Efficient and Effective Algorithms for A Family of Influence Maximization Problems with A Matroid Constraint

[Research track – Social Networks]

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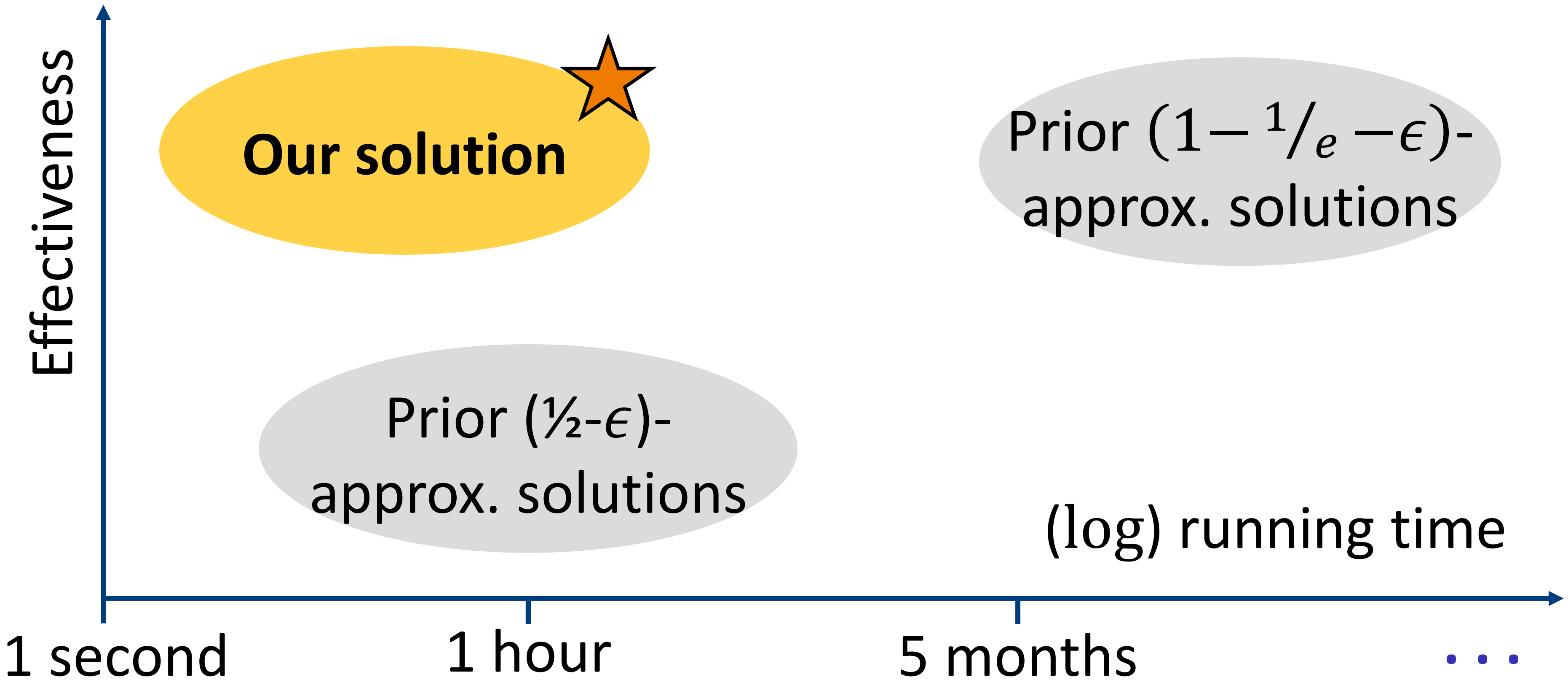
TL; DR

- We study **influence maximization** problems with a **matroid** constraint
- We give a **fast** solution with a $(\mathbf{1 - 1/e - \epsilon})$**-approximation**



Roadmap

- Influence Maximization (IM)
- Why matroid constraints
- Our idea & guarantees
- Experiments

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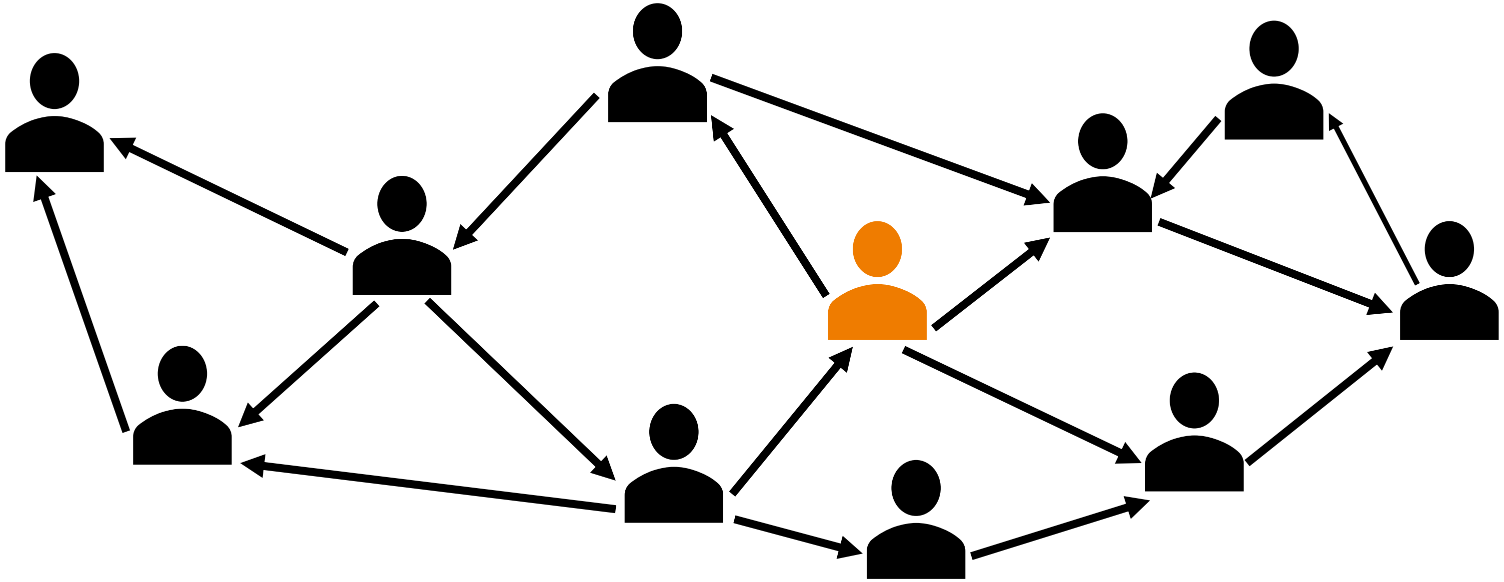
Influence Maximization (IM)

- In a social network, each user (say ) has *influence*



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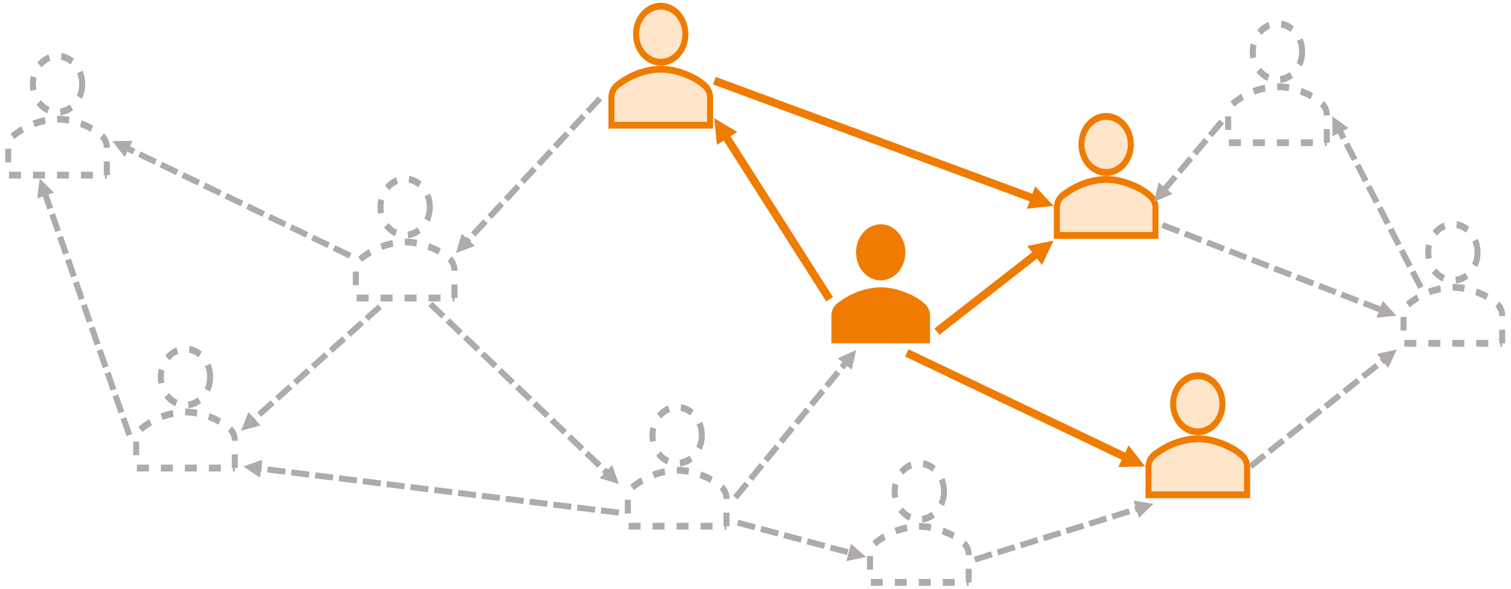
- Think of the influence as a (random) set of users they can reach (i.e., 👤)



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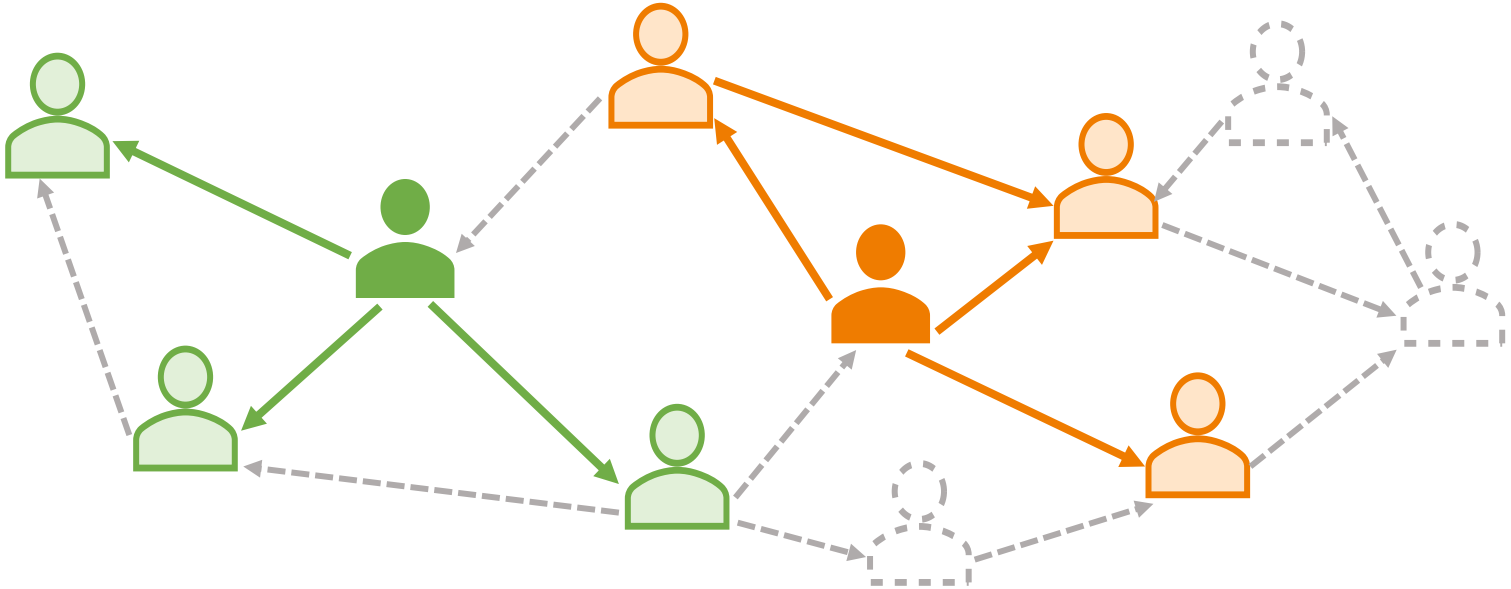
- IM aims to select k users to **maximize their joint influence**



Example ($k = 2$; result of selecting 👤 👤)

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- Applied in *viral marketing*, *recommendation*, *power grids management*, etc.
- **Example (viral marketing with ad promotion):**

= TikTok influencers who we want to endorse an ad

= Others who will watch the ad **(the more, the better for us!)**

k = our budget for launching the campaign

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- Real-world scenarios often need **multiple** such sets
- A **matroid** $(U, \mathbb{I})$ can model constraints across them

Why Matroid Constraints

- **Example 1 (Classical IM):**

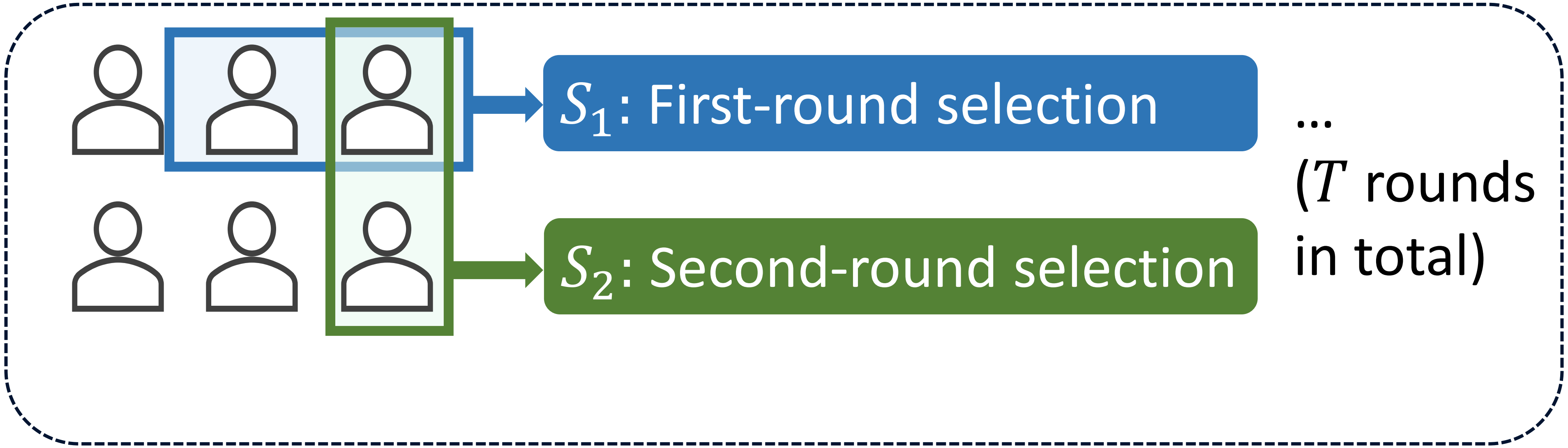
A *uniform matroid* $(U, \mathbb{I})$ specifies:

U: **all users**

$\mathbb{I}$: $|\boldsymbol{S}| \leq \boldsymbol{k}$ (at most k users)

Why Matroid Constraints

- **Example 2 (Multi-round):**

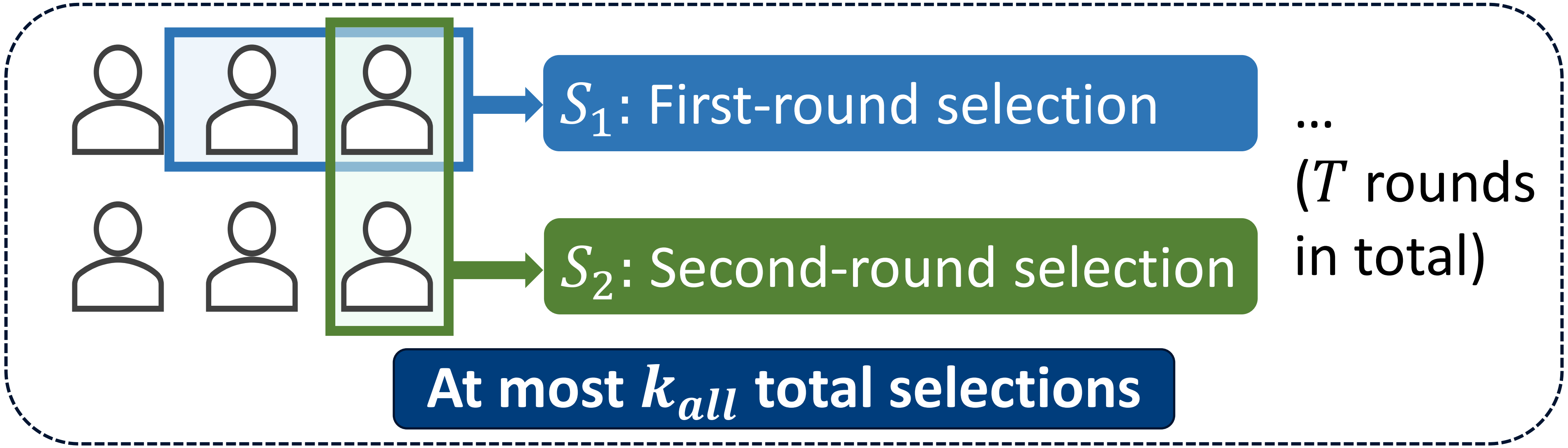


A *partition matroid* $(U, \mathbb{I})$ specifies:

U: **all users × [T rounds]** $\mathbb{I}$: $\forall \boldsymbol{i}, |\boldsymbol{S_i}| \leq \boldsymbol{k_i}$ (at most k_i users in round i)

Why Matroid Constraints

- **Example 3 (Multi-round + an overall budget):**



A *(general) matroid* $(U, \mathbb{I})$ specifies:

U: **all users** $\times$ **[T rounds]** $\mathbb{I}$: $\forall \boldsymbol{i}, |\boldsymbol{S_i}| \leq \boldsymbol{k_i}$ **and** $\sum_{i=1}^{T} |\boldsymbol{S_i}| \leq \boldsymbol{k_{all}}$

Why Matroid Constraints

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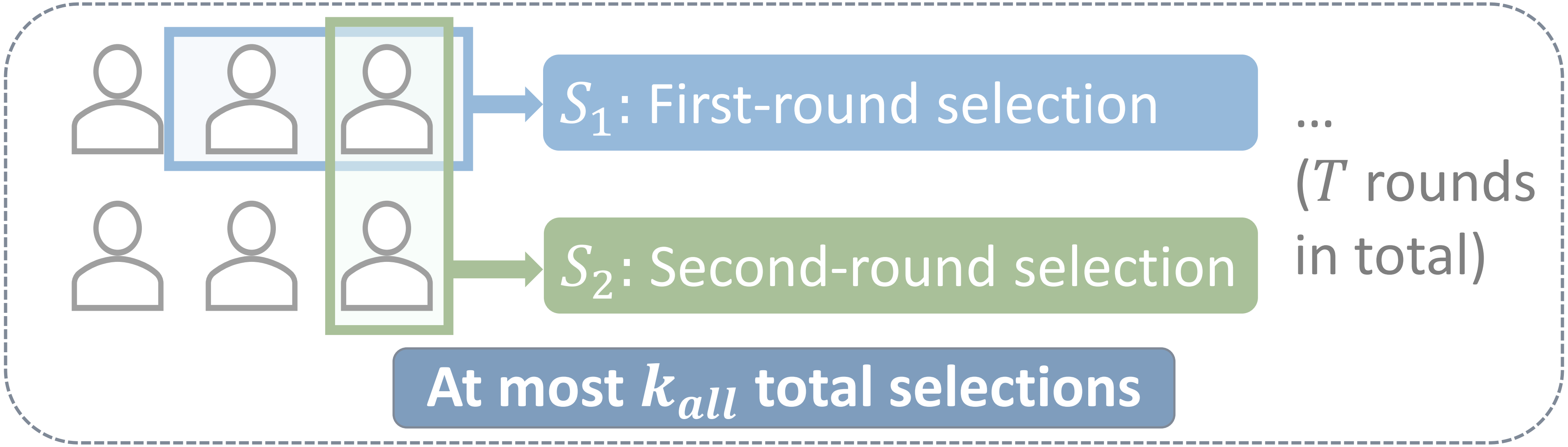

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- **Problem definition:**

Maximize $\sigma(S)$ *(expected influence)* s.t. $S \subseteq U, S \in \mathbb{I}$.

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TL; DR (Recap)

- We study influence maximization problems with a matroid constraint
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Let's get into some math.

Recap of Existing Results

(1) This problem is an instance of **monotone submodular maximization under a matroid constraint** *(thus NP-Hard)*

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(2) Existing $(1 - 1/e - \epsilon)$-approximation algorithms adopt hill-climbing on the following *multilinear extension* F of $\sigma(\cdot)$:

$$F([x_1 \ \ldots \ x_{|U|}]) = \sum_{S \subseteq U} \prod_{i \in S} x_i \prod_{j \notin S} (1 - x_j) \cdot \sigma(S).$$

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- Prior work (Calinescu et al. *SIAM J. Comput.'11*; Badanidiyuru et al. *SODA'14*) *estimates* $F([x_1 \ \dots \ x_{|U|}])$ by sampling, as it involves all $2^{|U|}$ possible S

Gruia Calinescu, Chandra Chekuri, Martin Pal, and Jan Vondrák. 2011. Maximizing a monotone submodular function subject to a matroid constraint. SIAM J. Comput. 40, 6 (2011), 1740–1766.
Ashwinkumar Badanidiyuru and Jan Vondrák. 2014. Fast algorithms for maximizing submodular functions. In SODA. 1497–1514.

Our Idea & Guarantees

(1) Adapt the previous approach (RR sets) to rewrite $\sigma(S)$ to set cover:

$$\sigma(S) \approx \mathrm{C} \cdot \sum_R \mathbf{1}\{S \cap R \neq \emptyset\}$$ *(R denotes RR set; C is constant)*

(2) Hill-climbing on $F(\cdot)$, with **fast & accurate assessment of its value**.

How?

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(2) Hill-climbing on $F(\cdot)$, with **<u>fast & accurate assessment of its value</u>.**

<u>How?</u> Plug the set cover into $F(\cdot)$:

$$F([x_1 \; \dots \; x_{|U|}]) \approx \mathrm{C} \cdot \sum_R \underbrace{\sum_{S \subseteq U} \prod_{i \in S} x_i \prod_{j \notin S} (1 - x_j)\, \mathbf{1}\{S \cap R \neq \emptyset\}}_{\spadesuit}.$$

Observation: $\spadesuit = (1 - \prod_{i \in R}(1 - x_i))$ **(a computable form)**.

Our Idea & Guarantees

- Example: Multi-Round IM (n users; T rounds in total, k users per round)
- Approximating $\sigma(\cdot)$ needs $L = \sum_R |\mathrm{R}|$ time in total (Tang et al. *SIGMOD'14*)

Algorithm	Time Complexity	Approximation
Calinescu et al. (*SIAM J. Comput.'11*)	$O(n^7 \cdot L)$	$1 - 1/e - \epsilon$
Badanidiyuru et al. (*SODA'14*)	$\mathrm{O}(nkT \cdot \epsilon^{-4} \cdot \mathrm{L})$	
Our algorithm **(AMP)**	$O(k \cdot \epsilon^{-1} \cdot L)$	

Tang Youze, Xiaokui Xiao, and Yanchen Shi. 2014. Influence maximization: Near-optimal time complexity meets practical efficiency. SIGMOD.

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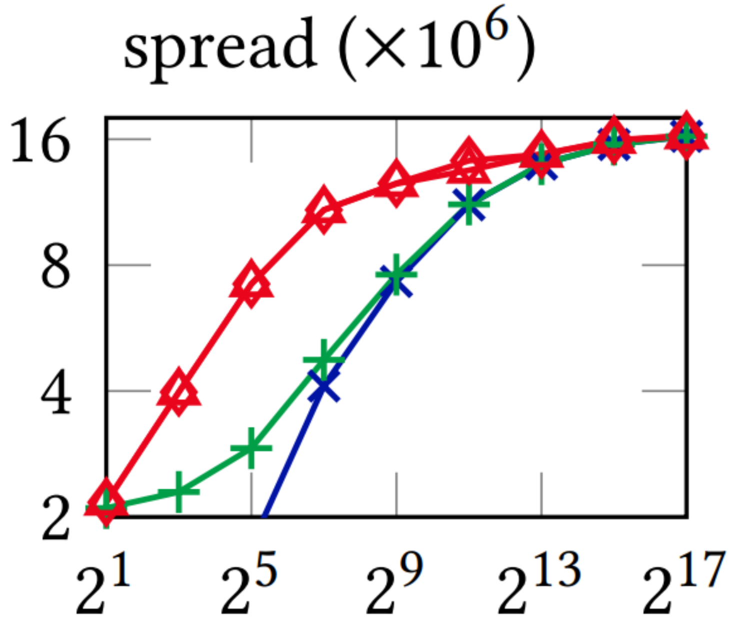
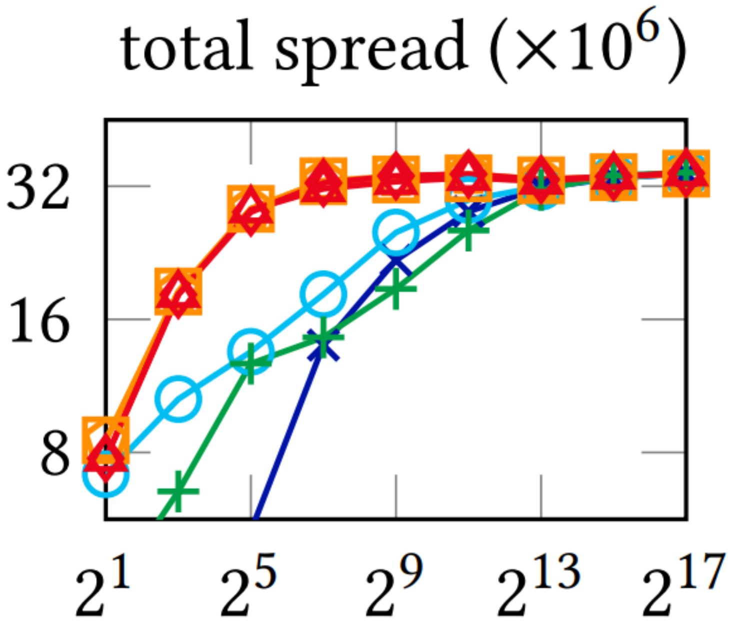
Experiments

- **Offline experiment:** 7 public datasets and 4 problem instances
- Our solution **AMP** (main algorithm *given RR sets;* **red△** & **orange□**):



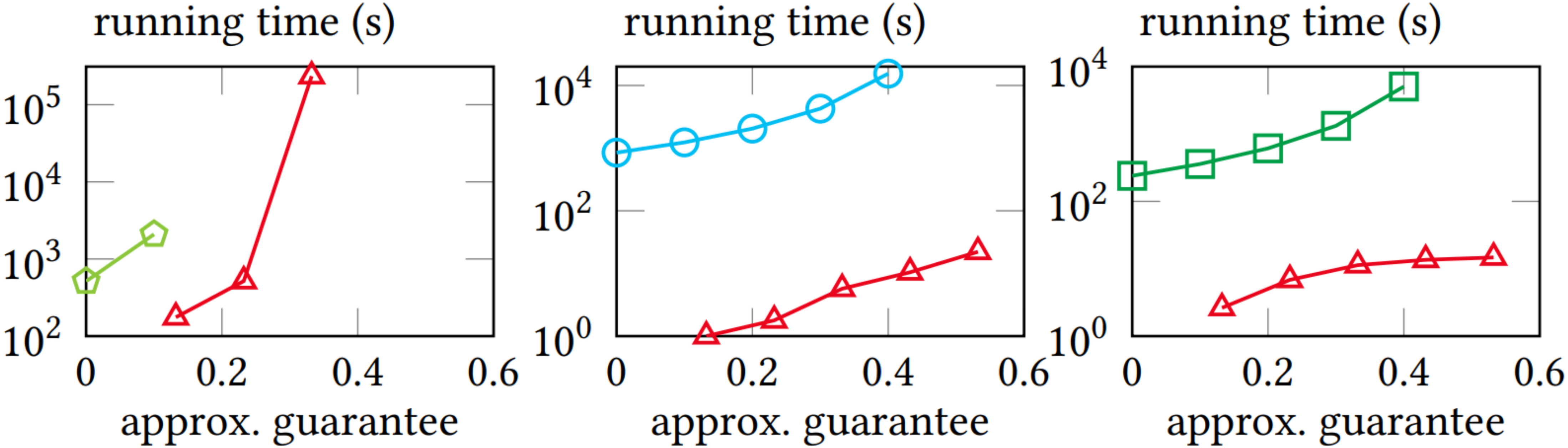
(a) IM



(b) Multi-round IM

Experiments

- **Offline experiment:** 7 public datasets and 4 problem instances
- Our solution **RAMP** (RR set generation + main algorithm; **red△**):



(a) RM (Twitter-large) **(b) MRIM (Wiki)** **(c) AdvIM (Wiki)**

(n=41.7M m=1.5B) (n=7.1K m=103.6K)

Experiments

- **Online deployment:** a Tencent game
- **Task:** recommend in-game products (maps) to each user
- **Scale:** 0.7 million users and 400 maps

Metric: Increase in engaged user–map pairs
(vs. control group, $\mathrm{K} = 10^3$)

Algorithm	RM-A	RAMP (ours)
Δ of pairs	20K	**160K**

Thanks!

Our paper

Definition Of Matroid

- A matroid is a pair $(U, \mathbb{I})$ where U is a finite set and $\mathbb{I}$ is a collection of subsets of U satisfying the following axioms:

1. $\mathbb{I} \neq \emptyset$
2. If $I \in \mathbb{I}$ and $J \subseteq I$, then $J \in \mathbb{I}$
3. If $I, J \in \mathbb{I}$ and $|I| < |J|$, then there exists $\mathrm{x} \in J \setminus I$ such that $I \cup \{\mathrm{x}\} \in \mathbb{I}$