	70	true	Don't
Rain	75	80	false	Play
Rain	68	80	false	Play
Rain	70	96	false	Play

$$gini_f^D(S) = 1 - \sum_{C_i \in \mathcal{U}} \left( \frac{|\{d \in D \mid d \in C_i, d[f] \in S\}|}{|D|} \right)^2$$

$$gini_f^D(S_1, S_2) = \sum_{S \in \{S_1, S_2\}} \frac{|\{d \in D \mid d[f] \in S\}|}{|D|} * gini_f^D(S)$$

- $gini(\text{Sunny}) = 1 - (2/5)^2 - (3/5)^2 = 0.48$
- $gini(\text{Overcast}) = 1 - (4/4)^2 - (0/5)^2 = 0$
- $gini(\text{Rain}) = 1 - (3/5)^2 - (2/5)^2 = 0.48$
- $gini(\text{Outlook}) = 5/14 * 0.48 + 4/14 * 0 + 5/14 * 0.48 = 0.34$

# Characteristics of C4.5/CART Trees

- **Single coverage of training data (elegance)**
- **Divide-and-conquer splitting strategy**
- **Fragmentation problem  $\Rightarrow$  Locally reliable but globally insignificant rules**

- **Miss many globally significant rules; mislead system**



Example Use of Decision Tree Methods: **Proteomics**  
**Approaches to Biomarker Discovery**

- In prostate and bladder cancers (Adam et al. *Proteomics*, 2001)
- In serum samples to detect breast cancer (Zhang et al. *Clinical Chemistry*, 2002)
- In serum samples to detect ovarian cancer (Petricoin et al. *Lancet*; Li & Rao, *PAKDD* 2004)

# Decision Tree Ensembles



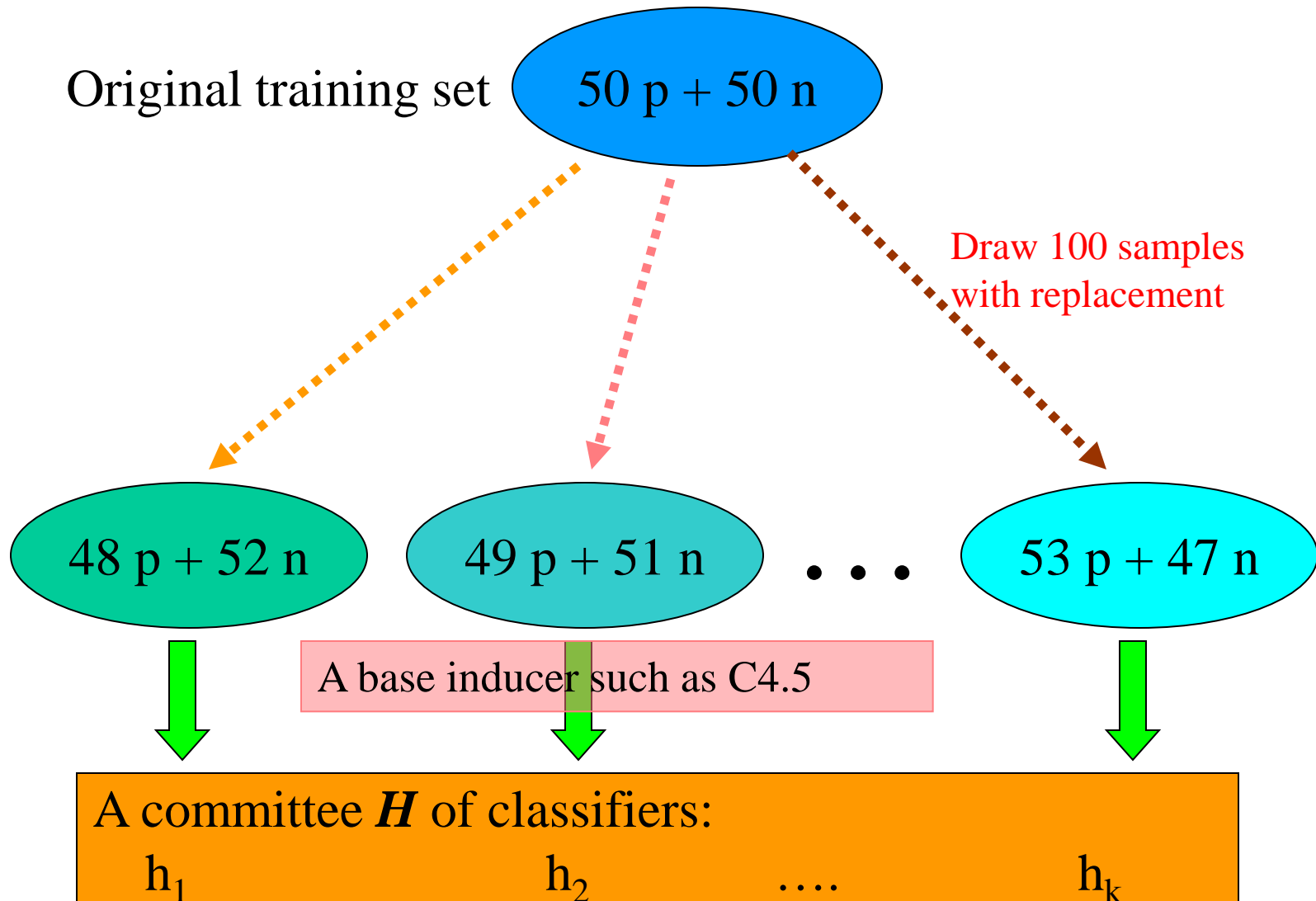
# Motivating Example

- $h_1, h_2, h_3$  are indep classifiers w/ accuracy = 60%
- $C_1, C_2$  are the only classes
- $t$  is a test instance in  $C_1$
- $h(t) = \operatorname{argmax}_{C \in \{C_1, C_2\}} |\{h_j \in \{h_1, h_2, h_3\} \mid h_j(t) = C\}|$
- Then  $\operatorname{prob}(h(t) = C_1)$ 
  - =  $\operatorname{prob}(h_1(t)=C_1 \ \& \ h_2(t)=C_1 \ \& \ h_3(t)=C_1) +$   
 $\operatorname{prob}(h_1(t)=C_1 \ \& \ h_2(t)=C_1 \ \& \ h_3(t)=C_2) +$   
 $\operatorname{prob}(h_1(t)=C_1 \ \& \ h_2(t)=C_2 \ \& \ h_3(t)=C_1) +$   
 $\operatorname{prob}(h_1(t)=C_2 \ \& \ h_2(t)=C_1 \ \& \ h_3(t)=C_1)$
  - =  $60\% * 60\% * 60\% + 60\% * 60\% * 40\% +$   
 $60\% * 40\% * 60\% + 40\% * 60\% * 60\% = 64.8\%$

# Bagging

- Proposed by Breiman (1996)
- Also called **Bootstrap aggregating**
- Make use of randomness injected to training data

# Main Ideas



# Decision Making by Bagging

Given a new test sample  $T$

$$\text{bagged}(T) = \operatorname{argmax}_{C_j \in \mathcal{U}} |\{h_i \in \mathcal{H} \mid h_i(T) = C_j\}|$$

$$\text{where } \mathcal{U} = \{C_1, \dots, C_r\}$$

Exercise: What does the above formula mean?

# Summary of Ensemble Classifiers

Bagging

Random  
Forest

AdaBoost.M1

Rules may  
not be correct  
when  
applied to  
training data

Randomization  
Trees

CS4

Rules correct

Exercise: Describe the decision tree  
ensemble classifiers not explained in this ppt

# Other Machine Learning Approaches





# Outline

- **K-Nearest Neighbour**
- **Support Vector Machines**
- **Bayesian Approach**
- **Hidden Markov Models**

Exercise: Name and describe one other commonly used machine learning method

# K-Nearest Neighbours



# How kNN Works

- Given a new case
- Find k “nearest” neighbours, i.e., k most similar points in the training data set
- Assign new case to the same class to which most of these neighbours belong
- A common “distance” measure betw samples  $x$  and  $y$  is
$$\sqrt{\sum_f (x[f] - y[f])^2}$$
where  $f$  ranges over features of the samples

Exercise: What does the formula above mean?

# Illustration of kNN (k=8)

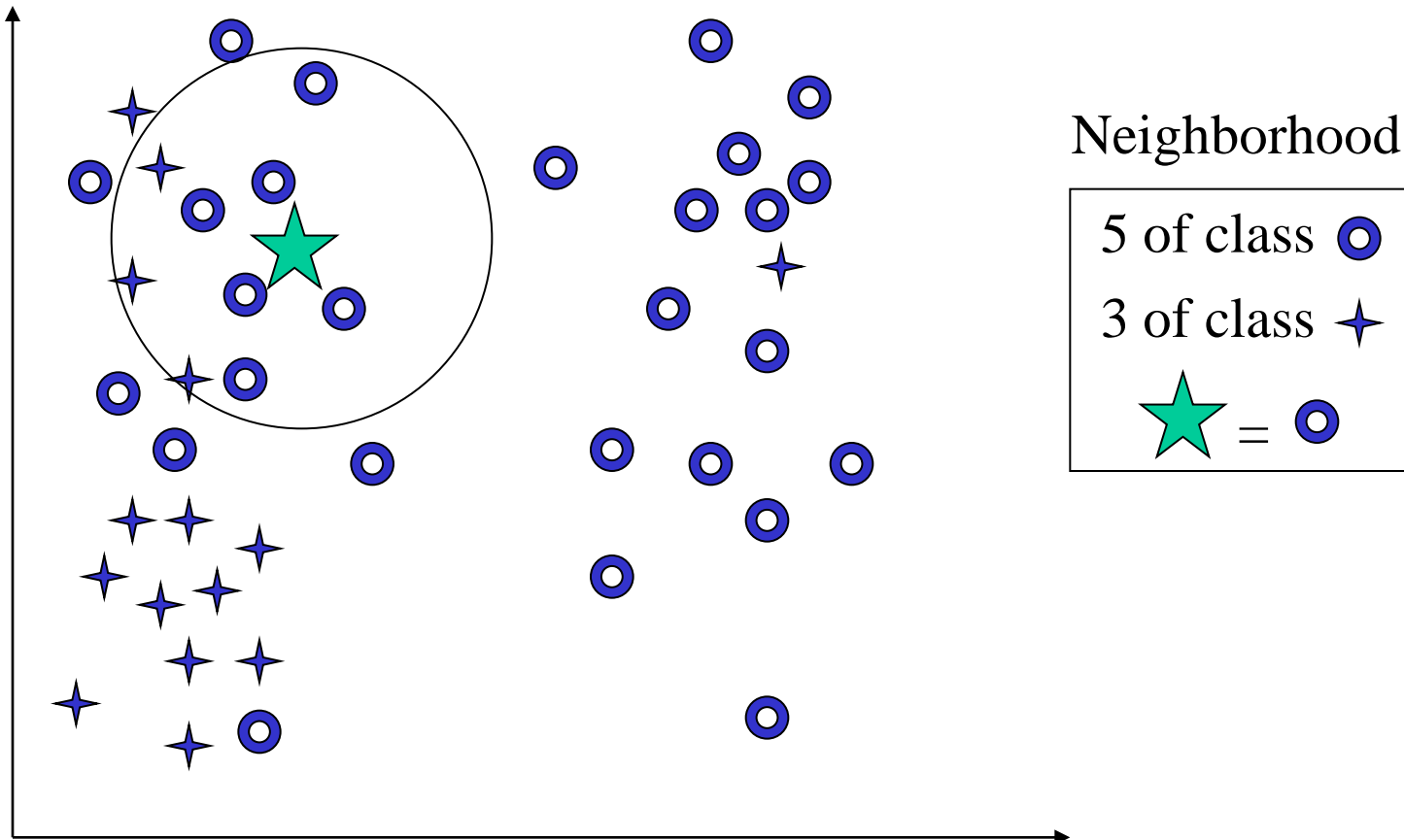


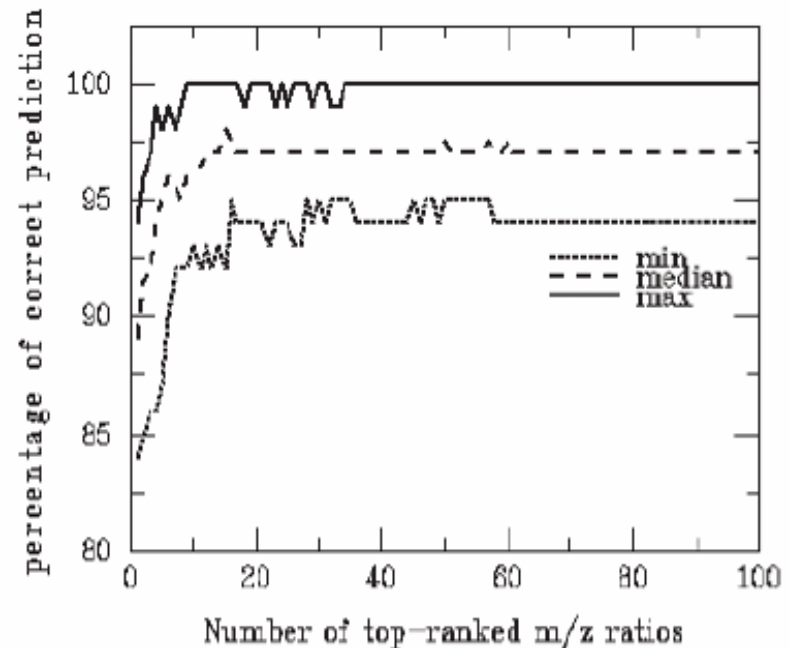
Image credit: Zaki

## Some Issues

- **Simple to implement**
- **Must compare new case against all training cases**  
⇒ **May be slow during prediction**
  
- **No need to train**
- **But need to design distance measure properly**  
⇒ **May need expert for this**
  
- **Can't explain prediction outcome**  
⇒ **Can't provide a model of the data**

# Example Use of kNN: Ovarian Cancer Diagnosis Based on SELDI Proteomic Data

- Li et al, *Bioinformatics* 20:1638-1640, 2004
- Use kNN to diagnose ovarian cancers using proteomic spectra
- Data set is from Petricoin et al., *Lancet* 359:572-577, 2002



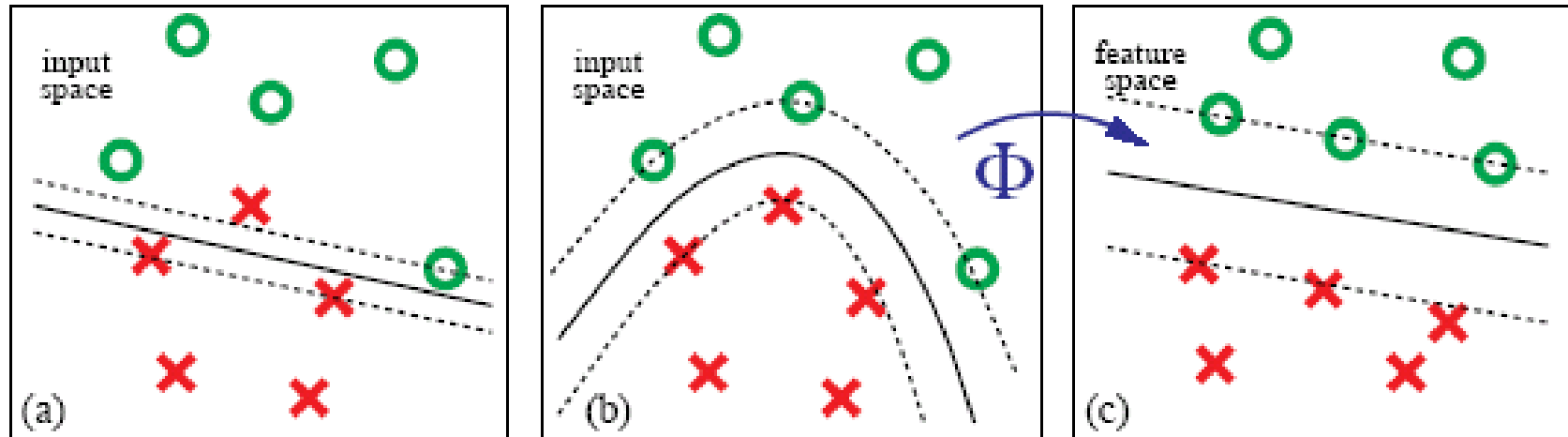
**Fig. 1.** Minimum, median and maximum of percentages of correct prediction as a function of the number of top-ranked  $m/z$  ratios in 50 independent partitions into learning and validation sets.

# Support Vector Machines



# Basic Idea

Image credit: Zien

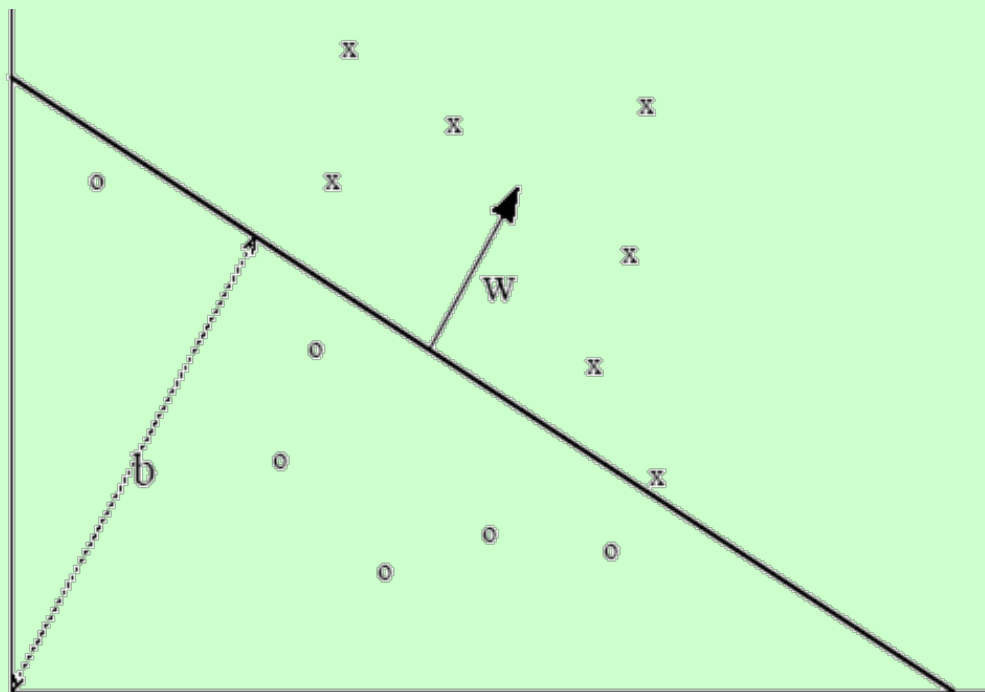


- (a) Linear separation not possible w/o errors
- (b) Better separation by nonlinear surfaces in input space
- (c) Nonlinear surface corr to linear surface in feature space.  
 Map from input to feature space by “kernel” function  $\Phi$   
 $\Rightarrow$  “Linear learning machine” + kernel function as classifier



# Linear Learning Machines

- Hyperplane separating the x's and o's points is given by  $(W \cdot X) + b = 0$ , with  $(W \cdot X) = \sum_j W[j] * X[j]$
- ⇒ Decision function is  $\text{llm}(X) = \text{sign}((W \cdot X) + b)$



# Linear Learning Machines

- Solution is a linear combination of training points  $X_k$  with labels  $Y_k$

$$W = \sum_k \alpha_k * Y_k * X_k,$$

with  $\alpha_k > 0$ , and  $Y_k = \pm 1$

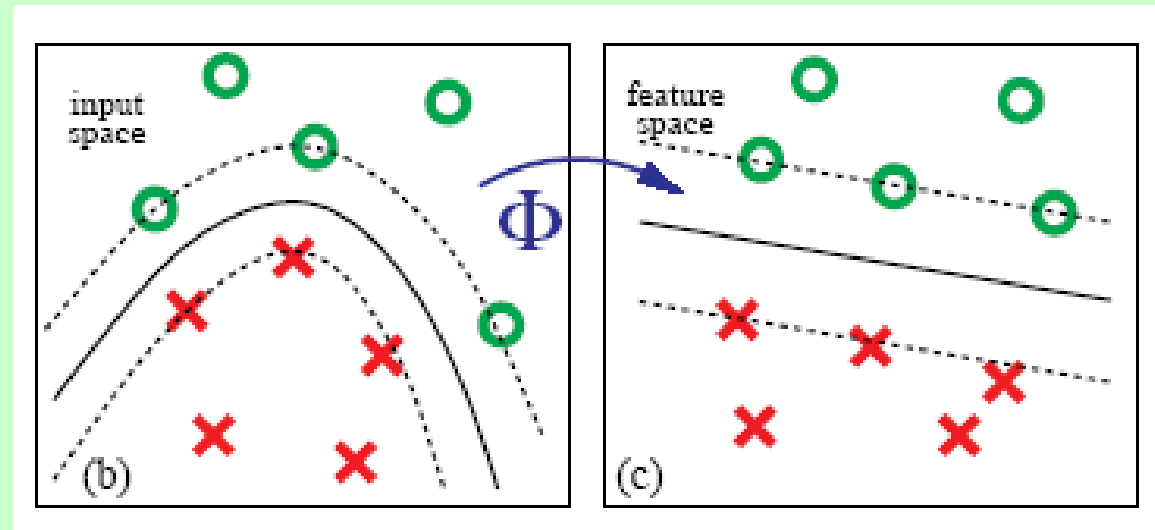
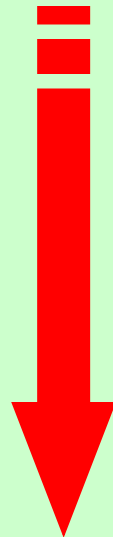
$$\Rightarrow \text{llm}(X) = \text{sign}(\sum_k \alpha_k * Y_k * (X_k \bullet X) + b)$$



“data” appears only in dot product!

# Kernel Function

- $\text{llm}(X) = \text{sign}(\sum_k \alpha_k * Y_k * (X_k \bullet X) + b)$



- $\text{svm}(X) = \text{sign}(\sum_k \alpha_k * Y_k * (\Phi X_k \bullet \Phi X) + b)$

$$\Rightarrow \text{svm}(X) = \text{sign}(\sum_k \alpha_k * Y_k * K(X_k, X) + b)$$

$$\text{where } K(X_k, X) = (\Phi X_k \bullet \Phi X)$$

# Kernel Function

- $\text{svm}(X) = \text{sign}(\sum_k \alpha_k * Y_k * K(X_k, X) + b)$   
 $\Rightarrow K(A, B)$  can be computed w/o computing  $\Phi$
- In fact replace it w/ lots of more “powerful” kernels besides  $(A \cdot B)$ . E.g.,
  - $K(A, B) = (A \cdot B)^d$
  - $K(A, B) = \exp(- \|A - B\|^2 / (2 * \sigma))$ , ...

## How SVM Works

- $\text{svm}(X) = \text{sign}(\sum_k \alpha_k * Y_k * K(X_k, X) + b)$
- To find  $\alpha_k$  is a quadratic programming problem

$$\text{max: } \sum_k \alpha_k - 0.5 * \sum_k \sum_h \alpha_k * \alpha_h * Y_k * Y_h * K(X_k, X_h)$$

$$\text{subject to: } \sum_k \alpha_k * Y_k = 0$$

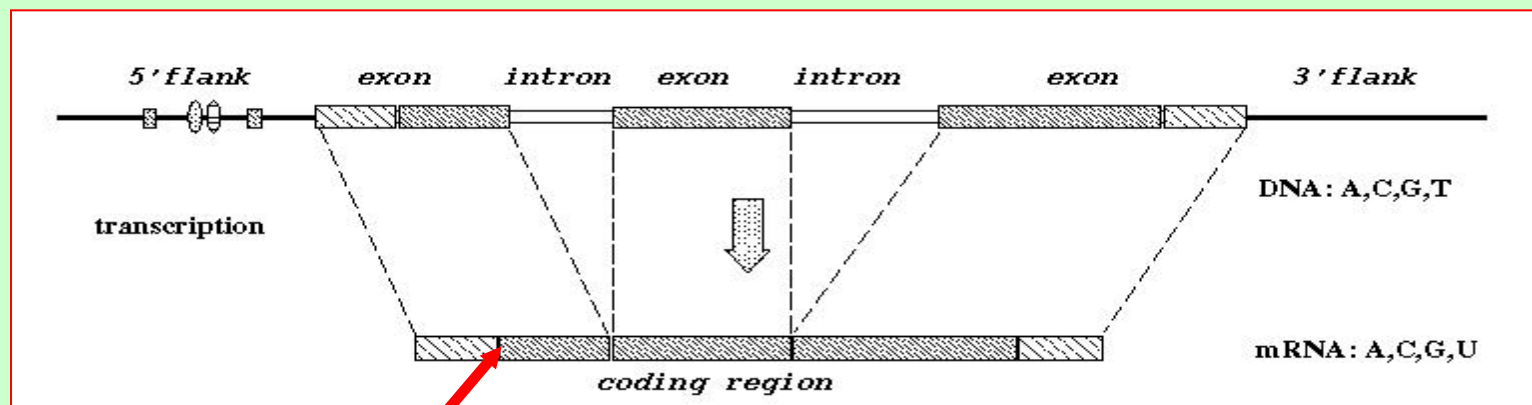
$$\text{and for all } \alpha_k, C \geq \alpha_k \geq 0$$

- To find  $b$ , estimate by averaging

$$Y_h - \sum_k \alpha_k * Y_k * K(X_h, X_k)$$

$$\text{for all } \alpha_h \geq 0$$

# Example Use of SVM: Recognition of Protein Translation Initiation Sites



**TIS**

- Zien et al., *Bioinformatics* 16:799-807, 2000
- Use SVM to recognize protein translation initiation sites from genomic sequences
- Raw data set is same as Liu & Wong, *JBCB* 1:139-168, 2003

# Bayesian Approach



# Bayes Theorem

$$P(h|d) = \frac{P(d|h) * P(h)}{P(d)}$$

- $P(h)$  = prior prob that hypothesis  $h$  holds
- $P(d|h)$  = prob of observing data  $d$  given  $h$  holds
- $P(h|d)$  = posterior prob that  $h$  holds given observed data  $d$



# Bayesian Approach

- Let  $H$  be all possible classes. Given a test instance w/ feature vector  $\{f_1 = v_1, \dots, f_n = v_n\}$ , the most probable classification is given by

$$\operatorname{argmax}_{h_j \in H} P(h_j | f_1 = v_1, \dots, f_n = v_n)$$

- Using Bayes Theorem, rewrites to

$$\operatorname{argmax}_{h_j \in H} \frac{P(f_1 = v_1, \dots, f_n = v_n | h_j) * P(h_j)}{P(f_1 = v_1, \dots, f_n = v_n)}$$

- Since denominator is independent of  $h_j$ , this simplifies to

$$\operatorname{argmax}_{h_j \in H} P(f_1 = v_1, \dots, f_n = v_n | h_j) * P(h_j)$$

# Naïve Bayes

- But estimating  $P(f_1=v_1, \dots, f_n=v_n/h_j)$  accurately may not be feasible unless training data set is large
- “Solved” by assuming  $f_1, \dots, f_n$  are conditionally independent of each other
- Then  $\operatorname{argmax}_{h_j \in H} P(f_1 = v_1, \dots, f_n = v_n | h_j) * P(h_j)$

$$= \operatorname{argmax}_{h_j \in H} \prod_i P(f_i = v_i | h_j) * P(h_j)$$

where  $P(h_j)$  and  $P(f_i=v_i/h_j)$  can often be estimated reliably from typical training data set

Exercise: How do you estimate  $P(h_j)$  and  $P(f_j=v_j|h_j)$ ?

Abstractly, the probability model for a classifier is a conditional model

$$p(C|F_1, \dots, F_n)$$

over a dependent class variable  $C$  with a small number of outcomes or *classes*, conditional on several feature variables  $F_1$  through  $F_n$ . The problem is that if the number of features  $n$  is large or when a feature can take on a large number of values, then basing such a model on probability tables is infeasible. We therefore reformulate the model to make it more tractable.

Using [Bayes' theorem](#), we write

$$p(C|F_1, \dots, F_n) = \frac{p(C) p(F_1, \dots, F_n|C)}{p(F_1, \dots, F_n)}.$$

In practice we are only interested in the numerator of that fraction, since the denominator does not depend on  $C$  and the values of the features  $F_i$  are given, so that the denominator is effectively constant. The numerator is equivalent to the [joint probability](#) model

$$p(C, F_1, \dots, F_n)$$

which can be rewritten as follows, using repeated applications of the definition of [conditional probability](#):

$$\begin{aligned} p(C, F_1, \dots, F_n) &= p(C) p(F_1, \dots, F_n|C) \\ &= p(C) p(F_1|C) p(F_2, \dots, F_n|C, F_1) \\ &= p(C) p(F_1|C) p(F_2|C, F_1) p(F_3, \dots, F_n|C, F_1, F_2) \\ &= p(C) p(F_1|C) p(F_2|C, F_1) p(F_3|C, F_1, F_2) p(F_4, \dots, F_n|C, F_1, F_2, F_3) \end{aligned}$$

and so forth. Now the "naive" conditional independence assumptions come into play: assume that each feature  $F_i$  is conditionally [independent](#) of every other feature  $F_j$  for  $j \neq i$ . This means that

$$p(F_i|C, F_j) = p(F_i|C)$$

and so the joint model can be expressed as

$$\begin{aligned} p(C, F_1, \dots, F_n) &= p(C) p(F_1|C) p(F_2|C) p(F_3|C) \dots \\ &= p(C) \prod_{i=1}^n p(F_i|C). \end{aligned}$$

Source: Wikipedia

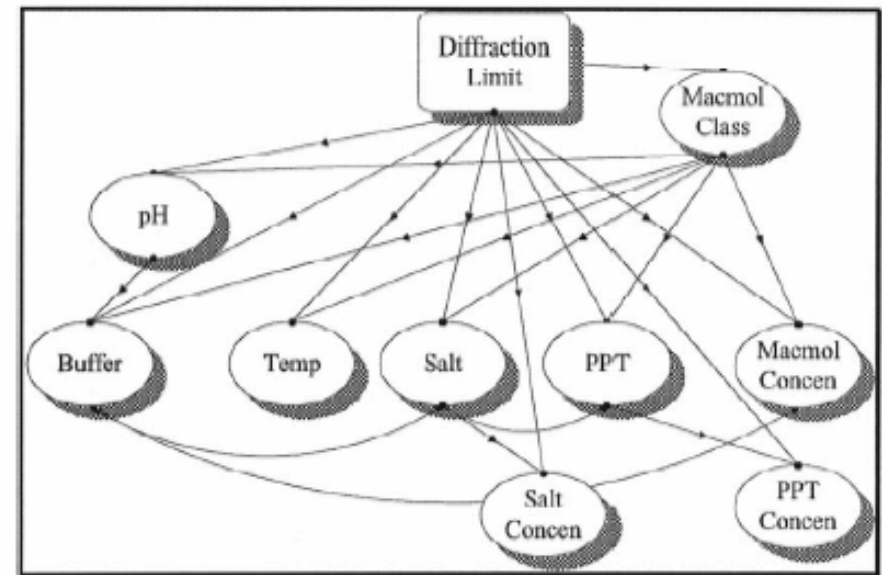
# Independence vs Conditional Independence

- Independence:  $P(A,B) = P(A) * P(B)$
- Conditional Independence:  $P(A,B|C) = P(A|C) * P(B|C)$
- Indep does not imply conditional indep
  - Consider tossing a fair coin twice
    - A is event of getting head in 1st toss
    - B is event of getting head in 2nd toss
    - C is event of getting exactly one head
  - Then  $A=\{HT, HH\}$ ,  $B=\{HH, TH\}$  and  $C=\{HT, TH\}$
  - $P(A,B|C) = P(\{HH\}|C)=0$
  - $P(A|C) = P(A,C)/P(C) = P(\{HT\})/P(C) = (1/4)/(1/2) = 1/2$
  - Similarly,  $P(B|C) = 1/2$

Source: Choi Kwok Pui

# Example Use of Bayesian: Design of Screens for Macromolecular Crystallization

- Hennessy et al., *Acta Cryst* D56:817-827, 2000
- Crystallization of proteins requires search of expt settings to find right conditions for diffraction-quality xtals
- BMCD is a db of known crystallization conditions
- Use Bayes to determine prob of success of a set of expt conditions based on BMCD



**Figure 1**

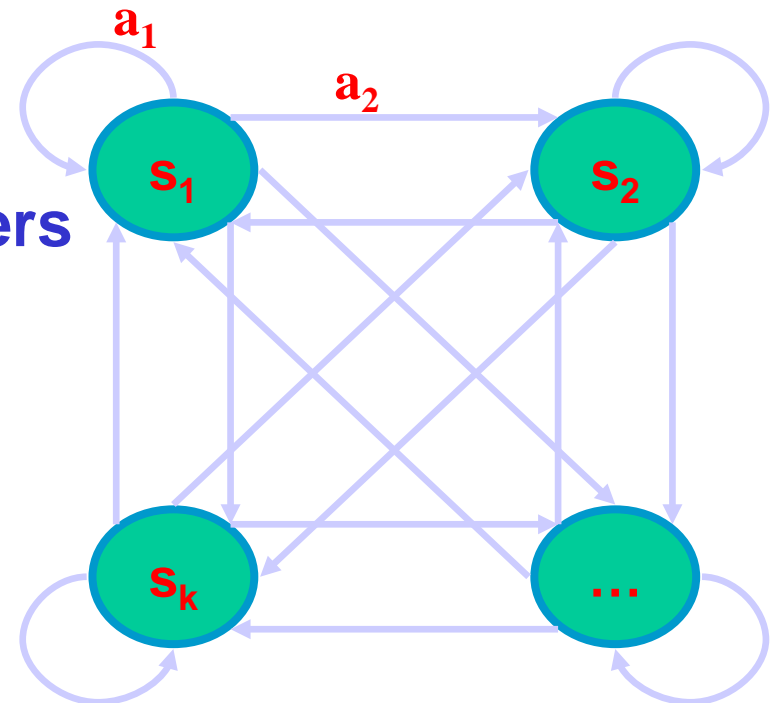
Crystallization parameter dependency graph. The graph represents the parameters included in the calculation of the estimated probability of success and their dependencies. A connecting arc from pH to buffer indicates that the probability distribution for the buffer may depend on the value of the pH. The lack of a connecting arc between two parameters reflects conditional independence (the probability distribution for a parameter is independent of the value of the other parameter).

# Hidden Markov Models



# What is a HMM

- HMM is a stochastic generative model for seqs
- Defined by model parameters
  - finite set of states  $S$
  - finite alphabet  $A$
  - transition prob matrix  $T$
  - emission prob matrix  $E$
- Move from state to state as per  $T$  while emitting symbols as per  $E$



## Order of a HMM

- In  $n$ th order HMM,  $T$  &  $E$  depend on all  $n$  previous states
- E.g., for 1st order HMM, given emissions  $X = x_1, x_2, \dots$ , & states  $S = s_1, s_2, \dots$ , the prob of this seq is

$$Prob(X, S) = \prod_i Prob(x_i | s_i) = \prod_i E(x_i | s_i) * T(s_{i-1}, s_i)$$



## Using HMM

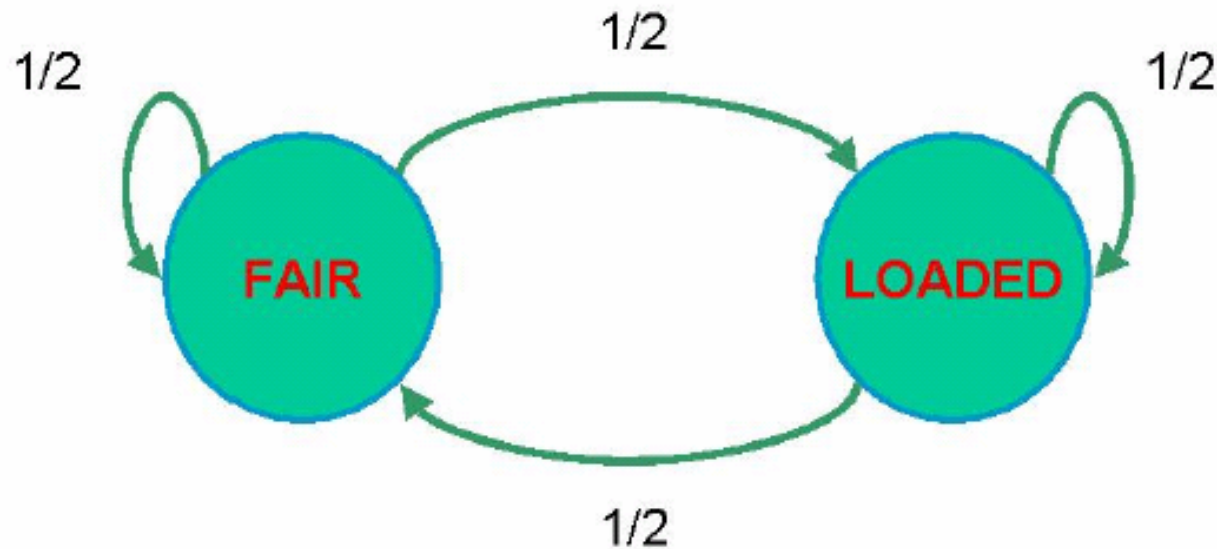
- Given the model parameters, compute the probability of a particular output sequence. Solved by the **forward algorithm**
- Given the model parameters, find the most likely sequence of (hidden) states which could have generated a given output sequence. Solved by the **Viterbi algorithm**
- Given an output sequence, find the most likely set of state transition and output probabilities. Solved by the **Baum-Welch algorithm**

Exercise: Describe these algorithms

# Example: Dishonest Casino

- **Casino has two dices:**
  - Fair dice
    - $P(i) = 1/6, i = 1..6$
  - Loaded dice
    - $P(i) = 1/10, i = 1..5$
    - $P(i) = 1/2, i = 6$
- **Casino switches betw fair & loaded die with prob 1/2. Initially, dice is always fair**
- **Game:**
  - You bet \$1
  - You roll
  - Casino rolls
  - Highest number wins \$2
- **Question: Suppose we played 2 games, and the sequence of rolls was 1, 6, 2, 6. Were we likely to have been cheated?**

# “Visualization” of Dishonest Casino



## Emission Matrix

$E(1 Fair) = 1/6$	$E(1 Loaded) = 1/10$
$E(2 Fair) = 1/6$	$E(2 Loaded) = 1/10$
$E(3 Fair) = 1/6$	$E(3 Loaded) = 1/10$
$E(4 Fair) = 1/6$	$E(4 Loaded) = 1/10$
$E(5 Fair) = 1/6$	$E(5 Loaded) = 1/10$
$E(6 Fair) = 1/6$	$E(6 Loaded) = 1/2$

## Transition Matrix

$T(Loaded, Loaded) = 1/2$
$T(Loaded, Fair) = 1/2$
$T(Fair, Fair) = 1/2$
$T(Fair, Loaded) = 1/2$
$T(? , Fair) = 1.0$
$T(? , Loaded) = 0.0$

# 1, 6, 2, 6?

## We were probably cheated...

$$\begin{aligned}
 \text{Prob}(X, S = \text{Fair}, \text{Fair}, \text{Fair}, \text{Fair}) &= E(1|\text{Fair}) * T(?, \text{Fair}) * \\
 &E(6|\text{Fair}) * T(\text{Fair}, \text{Fair}) * \\
 &E(2|\text{Fair}) * T(\text{Fair}, \text{Fair}) * \\
 &E(6|\text{Fair}) * T(\text{Fair}, \text{Fair}) \\
 &= \frac{1}{6} * 1 * \frac{1}{6} * \frac{1}{2} * \frac{1}{6} * \frac{1}{2} * \frac{1}{6} * \frac{1}{2} \\
 &= 9.6451 * 10^{-5}
 \end{aligned}$$

$$\begin{aligned}
 \text{Prob}(X, S = \text{Fair}, \text{Loaded}, \text{Fair}, \text{Loaded}) &= E(1|\text{Fair}) * T(?, \text{Fair}) * \\
 &E(6|\text{Loaded}) * T(\text{Fair}, \text{Loaded}) * \\
 &E(2|\text{Fair}) * T(\text{Loaded}, \text{Fair}) * \\
 &E(6|\text{Loaded}) * T(\text{Fair}, \text{Loaded}) \\
 &= \frac{1}{6} * 1 * \frac{1}{2} * \frac{1}{2} * \frac{1}{6} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} \\
 &= 8.6806 * 10^{-4}
 \end{aligned}$$

## Example Use of HMM: Protein Families Modelling

- Baldi et al., *PNAS* 91:1059-1063, 1994
- HMM is used to model families of biological sequences, such as kinases, globins, & immunoglobulins
- Bateman et al., *NAR* 32:D138-D141, 2004
- HMM is used to model 6190 families of protein domains in Pfam

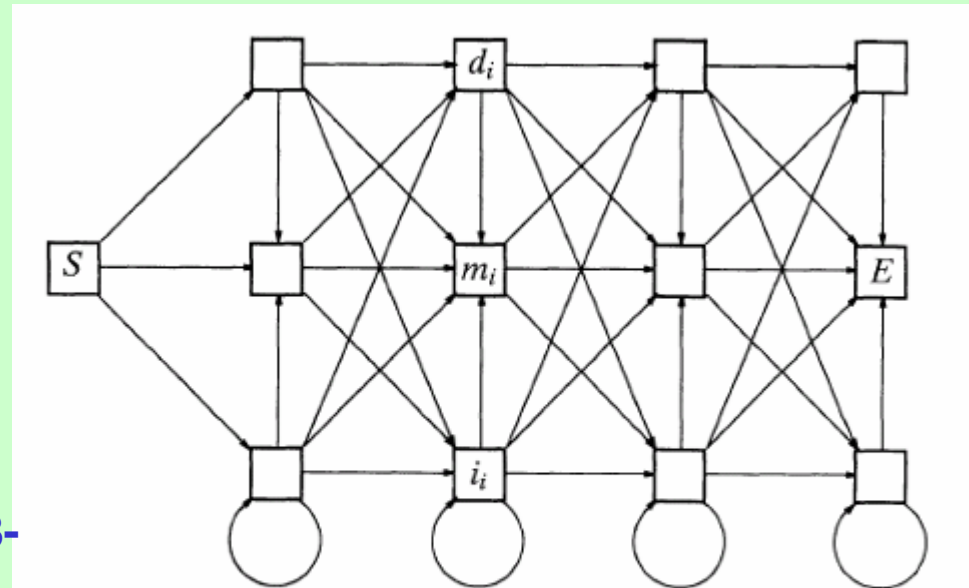


FIG. 1. HMM architecture.  $S$  and  $E$  are the start and end states. Sequence of main states  $m_i$  is the backbone. Side states  $d_i$  (resp.  $i_i$ ) correspond to deletions (resp. insertions).

# Concluding Remarks...



# What have we learned?

- **Decision Trees**
- **Decision Trees Ensembles**
  - Bagging
- **Other Methods**
  - K-Nearest Neighbour
  - Support Vector Machines
  - Bayesian Approach
  - Hidden Markov Models

Any Question?







- <http://www.cs.waikato.ac.nz/ml/weka>
- **Weka is a collection of machine learning algorithms for data mining tasks. The algorithms can either be applied directly to a dataset or called from your own Java code. Weka contains tools for data pre-processing, classification, regression, clustering, association rules, and visualization.**

Exercise: Download a copy of WEKA. What are the names of classifiers in WEKA that correspond to C4.5 and SVM?

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- **The “indep vs conditional indep” example came from Kwok Pui Choi**

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